The Primal Approach to Optimal Taxation[1]

Substitute out prices and solve directly for optimal allocations

- Large number of identical infinitely lived consumers
- $t = 0, 1, ...$
- Consumption / capital good and labor
- Consumer preferences

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

- Resource constraint

$$c_t + g + k_{t+1} = F(k_t, l_t) + (1 - \delta)k_t$$

- Consumers budget constraint

$$\sum_{t=0}^{\infty} p_t((1 + \tau_t^c)c_t + k_{t+1}) = \sum_{t=0}^{\infty} p_t \left[(1 - \tau_t^l) w_t l_t + \left[1 + (1 - \tau_t^k)(r_t - \delta) \right] k_t \right]$$

  where

p_t is the (pre-tax) price of consumption in t ($p_0 = 1, p_1 = \left[1 + (1 - \tau_1^k)(r_1 - \delta) \right]^{-1}$,etc)
τ_t^k is the tax rate on capital income (τ_0^k given)
τ_t^l is the tax rate on labor income
τ_t^c is the tax rate on consumption
k_0 is the initial capital stock (given)

Lagrangian for households' problem:

$$\begin{aligned}
\mathcal{L} &= \max_{\{c_t\},\{l_t\},\{k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) + \\
&\lambda \left(\sum_{t=0}^{\infty} p_t \left[(1 - \tau_t^l) w_t l_t + \left[1 + (1 - \tau_t^k)(r_t - \delta) \right] k_t \right] - \sum_{t=0}^{\infty} p_t((1 + \tau_t^c)c_t + k_{t+1}) \right)
\end{aligned}$$

First order conditions:

[1] Jonathan Heathcote, January 17th 2012 (notes largely based on Atkeson, Chari, Kehoe: Minn. Fed. QR 1999)

- wrt c_t :
$$\beta^t U_{c_t} - \lambda(1+\tau_t^c)p_t = 0$$
- wrt k_{t+1} :
$$-\lambda p_t + \lambda p_{t+1}\left[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right] = 0$$
- wrt l_t :
$$\beta^t U_{l_t} + \lambda p_t(1-\tau_t^l)w_t = 0$$

Combining these gives

$$\begin{aligned} -U_{l_t} &= U_{c_t}\frac{(1-\tau_t^l)}{(1+\tau_t^c)}w_t \\ \frac{U_{c_t}}{(1+\tau_t^c)} &= \beta\frac{U_{c_{t+1}}}{(1+\tau_{t+1}^c)}\left[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right] \end{aligned}$$

These tax instruments are not all independent: without loss of generality we can set either $\tau_t^c = 0$ for all t, or $\tau_t^k = 0$ for all t. For example, suppose we take some feasible policy $\left\{\tau_t^c, \tau_t^l, \tau_t^k\right\}$ with $\tau_t^k > 0$ for at least some t. Now suppose we introduce a constitutional amendment which mandates $\widetilde{\tau}_t^k = 0$ for all t. We can come up with a new policy $\left\{\widetilde{\tau}_t^c, \widetilde{\tau}_t^l, \widetilde{\tau}_t^k = 0\right\}$ that replicates the old competitive equilibrium.

First, take the inter-temporal FOC for households. We need to set the sequence $\widetilde{\tau}_t^c$ to satisfy

$$\frac{(1+\widetilde{\tau}_{t+1}^c)}{(1+\widetilde{\tau}_t^c)\left[1+(r_{t+1}-\delta)\right]} = \frac{(1+\tau_{t+1}^c)}{(1+\tau_t^c)\left[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right]}$$

(note this does not restrict $\widetilde{\tau}_0^c$)

Given a sequence $\widetilde{\tau}_t^c$, we then set the sequence $\widetilde{\tau}_t^l$ to satisfy

$$\frac{(1-\widetilde{\tau}_t^l)}{(1+\widetilde{\tau}_t^c)} = \frac{(1-\tau_t^l)}{(1+\tau_t^c)}$$

Now we have ensured that our new tax scheme satisfies the agent's FOCs at the initial allocations. What about budget balance? We simply need to set $\widetilde{\tau}_0^c$ to satisfy

$$\sum_{t=0}^{\infty}\widetilde{p}_t((1+\widetilde{\tau}_t^c)c_t + k_{t+1}) = \sum_{t=0}^{\infty}\widetilde{p}_t\left((1-\widetilde{\tau}_t^l)w_t l_t + \left[1+(r_t-\delta)\right]k_t\right)$$

(if the household's budget constraint is satisfied, then so is the government's, by Walras Law)

This is important, because it means that when we talk about policies it will be more useful to talk about which margins (savings versus labor supply) are being distorted, rather than about the levels of particular taxes.

Note one more thing. Suppose that according to the original policy, $\tau_t^k = \tau^k > 0$ while $\tau_t^c = \tau^c$. Then to implement the same allocations with $\tilde{\tau}^k = 0$ will require

$$(1 + \tilde{\tau}_{t+1}^c) = \frac{1 + (r_{t+1} - \delta)}{1 + (1 - \tau^k)(r_{t+1} - \delta)}(1 + \tilde{\tau}_t^c)$$

This teaches us that positive capital tax rates are equivalent to consumption taxes that rise over time. This is a clue that capital taxes - more precisely inter-temporal distortions - will not be a good idea. They are not a good idea because if we think of consumption at different dates as simply consumption of different goods, these different goods essentially enter preferences symmetrically, so it will not make sense to tax them at different rates.

Given that one tax instrument is superfluous, from now on we will follow Atkeson, Chari and Kehoe, and set $\tau_t^c = 0$ for all t.

The households FOCs are (from above)

- wrt c_t :
$$\beta^t U_{c_t} - \lambda p_t = 0$$
- wrt k_{t+1} :
$$-\lambda p_t + \lambda p_{t+1}\left[1 + (1 - \tau_{t+1}^k)(r_{t+1} - \delta)\right] = 0$$
- wrt l_t :
$$\beta^t U_{l_t} + \lambda p_t(1 - \tau_t^l)w_t = 0$$

Firms solve

$$\max F(k_t, l_t) - w_t l_t - r_t k_t$$

First order conditions:

- wrt k_t :
$$F_{k_t} - r_t = 0$$

- wrt l_t :

$$F_{l_t} - w_t = 0$$

Government budget constraint:

$$\sum_{t=0}^{\infty} p_t g = \sum_{t=0}^{\infty} p_t \left[\tau_t^l w_t l_t + \tau_t^k (r_t - \delta) k_t\right]$$

Let $\pi = \{\pi_t\}_{t=0}^{\infty} = \left\{\tau_t^l, \tau_t^k\right\}_{t=0}^{\infty}$ denote a policy
Let $x = \{x_t\}_{t=0}^{\infty} = \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ describe an allocation
Let (w, r, p) describe a price system
Suppose the government can commit itself to any feasible sequence of policies

Ramsey equilibrium:

A policy π, an allocation rule $x(\pi)$, and price rules $w(\pi)$ and $r(\pi)$ such that π maximizes

$$\sum_{t=0}^{\infty} \beta^t U(c_t(\pi), l_t(\pi))$$

subject to

- the GBC is satisfied with allocations and prices given by $x(\pi)$, $w(\pi)$ and $r(\pi)$
- a competitive equilibrium exists for π, and
- the initial capital income tax rate τ_0^k is fixed

Conditions characterizing competitive equilibrium

- the resource constraint
- the implementability constraint:

$$\sum_{t=0}^{\infty} \beta^t \left(U_{c_t} c_t + U_{l_t} l_t\right) = U_{c_0} \left(1 + (1 - \tau_0^k)(F_{k_0} - \delta)\right) k_0$$

Where does the implementability constraint come from, and why must it be satisfied in competitive equilibrium?

Take the consumer's budget constraint, and substitute out prices:

$$\sum_{t=0}^{\infty}\left[p_t c_t - p_t(1-\tau_t^l)w_t l_t\right] = \sum_{t=0}^{\infty} p_t\left[-k_{t+1} + \left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t\right]$$

$$\sum_{t=0}^{\infty}\left[\frac{\beta^t U_{c_t}}{\lambda}c_t + \frac{\beta^t U_{l_t}}{\lambda}l_t\right] = \sum_{t=0}^{\infty}\left[-p_t k_{t+1} + p_{t-1}k_t\right]$$

$$\sum_{t=0}^{\infty}\left[\beta^t U_{c_t}c_t + \beta^t U_{l_t}l_t\right] = \lambda p_{-1}k_0$$

$$\sum_{t=0}^{\infty}\left[\beta^t U_{ct}c_t + \beta^t U_{lt}l_t\right] = \lambda p_0\frac{p_{-1}}{p_0}k_0$$

$$\sum_{t=0}^{\infty}\left[\beta^t U_{c_t}c_t + \beta^t U_{lt}l_t\right] = U_{c_0}\left[1+(1-\tau_0^k)(F_{k0}-\delta)\right]k_0$$

Why must an allocation satisfying the resource and implementability constraints be a competitive equilibrium allocation?

Just use the 5 household and firm first order conditions to define 5 prices and tax rates: p_t, r_t, w_t, τ_t^l, and τ_t^k.Then substitute these into the implementability constraint to get the household budget constraint. Thus all the competitive equilibrium conditions are satisfied.

Solving the Ramsey problem

$$\begin{aligned}
\mathcal{L} &= \max\sum_{t=0}^{\infty}\beta^t\left[U(c_t,l_t)+\lambda\left(U_{ct}c_t+U_{lt}l_t\right)\right] \\
&\quad+\sum_{t=0}^{\infty}\beta^t\mu_t\left[F(k_t,l_t)+(1-\delta)k_t-c_t-g-k_{t+1}\right] \\
&\quad-\lambda U_{c0}\left(1+(1-\tau_0^k)(F_{k0}-\delta)\right)k_0
\end{aligned}$$

Let

$$W(c_t,l_t) = U(c_t,l_t)+\lambda\left(U_{c_t}c_t+U_{lt}l_t\right)$$

First order conditions

- wrt c_t $(t\geq 1)$:

$$\beta^t W_{ct} - \beta^t\mu_t = 0$$

- wrt l_t $(t \geq 1)$:
$$\beta^t W_{lt} + \beta^t \mu_t F_{lt} = 0$$
Combining these two we get, for $t \geq 1$
$$\frac{-W_{lt}}{W_{ct}} = F_{lt}$$
- wrt c_0 :
$$W_{c0} - \mu_0 - \lambda U_{cc0}\left[1 + (1-\tau_0^k)(F_{k0} - \delta)\right] k_0 = 0$$
- wrt l_0 :
$$W_{l0} + \mu_0 F_{l0} - \lambda U_{cl0}\left[1 + (1-\tau_0^k)(F_{k0} - \delta)\right] k_0 - \lambda U_{c0}(1-\tau_0^k) F_{kl0} k_0 = 0$$

Combining these two we get, for $t = 0$

$$\begin{aligned} & W_{c0} - \lambda U_{cc0}\left[1 + (1-\tau_0^k)(F_{k0} - \delta)\right] k_0 \\ = \; & \frac{1}{F_{l0}}\left[\lambda U_{cl0}\left[1 + (1-\tau_0^k)(F_{k0} - \delta)\right] k_0 + \lambda U_{c0}(1-\tau_0^k) F_{kl0} k_0 - W_{l0}\right] \end{aligned}$$

- wrt k_{t+1} $(t \geq 0)$
$$-\beta^t \mu_t + \beta^{t+1} \mu_{t+1}\left(F_{kt+1} + (1-\delta)\right) = 0$$
or for $t \geq 1$
$$-W_{ct} + \beta W_{ct+1}\left(F_{kt+1} + (1-\delta)\right) = 0$$
and for $t = 0$
$$-W_{c0} + \lambda U_{cc0}\left[1 + (1-\tau_0^k)(F_{k_0} - \delta)\right] k_0 + \beta W_{c1}\left(F_{k1} + (1-\delta)\right) = 0$$

Characterizing optimal taxes

Optimal capital taxes. Note that if
$$\frac{W_{ct}}{U_{ct}} = \frac{W_{ct+1}}{U_{ct+1}}$$

then for $t \geq 1$

$$-U_{ct} + \beta U_{ct+1} \left(F_{kt+1} + (1-\delta) \right) = 0$$

and thus the optimal τ^k_{t+1} (τ^{k*}_{t+1}) is zero.

When will it be the case that

$$\frac{W_{ct}}{U_{ct}} = \frac{W_{ct+1}}{U_{ct+1}}$$

- In steady state
- If the utility function has various particular forms

Consider, for example,

$$U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + V(l_t)$$

Now

$$\begin{aligned} W(c_t, l_t) &= U(c_t, l_t) + \lambda \left(U_{ct} c_t + U_{lt} l_t \right) \\ &= \frac{c_t^{1-\sigma}}{1-\sigma} + V(l_t) + \lambda \left[c_t^{1-\sigma} + V'(l_t) l_t \right] \end{aligned}$$

$$W_{ct} = \left(1 + \lambda \left(1-\sigma\right)\right) c_t^{-\sigma}$$

Thus it is clear that for this utility function that

$$\frac{W_{ct}}{U_{ct}} = \frac{W_{ct+1}}{U_{ct+1}}$$

and therefore that $\tau^{k*}_{t+1} = 0$ from $t = 1$ and on (which means $\tau^{k*}_2, \tau^{k*}_3, ... = 0$)

What are optimal capital taxes in $t = 1$ (assuming the same functional form)? The sign of τ^{k*}_1 may be deduced by substituting the expression for W_{c0} into the FOC for k_1 from the Ramsey problem:

$$-U_{c0} + \frac{\lambda U_{cc0} \left(1 + (1-\theta_0)(F_{k_0} - \delta)\right) k_0}{(1 + \lambda(1-\sigma))} + \beta U_{c1} \left(F_{k1} + (1-\delta) \right) = 0$$

The second term is negative. Thus

$$U_{c0} < \beta U_{c1} \left(1 + F_{k1} - \delta \right)$$

But we know that

$$U_{c0} = \beta U_{c1}\left(1 + \left(1 - \tau_1^{k*}\right)(F_{k1} - \delta)\right)$$

Thus $\tau_1^{k*} > 0$.

What are optimal labor taxes? Suppose

$$\begin{aligned}
U(c,l) &= \frac{c^{1-\sigma}}{1-\sigma} - \varphi\frac{h^{1+\chi}}{1+\chi} \\
U_c c &= c^{1-\sigma} \\
U_l l &= -\varphi h^{1+\chi} \\
W(c,l) &= c^{1-\sigma}\left(\frac{1}{1-\sigma} + \lambda\right) - \varphi h^{1+\chi}\left(\frac{1}{1+\chi} + \lambda\right) \\
W_c &= (1-\sigma)\left(\frac{1}{1-\sigma} + \lambda\right) c^{-\sigma} \\
W_l &= -\varphi(1+\chi)\left(\frac{1}{1+\chi} + \lambda\right) h^{\chi}
\end{aligned}$$

Recall the static first order condition for $t \geq 1$ is

$$\frac{-W_{lt}}{W_{ct}} = \frac{\varphi(1+\chi)\left(\frac{1}{1+\chi} + \lambda\right) h_t^{\chi}}{(1-\sigma)\left(\frac{1}{1-\sigma} + \lambda\right) c_t^{-\sigma}} = F_{lt}$$

The agent's FOC is

$$c_t^{-\sigma}(1 - \tau_t^l) = \varphi h_t^{\chi}$$

Thus the implied optimal tax rate is

$$(1 - \tau_t^{l*}) = \frac{(1-\sigma)\left(\frac{1}{1-\sigma} + \lambda\right)}{(1+\chi)\left(\frac{1}{1+\chi} + \lambda\right)} = \frac{1 + (1-\sigma)\lambda}{1 + (1+\chi)\lambda}$$

Since σ, χ and λ are all positive, the optimal tax rate on labor is constant and positive in this example.