This is a simple example economy to illustrate how government spending that seems costless – because it is not financed through higher taxes – really is costly.

Example Economy

- Infinite horizon
- Infinitely-lived rep agent
- Closed economy
- Fixed aggregate endowment 1 each period
- Resource constraint in each period

$$C_t + G_t = 1$$

- Preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

Suppose the government spends $G_0 > 0$ at $t = 0$, $G_t = 0$ for $t \geq 1$

Suppose taxes are zero in every period

Spending is financed by issuing real one period debt: sold at price Q_t, delivers one unit of consumption at $t + 1$

Initial debt is zero

Allocations are

$$\begin{aligned} C_0 &= 1 - G_0 \\ C_t &= 1 \text{ for } t \geq 1 \end{aligned}$$

FOC for bonds is

$$Q_t C_t^{-\gamma} = \beta C_{t+1}^{-\gamma}$$

which implies

$$\begin{aligned} Q_0 &= \beta(1 - G_0)^{\gamma} \\ Q_t &= \beta \text{ for } t \geq 1 \end{aligned}$$

Govt budget constraints are

$$\begin{aligned} G_0 &= Q_0 B_1 \\ B_t &= Q_t B_{t+1} \text{ for } t \geq 1 \end{aligned}$$

Suppose $G_0 = 0.1$ and $\beta = 1.05$ and $\gamma = 2$

We have $Q_0 = 1.05(1 - 0.1)^2 = 0.8505$ and thus $B_1 = \frac{0.1}{0.8505} = 0.11758$

After that we have

$$\frac{B_{t+1}}{B_t} = \frac{1}{Q_t} = \frac{1}{1.05} = 0.9524$$

So debt will steadily decline over time.

But what about the present value of debt? The date zero price of debt issued at t is $\Pi_{j=0}^{t} Q_t = Q_0 \frac{B_1}{B_2} \frac{B_2}{B_3} ... \frac{B_t}{B_{t+1}} = Q_0 \beta^t$

At the same time

$$\begin{aligned} B_{t+1} &= \frac{B_{t+1}}{B_t} \frac{B_t}{B_{t-1}} ... \frac{B_2}{B_1} B_1 \\ &= \frac{1}{\beta^t} B_1 \end{aligned}$$

So the date zero price of debt issued at t is

$$B_{t+1} \Pi_{j=0}^{t} Q_t = \frac{1}{\beta^t} B_1 Q_0 \beta^t = G_0$$

So (not surprisingly) if the govt never levies any taxes, the present value of debt remains equal to G_0. Debt declines over time, but it is an illusion that debt is being paid off: in present value terms it isn't.

What do we learn from this simple example?

1. Deficit financed spending can be perfectly feasible – debt eventually disappears without default, taxes, or inflation

2. It is not true that the representative agent is not hurt by the deficit spending just because he never pays any taxes – the representative agent ends up consuming less in the first period, so (assuming he does not value G) his lifetime utility is reduced

3. Could a single atomistic agent could decide to consume his entire endowment at every date and never trade bonds? The answer is no. If an atomistic agent doesn't buy bonds in the first period, he will have lower future wealth, in a situation in which the government has debts and will eventually impose taxes to pay them off. If the govt. never pays off its debts, it is effectively running a Ponzi scheme.

To see this more clearly, let's suppose the economy ends at some date T and at date T all outstanding debt is paid off via a lump-sum tax (T could be very large).

The real allocation $\{C_t, G_t\}_{t=0}^{T}$ will be exactly unchanged. So will the path for bond prices Q_t.

The budget constraint for the government at date T is

$$B_T = Tax_T$$

For the rep agent, we have

$$C_T = 1 + B_T - Tax_T = 1$$

But now suppose that one atomistic agent decides never to buy any debt. That agent enjoys

$$\tilde{C}_t = 1 \text{ for } t = 0, ..., T-1$$

but in period T they take a hit

$$\tilde{C}_T = 1 - Tax_T$$

That tax will be small if T is large, but consumption far in the future is extremely valuable with $\beta > 1$. Thus we can see why buying the debt at date 0 is optimal.

So my final takeaway from this simple example is that even if real interest rates are very low or negative, deficits today will eventually require increases in taxation, and those tax increases may be quite painful.

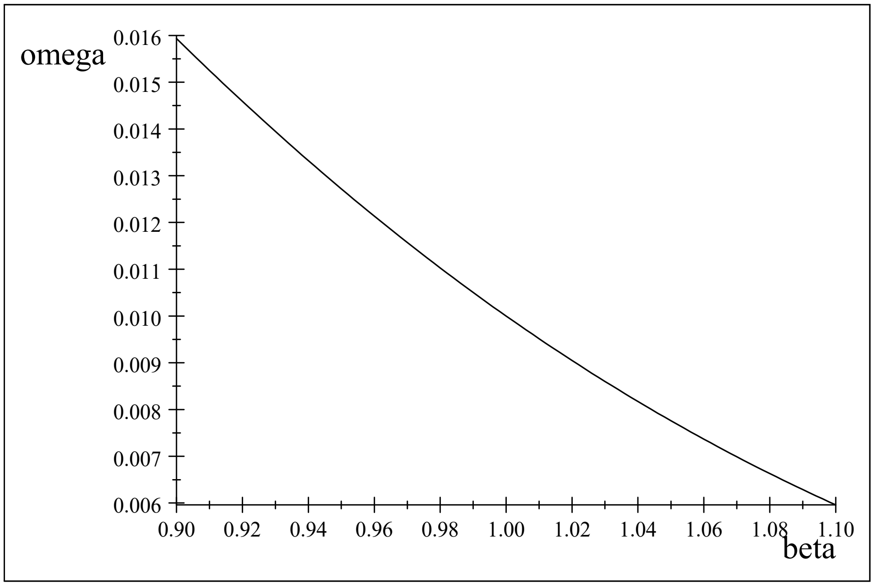
Of course we sort of knew this immediately (in an endowment economy more G means less C) but it was perhaps worth working it out.

One last question. Are deficit financed increases in G_0 more costly when β is high or when β is low?

The welfare cost of an increase in G_0 in this economy is the value for ω that satisfies

$$\begin{aligned}
\sum_{t=0}^{T} \beta^t \frac{(1-\omega)^{1-\gamma}}{1-\gamma} &= \frac{(1-G_0)^{1-\gamma}}{1-\gamma} + \sum_{t=1}^{T} \beta^t \frac{1}{1-\gamma} \\
(1-\omega)^{1-\gamma} \left(\frac{1-\beta^{T+1}}{1-\beta} \right) &= (1-G_0)^{1-\gamma} + \left(\frac{1-\beta^{T+1}}{1-\beta} \right) - 1 \\
\omega &= 1 - \left(\frac{(1-G_0)^{1-\gamma} + \left(\frac{1-\beta^{T+1}}{1-\beta} \right) - 1}{\left(\frac{1-\beta^{T+1}}{1-\beta} \right)} \right)^{\frac{1}{1-\gamma}}
\end{aligned}$$

Let's set $G_0 = 0.1$ and $T = 10$ and $\gamma = 2$



So the larger is β, the smaller is the welfare cost (measured this way) of an increase in G_0.

I think the intuition is just that when β is large, the agents care more about future relative to current consumption. So they don't mind so much that the government is reducing current consumption. But there is nothing magic that happens at $\beta = 1$: deficit spending is always costly, whether or not real interest rates are positive.

1 An OG economy

How does the analysis change in an OG economy?

Consider a two period OG setup (each period is say 30 years)

Suppose a tree drops one unit of fruit each period

Fraction $1-\theta$ is labor income for the young

Fraction θ accrues to the owner of the tree, which has a price P_t

Absent the government, budget constraints are

$$\begin{aligned} C_t^Y &= (1-\theta) - P_t S_{t+1} \\ C_t^o &= (P_t + \theta) S_t \end{aligned}$$

where S_{t+1} is the number of shares the young purchase (which must be one in equilibrium)

Household preferences are

$$\log C_t^Y + \beta \log C_{t+1}^o$$

Allocations must satisfy

$$\begin{aligned}
\frac{1}{C_t^Y} &= \beta R_{t+1} \frac{1}{C_{t+1}^o} \\
R_{t+1} &= \frac{\theta + P_{t+1}}{P_t}
\end{aligned}$$

$$C_t^Y + \frac{C_{t+1}^o}{R_{t+1}} = (1-\theta)$$

Thus, in steady state, absent government intervention, we have

$$C_t^Y (1+\beta) = (1-\theta)$$

which implies

$$\begin{aligned}
P_t &= (1-\theta) - \frac{(1-\theta)}{1+\beta} \\
&= \frac{\beta(1-\theta)}{1+\beta}
\end{aligned}$$

and

$$\begin{aligned}
R &= \frac{\theta(1+\beta)}{(1-\theta)\beta} + 1 \\
&= \frac{\theta + \beta}{(1-\theta)\beta}
\end{aligned}$$

Note that reducing β pushes up R. But note also that θ plays an important role: a larger θ (larger supply of assets) raises R.

Note that we cannot get negative real rates in this economy. Why?

Now introduce government debt and government spending

Debt and the tree must offer the same return (absent uncertainty)

Suppose the government spends G_0 at date 0 financed by issuing debt (a surprise)

The young are the only possible buyers of this debt

Their budget constraint is

$$C_0^Y = (1-\theta) - P_0 S_0 - Q_0 B_1$$

where we know that

$$G_0 = Q_0 B_1$$

We also know, that in this particular economy, the young optimally choose the same consumption irrespective of the interest rate (as long as they never face any taxes)

So we have, in equilibrium

$$C_0^Y = \frac{(1-\theta)}{(1+\beta)} = (1-\theta) - P_0 - G_0$$

which implies

$$P_0 = \frac{\beta(1-\theta)}{(1+\beta)} - G_0$$

so deficit public spending at date 0 reduces tree prices one-for-one. What is the intuition? Only the young buy assets, and their total demand for saving is fixed. So more asset supply (i.e., more debt issued) means lower asset prices.

We also know that

$$\begin{aligned} C_0^o &= \theta + P_0 \\ &= \theta + \frac{\beta(1-\theta)}{(1+\beta)} - G_0 \end{aligned}$$

which tells us that the old take the entire hit from extra government purchases at date 0.

Let's assume that at date 1 the debt is paid off with a lump-sum tax on the young

$$T_1 = B_1$$

For the young, we get the usual decision to consume a fraction of permanent income, which is now reduced by T_1 :

$$C_1^Y = \frac{1-\theta-T_1}{1+\beta}$$

From their budget constraint, consumption of the old is

$$\begin{aligned} C_1^o &= P_1 + \theta + B_1 \\ &= P_1 + \theta + T_1 \end{aligned}$$

From the resource constraint

$$C_1^o = 1 - C_1^Y$$

We also have the FOC for the young at date 0

$$\begin{aligned} \frac{1}{C_0^Y} &= \beta \frac{P_1+\theta}{P_0} \frac{1}{C_1^o} \\ &= \beta \frac{1}{Q_0} \frac{1}{C_1^o} \end{aligned}$$

where

$$Q_0 B_1 = Q_0 T_1 = G_0$$

Combining we know that

$$\begin{aligned} \frac{P_0}{P_1+\theta} B_1 &= G_0 \\ B_1 &= G_0 \frac{P_1+\theta}{P_0} \end{aligned}$$

Can we pin down P_1 from the FOC for the young at $t = 0$?

$$
\begin{aligned}
\frac{1}{C_0^Y} &= \beta \frac{P_1 + \theta}{P_0} \frac{1}{P_1 + \theta + G_0 \frac{P_1 + \theta}{P_0}} \\
\frac{1}{\frac{(1-\theta)}{(1+\beta)}} &= \beta R_1 \frac{1}{P_0 R_1 + G_0 R_1} \\
\frac{1}{\frac{(1-\theta)}{(1+\beta)}} &= \beta R_1 \frac{1}{P_0 R_1 + G_0 R_1} \\
\frac{(1+\beta)}{(1-\theta)} &= \beta \frac{1}{P_0 + G_0} = \beta \frac{1}{\frac{\beta(1-\theta)}{(1+\beta)}}
\end{aligned}
$$

so this equation is automatically satisfied for any R_1 (equivalently, for any P_1). The logic is that a higher R boosts returns and consumption C_1^o by the same amount

So what does pin down P_1?

At date 2 (and thereafter) there is no debt and no taxes. So from optimality for the young and the resource constraint, we have

$$
\begin{aligned}
C_2^Y &= \frac{1-\theta}{1+\beta} \\
C_2^o &= \frac{\beta + \theta}{1+\beta}
\end{aligned}
$$

And we also know that

$$
C_2^Y + P_2 = (1-\theta)
$$

which implies

$$
\begin{aligned}
P_2 &= (1-\theta) - \frac{1-\theta}{1+\beta} \\
&= \frac{\beta(1-\theta)}{1+\beta}
\end{aligned}
$$

which is the steady state price

So the price sequence P_1 must satisfy the FOC for the young at $t = 1$

$$
\begin{aligned}
\frac{1}{\frac{1-\theta-G_0 \frac{P_1+\theta}{P_0}}{1+\beta}} &= \beta \frac{P_2 + \theta}{P_1} \frac{1}{\frac{\beta+\theta}{1+\beta}} \\
\frac{1}{1 - \theta - G_0 \frac{P_1 + \theta}{\frac{\beta(1-\theta)}{(1+\beta)} - G_0}} &= \beta \frac{\frac{\beta(1-\theta)}{1+\beta} + \theta}{P_1} \frac{1}{\beta + \theta} \\
\frac{\frac{\beta(1-\theta)}{(1+\beta)} - G_0}{(1-\theta)\left(\frac{\beta(1-\theta)}{(1+\beta)} - G_0\right) - G_0(P_1 + \theta)} &= \beta \frac{\frac{\beta(1-\theta)}{1+\beta} + \theta}{P_1} \frac{1}{\beta + \theta}
\end{aligned}
$$

which can be solved for P_1

Consider an example. Suppose $\theta = 0.2$ $\beta = 0.6$ (which implies that in steady state $C^Y = C^o$)

In steady state, we have $R = \frac{\theta+\beta}{(1-\theta)\beta} = 1\frac{2}{3}$ (so $\beta R = \frac{5}{3}\frac{3}{5} = 1$) $R = \frac{5}{3}$

Suppose $G_0 = 0.1$
That implies
$P_0 = 0.2$
$P_1 = 0.221$
$P_2 = 0.3$
$P_3 = 0.3$
...
This implies

$$\begin{aligned} C_0^Y &= \frac{1-\theta}{1+\beta} = 0.5 \\ C_0^o &= 1 - C_0^o - G_0 = 0.4 \end{aligned}$$

$$\begin{aligned} C_1^Y &= \frac{1-\theta-G_0\frac{P_1+\theta}{P_0}}{1+\beta} = 0.368 \\ C_1^o &= P_1 + \theta + G_0\frac{P_1+\theta}{P_0} = 0.632 \end{aligned}$$

$$\begin{aligned} C_2^Y &= 0.5 \\ C_2^o &= 0.5 \end{aligned}$$

So this particular scheme:
(1) hurts the old at $t = 0$
(2) hurts the newborn at $t = 1$ (they pay more taxes)
(3) benefits the newborn at $t = 0$ (who consume more at $t = 1$)
Interest rates are high for two periods:
high at date 0 because the young have to buy the debt
high at date 1 because the young at $t = 1$ have to pay off the old debt

$$\begin{aligned} R_1 &= \frac{P_1+\theta}{P_0} = 2.105 \\ R_2 &= \frac{P_2+\theta}{P_1} = 2.262 \\ R_3 &= \frac{P_3+\theta}{P_2} = 1.667 \end{aligned}$$

Homework:

Work things out for the case in which the increase in spending is financed by a tax on the old at $t = 1$?