This is a simple example economy to illustrate how government spending that seems costless – because it is not financed through higher taxes – really is costly.

Example Economy

- Infinite horizon
- Infinitely-lived rep agent
- Closed economy
- Fixed aggregate endowment 1 each period
- Resource constraint in each period

$$C_t + G_t = 1$$

- Preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

Suppose the government spends $G_0 > 0$ at $t = 0$, $G_t = 0$ for $t \geq 1$

Suppose taxes are zero in every period

Spending is financed by issuing real one period debt: sold at price Q_t, delivers one unit of consumption at $t + 1$

Initial debt is zero

Allocations are

$$\begin{aligned} C_0 &= 1 - G_0 \\ C_t &= 1 \text{ for } t \geq 1 \end{aligned}$$

FOC for bonds is

$$Q_t C_t^{-\gamma} = \beta C_{t+1}^{-\gamma}$$

which implies

$$\begin{aligned} Q_0 &= \beta(1 - G_0)^\gamma \\ Q_t &= \beta \text{ for } t \geq 1 \end{aligned}$$

Govt budget constraints are

$$\begin{aligned} G_0 &= Q_0 B_1 \\ B_t &= Q_t B_{t+1} \text{ for } t \geq 1 \end{aligned}$$

Suppose $G_0 = 0.1$ and $\beta = 1.05$ and $\gamma = 2$

We have $Q_0 = 1.05(1 - 0.1)^2 = 0.8505$ and thus $B_1 = \frac{0.1}{0.8505} = 0.11758$

After that we have

$$\frac{B_{t+1}}{B_t} = \frac{1}{Q_t} = \frac{1}{1.05} = 0.9524$$

So debt will steadily decline over time.

But what about the present value of debt? The date zero price of debt issued at t is $\Pi_{j=0}^{t} Q_t = Q_0 \frac{B_1}{B_2} \frac{B_2}{B_3} \cdots \frac{B_t}{B_{t+1}} = Q_0 \beta^t$

At the same time

$$\begin{aligned} B_{t+1} &= \frac{B_{t+1}}{B_t} \frac{B_t}{B_{t-1}} \cdots \frac{B_2}{B_1} B_1 \\ &= \frac{1}{\beta^t} B_1 \end{aligned}$$

So the date zero price of debt issued at t is

$$B_{t+1} \Pi_{j=0}^{t} Q_t = \frac{1}{\beta^t} B_1 Q_0 \beta^t = G_0$$

So (not surprisingly) if the govt never levies any taxes, the present value of debt remains equal to G_0. Debt declines over time, but it is an illusion that debt is being paid off: in present value terms it isn't.

What do we learn from this simple example?

1. Deficit financed spending can be perfectly feasible – debt eventually disappears without default, taxes, or inflation

2. It is not true that the representative agent is not hurt by the deficit spending just because he never pays any taxes – the representative agent ends up consuming less in the first period, so (assuming he does not value G) his lifetime utility is reduced

3. Could a single atomistic agent could decide to consume his entire endowment at every date and never trade bonds? The answer is no. If an atomistic agent doesn't buy bonds in the first period, he will have lower future wealth, in a situation in which the government has debts and will eventually impose taxes to pay them off. If the govt. never pays off its debts, it is effectively running a Ponzi scheme.

To see this more clearly, let's suppose the economy ends at some date T and at date T all outstanding debt is paid off via a lump-sum tax (T could be very large).

The real allocation $\{C_t, G_t\}_{t=0}^{T}$ will be exactly unchanged. So will the path for bond prices Q_t.

The budget constraint for the government at date T is

$$B_T = Tax_T$$

For the rep agent, we have

$$C_T = 1 + B_T - Tax_T = 1$$

But now suppose that one atomistic agent decides never to buy any debt. That agent enjoys

$$\tilde{C}_t = 1 \text{ for } t = 0, ..., T-1$$

but in period T they take a hit

$$\tilde{C}_T = 1 - Tax_T$$

That tax will be small if T is large, but consumption far in the future is extremely valuable with $\beta > 1$. Thus we can see why buying the debt at date 0 is optimal.

So my final takeaway from this simple example is that even if real interest rates are very low or negative, deficits today will eventually require increases in taxation, and those tax increases may be quite painful.

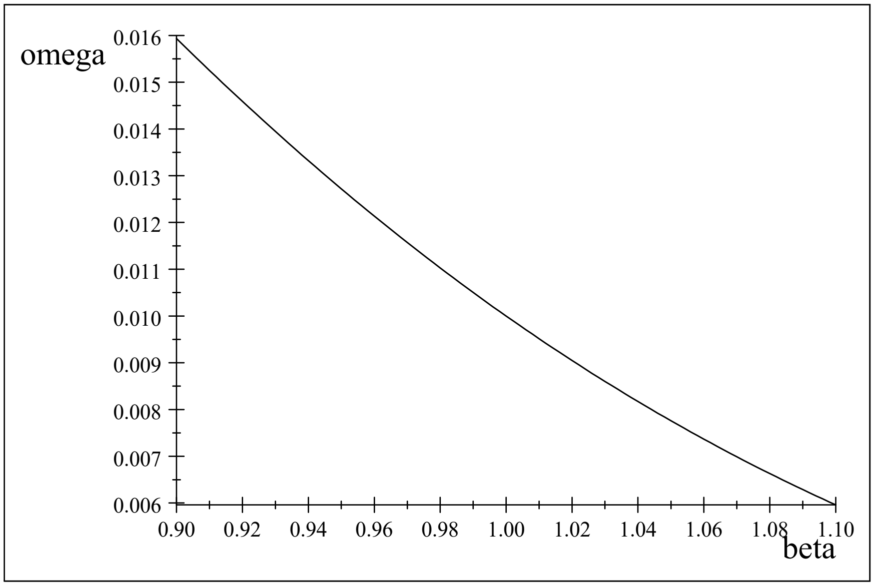
Of course we sort of knew this immediately (in an endowment economy more G means less C) but it was perhaps worth working it out.

One last question. Are deficit financed increases in G_0 more costly when β is high or when β is low?

The welfare cost of an increase in G_0 in this economy is the value for ω that satisfies

$$\begin{aligned}
\sum_{t=0}^{T} \beta^t \frac{(1-\omega)^{1-\gamma}}{1-\gamma} &= \frac{(1-G_0)^{1-\gamma}}{1-\gamma} + \sum_{t=1}^{T} \beta^t \frac{1}{1-\gamma} \\
(1-\omega)^{1-\gamma} \left(\frac{1-\beta^{T+1}}{1-\beta} \right) &= (1-G_0)^{1-\gamma} + \left(\frac{1-\beta^{T+1}}{1-\beta} \right) - 1 \\
\omega &= 1 - \left(\frac{(1-G_0)^{1-\gamma} + \left(\frac{1-\beta^{T+1}}{1-\beta} \right) - 1}{\left(\frac{1-\beta^{T+1}}{1-\beta} \right)} \right)^{\frac{1}{1-\gamma}}
\end{aligned}$$

Let's set $G_0 = 0.1$ and $T = 10$ and $\gamma = 2$



So the larger is β, the smaller is the welfare cost (measured this way) of an increase in G_0.

I think the intuition is just that when β is large, the agents care more about future relative to current consumption. So they don't mind so much that the government is reducing current consumption. But there is nothing magic that happens at $\beta = 1$: deficit spending is always costly, whether or not real interest rates are positive.