Optimal Income Taxation: Mirrlees Meets Ramsey

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
Introduction

- Roles of government:
  - Provide public goods
  - Redistribute to the poor

- Labor income taxation is main source of revenue

- Important question: How should taxes vary with income?

- Want to maximize redistribution, minimize distortions
Disagreement in the Literature

- Proposals for a flat tax system with transfers
  - Friedman (1962)
  - Mirrlees (1971)
  - Flavor of Universal Basic Income

- Others have argued for U-shaped marginal tax schedule
  - Diamond (1998)
  - Saez (2001)
  - Flavor of means-tested transfers

- Or should marginal tax rates be increasing with income, as in the U.S.?
Approaches to Studying Optimal Taxation

- **MIRRLEESIAN approach**
  - No restrictions on shape of tax / transfer schedule
  - Compute best possible tax scheme subject to constraint that taxes must be a function of market earnings $y$
  - $y = w \times h$ but planner cannot see productivity $w$ or hours $h$

- **RAMSEY approach**
  - Impose a parametric form for tax function
  - Less flexible, but easier to embed into richer models
This Paper

We compare 3 tax and transfer systems:

1. **Mirsleesian tax system**: Main focus
   - fully non-linear

2. **Ramsey 1**: Affine system: \( T(y) = \tau_0 + \tau_1 y \)
   - constant marginal rates with lump-sum transfers

3. **Ramsey 2**: HSV system: \( T(y) = y - \lambda y^{1-\tau} \)
   - increasing marginal rates without transfers
   - \( \tau \) indexes progressivity: \( 1 - \tau = \frac{1-T'(y)}{1-T(y)/y} \)
Novel Elements

1. Key innovation: **partial private insurance** ⇒ reduced role for redistribution through tax system

2. Explore how **fiscal pressure** to raise revenue shapes optimal tax system ⇒ reconcile disparate results in the literature

3. Emphasize role of planner’s **taste for redistribution** in shaping optimal tax schedule and welfare gains

4. Characterize **Pareto-improving** tax reforms
Main Findings

- Marginal tax rates in the United States should be increasing in income, NOT flat or U-shaped

- Increasing fiscal pressure $\rightarrow$ flatter, then U-shaped optimal tax schedule

- Current tax system close to optimal, given modest taste for redistribution

- Pareto-improving tax reforms imply no changes in taxes for most households
Environment

- Static model, labor supply only margin distorted by taxes
- Heterogeneous individual labor productivity with two stochastic components

\[ \log w = \alpha + \varepsilon \]

- \( \varepsilon \) is privately-insurable, \( \alpha \) is not

Interpretation:

- Individuals belong to large families
- \( \varepsilon \) idiosyncratic risks that can be pooled within family
- \( \alpha \) common across all family members \( \Rightarrow \) no private insurance
- Planner sees neither component of productivity
Environment 2

- Common preferences

\[ u(c, h) = \log(c) - \frac{h^{1+\sigma}}{1 + \sigma} \]

- Production linear in aggregate effective hours

\[ \int \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_\alpha dF_\varepsilon = \int \int c(\alpha, \varepsilon) dF_\alpha dF_\varepsilon + G \]
Planner’s Problem

- Seeks to maximize SWF denoted $W(\alpha)$

- Only sees total family income
  \[ y(\alpha) = \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_\varepsilon \]

- First Stage
  - Planner offers menu of contracts \( \{c(\tilde{\alpha}), y(\tilde{\alpha})\} \)
  - Family heads draw idiosyncratic $\alpha$ and report $\tilde{\alpha}$

- Second Stage
  - Family members draw idiosyncratic $\varepsilon$
  - Family head tells each member how much to work
  - Total earnings must deliver $y(\tilde{\alpha})$ to the planner
  - Must divide consumption $c(\tilde{\alpha})$ between family members
Nature of the Solution

- Planner cannot condition individual allocations on $\varepsilon$, given free within-family transfers
  - equally cheap for any family member to deliver income to the planner, and equally valuable to receive consumption

- Thus, planner cannot take over private insurance
  $\Rightarrow$ Distinct roles for public and private insurance

- Note: Extent of private risk-sharing is exogenous with respect the tax system
Planner's Problem

$$\max_{c(\alpha), y(\alpha)} \int W(\alpha) U(\alpha, \alpha) dF_\alpha$$

s.t. $$\int y(\alpha) dF_\alpha \geq \int c(\alpha) dF_\alpha + G$$

$$U(\alpha, \alpha) \geq U(\alpha, \tilde{\alpha}) \quad \forall \alpha, \forall \tilde{\alpha}$$

where $$U(\alpha, \tilde{\alpha}) \equiv$$

$$\left\{ \begin{array}{l}
\max \left\{ \frac{\log(c(\alpha, \tilde{\alpha}, \varepsilon)) - h(\alpha, \tilde{\alpha}, \varepsilon)^{1+\sigma}}{1+\sigma} \right\} dF_\varepsilon \\
\int c(\alpha, \tilde{\alpha}, \varepsilon) dF_\varepsilon = c(\tilde{\alpha}) \\
\int \exp(\alpha + \varepsilon) h(\alpha, \tilde{\alpha}, \varepsilon) dF_\varepsilon = y(\tilde{\alpha})
\end{array} \right.$$
Planner’s Problem: Ramsey

\[
\max_{\tau} \int W(\alpha) \left\{ \int u(c(\alpha, \varepsilon), h(\alpha, \varepsilon)) dF_\varepsilon \right\} dF_\alpha \\
\text{s.t.} \quad \int \int c(\alpha, \varepsilon) dF_\alpha dF_\varepsilon + G = \int \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_\alpha dF_\varepsilon
\]

where \( c(\alpha, \varepsilon) \) and \( h(\alpha, \varepsilon) \) are the solutions to

\[
\left\{ \begin{array}{l}
\max\{c(\alpha, \varepsilon), h(\alpha, \varepsilon)\} \\
\text{s.t.}
\end{array} \right. \int \left\{ \log c(\alpha, \varepsilon) - \frac{h(\alpha, \varepsilon)^{1+\sigma}}{1+\sigma} \right\} dF_\varepsilon \\
\int c(\alpha, \varepsilon) dF_\varepsilon = y(\alpha) - T(y(\alpha); \tau) \\
y(\alpha) = \int \exp(\alpha + \varepsilon) h(\alpha, \varepsilon) dF_\varepsilon
\]
Decomposing Welfare Effects of Tax Reform

- Contemplate raising $T'(y)$ at $y = \hat{y}$, using revenue to raise transfers

- "Distributional gain:" assuming no behavioral response

$$D(\hat{y}) = [1 - F(\hat{y})] - \frac{\int_{\hat{y}}^{\infty} u_c(y) dF(y)}{\int_{0}^{\infty} u_c(y) dF(y)}$$

  - Positive, assuming planner values redistributing downwards

- "Efficiency cost:" amount of hypothetical revenue from reform that leaks away due to behavioral responses

$$E(\hat{y}) = [1 - F(\hat{y})] - \Delta Tr(\hat{y})$$

  - Depends on substitution and income effects
  - Increases in $T'(y)$

- When tax system is optimal distributional gain equals efficiency cost at every $y$
Estimated on PSID data for 2000-2006
Households with head / spouse hours $\geq 260$ per year
Estimated value for $\tau = 0.181$, $R^2 = 0.96$
Calibration: Wage Distribution

1. $F_\alpha$: Exponentially Modified Gaussian $EMG(\mu_\alpha, \sigma^2_\alpha, \lambda_\alpha)$

2. $F_\epsilon$: Normal $N(\frac{-\sigma^2_\epsilon}{2}, \sigma^2_\epsilon)$

3. $\log(wh)$ and $\log(c)$ also $EMG$, given our utility function, private insurance model, and HSV tax system

4. Estimate earnings distribution parameters using cross-sectional income data from SCF

5. Decompose $\sigma^2_\alpha$ versus $\sigma^2_\epsilon$ to match cross-sectional dispersion in consumption

$$
\text{Var}(\log y) = \left(\frac{1 + \sigma}{\sigma}\right)^2 \sigma^2_\epsilon + \sigma^2_\alpha + \frac{1}{\lambda^2_\alpha}, \quad (1)
$$

$$
\text{Var}(\log c) = (1 - \tau)^2 \sigma^2_\alpha + \frac{(1 - \tau)^2}{\lambda^2_\alpha}. \quad (2)
$$
Distribution for Labor Income

Maximum likelihood \( \Rightarrow \lambda_\alpha = 2.2 \) and \( \sigma_y^2 = 0.4117 \)
Calibration

- Frisch elasticity $= 0.5 \Rightarrow \sigma = 2$
- Progressivity parameter $\tau = 0.181$ (HSV 2016)
- Govt spending $G$ s.t. $G/Y = 0.188$ (US, 2005)
- Extent of insurance: $\sigma_\alpha^2 = 0.142$ and $\sigma_\varepsilon^2 = 0.120$ to hit $\text{var}(\log c) = 0.234$
- Utilitarian social welfare function: $W(\alpha) = 1 \ \forall \alpha$
- Numerical implementation: 10,000 grid points for $\alpha$
Bottom of Wage Distribution

- Difficult to measure distribution of offered wages at the bottom, given selection into participation

- Low and Pistaferri (2015) estimate distribution of latent offered wages within a structural model in which workers face disability risk and choose participation

<table>
<thead>
<tr>
<th>Percentile Ratios</th>
<th>Model</th>
<th>LP</th>
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</thead>
<tbody>
<tr>
<td>P5/P1</td>
<td>1.46</td>
<td>1.48</td>
</tr>
<tr>
<td>P10/P5</td>
<td>1.23</td>
<td>1.20</td>
</tr>
<tr>
<td>P25/P10</td>
<td>1.42</td>
<td>1.40</td>
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</tbody>
</table>
Optimal Tax Policy

A: Marginal Tax Rate

B: Average Tax Rate

C: Distributional Gain $D(\alpha)$

D: $\alpha D(\alpha) \times \frac{1 - F_\alpha(\alpha)}{f_\alpha(\alpha)}$

E: Log Consumption

F: Hours Worked
Main Takeaways

- Optimal marginal rate monotone increasing in income
- Optimum much more redistributive than current system
- Efficiency / distributional gains rise with income
  - At 3 times average productivity, 71% of each hypothetical tax dollar leaks away
  - At 1/3 average productivity, only 3% does

- Why are things this way?
- Why did Diamond and Saez argue for a U-shaped marginal rate schedule?
Fiscal Pressure

- Always want high taxes at the top
  - Maximize tax collection (well understood)

- Optimal marginal rate at low income levels depends on fiscal pressure

- Fiscal pressure low in baseline calibration
  - Private insurance + public transfers from taxing the rich
  - ⇒ Modest consumption inequality at the bottom
  - ⇒ No desire for high taxes on the moderately poor to benefit the very poor
Increasing Fiscal Pressure

A: High Government Expenditures

B: $D_g/D_{\text{baseline}}$

C: No Insurable Shocks

D: $D/D_{\text{baseline}}$
Interpretation

- \( \uparrow G \) reduces revenue for transfers \( \Rightarrow \) \( \uparrow \) inequality at the bottom \( \Rightarrow \) \( \uparrow \) distributional gains \( \Rightarrow \) high marginal tax rates

- \( \downarrow \) private insurance \( \Rightarrow \) \( \uparrow \) consumption inequality at the bottom \( \Rightarrow \) higher marginal tax rates

- In both scenarios, planner faces more revenue pressure

- Low marginal rates in middle where productivity density largest

- Note Saez (2001) economy features high fiscal pressure:
  - total govt. spending at his optimum: 56% of GDP
  - c.f. 42% in our economy, 33.4% in the U.S.
Diamond’s (1998) U-shaped Example

- Assume

\[ u(c, h) = \log \left( c - \frac{h^{1+\sigma}}{1 + \sigma} \right) \]

- Simple expression for efficiency cost:

\[ E(\alpha) = \frac{1}{1 + \sigma} \frac{T'(\alpha)}{1 - T'(\alpha)} f_\alpha(\alpha) \]

- Let \( \alpha_m \) denote value for \( \alpha \) at which \( f_\alpha(\alpha) \) is maximized

- At optimum \( D(\alpha) = E(\alpha) \) so

\[ \frac{T'(\alpha)}{1 - T'(\alpha)} = (1 + \sigma) \frac{D(\alpha)}{f_\alpha(\alpha)} \]
Declining Marginal Rates?

\[ \frac{T'(\alpha)}{1 - T'(\alpha)} = (1 + \sigma) \frac{D(\alpha)}{f_\alpha(\alpha)} \]

\[ D(\alpha) = \int_\alpha^{\infty} \left\{ 1 - \frac{u_c(s)}{E[u_c(\alpha)]} \right\} dF_\alpha(s) \]

- \( u_c(\alpha) \) decreasing ⇒ \( \exists \alpha^* \) s.t. \( u_c(\alpha^*) = E[u_c(\alpha)] \), \( D(\alpha) \) maxed
- Does \( T'(\alpha) \) have a downward sloping portion?
  - Yes, if \( \alpha^* < \alpha_m \)
  - Diamond (1998): “This seems like the more interesting case, assuming the government would like to redistribute toward a fraction of the labor force well below one half”

- But in our calibration, \( \alpha^* > \alpha_m \)!
  - ⇒ govt. wants to redistribute to middle class as well as poor
  - ⇒ no downward-sloping portion for \( T'(\alpha) \)
Increasing $\rightarrow$ U-Shaped Rates

- Consider increase in $G$ paid for by reducing lump-sum transfers

- **Proposition**: This reform (i) has no effect on $E(\alpha)$, (ii) increases $D(\alpha)$ for all $\alpha$, and (iii) reduces $\alpha^*$

- Thus, higher fiscal pressure $\Rightarrow$ higher marginal rates, especially at low income levels

- Intuition: planner gives up on redistribution to middle class in order to focus on poorest

- Result: U-shaped system with flavor of means-tested transfers
Fiscal Pressure with GHH Preferences

A: Total Distributional Gain $\tilde{D}(\alpha)$

B: Marginal Tax Rate
Alternative Social Preferences

- Welfare gain for utilitarian planner moving to optimum sizable 2.1% of consumption

- But optimum is much more redistributive than current system \(\Rightarrow\) reform creates winners and losers

- Do welfare gains mostly reflect greater efficiency, or desire for less inequality?

- Are Pareto-improving reforms possible?
Empirically Motivated SWF

- Progressivity built into current tax system informative about taste for redistribution

- Assume planner (political system) choosing tax system in HSV class: 
\[ T(y) = y - \lambda y^{1-\tau} \]

- Assume planner has SWF in class 
\[ W(\alpha; \theta) = \exp(-\theta \alpha) \]

- What value for \( \theta \) gives observed \( \tau \) as solution to Ramsey problem?

- Empirically Motivated SWF 
\[ W(\alpha; \theta^*) \text{ s.t. } \tau^*(\theta^*) = \tau^{US} \]
Alternative SWFs

• Closed form expression for $\theta^*$!

$$\sigma^2_{\alpha} \theta^* - \frac{1}{\lambda_{\alpha} \theta^*} = -\frac{1}{\lambda_{\alpha} - 1 + \tau} - \sigma^2_{\alpha} (1 - \tau) + \frac{1}{1 + \sigma} \left\{ \frac{1}{(1 - g)(1 - \tau)} - 1 \right\}$$

• Simple in Normal case ($\lambda_{\alpha} \to \infty$)

$$\theta^* = -(1 - \tau) + \frac{1}{\sigma^2_{\alpha}} \frac{1}{1 + \sigma} \left\{ \frac{1}{(1 - g)(1 - \tau)} - 1 \right\}$$

• $\theta^*$ increasing in $\tau$ and $g$
• $\theta^*$ declining in $\sigma$ and $\sigma^2_{\alpha}$
• $\theta^*$ increasing in $\lambda_{\alpha}$ (holding fixed $\text{var}(\alpha) = \sigma^2_{\alpha} + \frac{1}{\lambda^2_{\alpha}}$)

• Also consider $\theta \to \infty$: Rawlsian

• And $\theta = -1$: Laissez-Faire Planner
Optimal Policy with Alternative Pareto Weight Functions

**A: Marginal Tax Rate**

- $\theta = -1$
- $\theta_{US} = -0.517$
- $\theta = 0$
- $\theta = 1$
- $\theta = \infty$

**B: $D_\theta/D_{baseline}$**

- $\alpha$ range from -2 to 2
- $D_\theta/D_{baseline}$ range from 0 to 10
Remarks

- Optimal marginal rates still increase in income given empirically-motivated SWF
- Optimal policy is sensitive to SWF (no surprise)
- With a strong desire to redistribute get downward-sloping / U-shaped marginal rates
  - Rawlsian case is the extreme – goal is simply to max revenue that can be used for transfers
- Welfare gains of tax reform vary enormously:
  - 662% of consumption for Rawlsian planner
  - 0.05% for planner with empirically-motivated SWF
Pareto-Improving Tax Reform

- Let’s insist tax reform cannot make anyone worse off

- Partially sidesteps question of what social welfare function to use

- Revisit original problem, with new constraints to ensure that each $\alpha$ type is weakly better off
Pareto Improving Tax Reform

A: Marginal Tax Rate

B: Welfare Gain (CEV, %)

C: Log Consumption

D: Hours Worked

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M: SVUS

Pareto-improving
Takeaways

• Pareto-improving constraints bind in the middle of the distribution

• Those in the tails gain
  • Lower marginal rates at the top
  • Larger transfers at the bottom

• Surprising theorem we stumbled across numerically:

• Allocations and marginal / average tax rates unchanged where Pareto-improving constraints bind
Mirrlees versus Ramsey

- Some advantages to simple parameter tax / transfer schedules

- But which form is best? Is it more important to have lump-sum transfers or increasing marginal rates?

- Compare Mirrlees to best-in-class affine and HSV schedules

- Focus here on utilitarian social welfare function
Mirrlees vs. Ramsey

A: Marginal Tax Rate

B: Average Tax Rate

C: Log Consumption

D: Hours Worked
### Comparison Summary

<table>
<thead>
<tr>
<th>Tax System</th>
<th>Tax Parameters</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \omega ) (%)</td>
<td>( \Delta Y ) (%)</td>
</tr>
<tr>
<td>HSV\textsuperscript{US}</td>
<td>( \lambda : 0.840 ) ( \tau : 0.181 )</td>
<td>–</td>
</tr>
<tr>
<td>HSV</td>
<td>( \lambda : 0.817 ) ( \tau : 0.331 )</td>
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<tr>
<td>Affine</td>
<td>( \tau_0 : -0.259 ) ( \tau_1 : 0.492 )</td>
<td>1.36</td>
</tr>
<tr>
<td>Mirrlees</td>
<td></td>
<td>2.07</td>
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</tbody>
</table>
Conclusions

• Optimal tax schedule likely features increasing marginal rates

• HSV approximation to current U.S. system might be close to optimal

• Optimal tax system less redistributive when there is private insurance

• Optimal system highly sensitive to desire for redistribution

• Pareto-improving tax systems likely similar to current one