

# Notes on Mirrlees Taxation (rough)

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Static problem:

Agents differ by productivity  $\theta$

$I$  values for productivity  $\theta_1, \dots, \theta_I$

Fraction  $\pi_i$  of each type

Preferences

$$U_i = u(c_i) - v\left(\frac{y_i}{\theta_i}\right)$$

Planner must raise revenue to finance  $G$

Planner puts weight  $W_i$  on type  $i$  s.t.  $\sum_i W_i \pi_i = 1$

An allocation is a vector  $\{(c_i, y_i)\}_{i=1}^I$

Social welfare is given by

$$\sum_i W_i \pi_i \left\{ u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \right\}$$

Planner can observe  $y$ , but not  $\theta$

So taxes must be a function of  $y$

Planner's problem is therefore to choose a tax function  $T(y)$  such that when agents take this schedule as given and solve

$$\begin{aligned} \max_{\{c_i, y_i\}} & \left\{ u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \right\} \\ \text{s.t.} & c_i = y_i - T(y_i) \end{aligned}$$

the resulting allocations maximize social welfare.

Suppose  $v(x) = (1 + \sigma)^{-1} x^{1+\sigma}$ . Model could be interpreted as model in which all agents have same wage, but different disutilities of work. Thus, interpretations in which workers have different labor productivity but identical preferences versus interpretation in which they are equally productive but have different disutility of work are formally identical. The idea is that in terms of individual choices it doesn't matter whether it is costly for a worker to deliver output because she must work a lot of hours or because she just hates working. (Though a planner might feel differently about these two people in terms of the planner's Pareto weights)

Now the problem is that the optimal  $T$  could be a very complicated non-parametric function. How are we supposed to solve for it?

Mirrlees' clever idea.

Instead of thinking of planner picking  $T$  think of planner picking allocations directly.

In particular think of planner as offering a menu of different choices  $\{(c_i, y_i)\}$  with one pair in this menu intended for each type. The planner can say:

"If you produce income  $y_i$  (which I can observe) then you must pay a tax  $y_i - c_i$ ."

But the planner cannot force agents to choose the pair intended for their type, because type is not observed

Thus the planner must incentivize choosing the appropriate allocation by making sure that each type weakly prefers to pick their intended allocation

Thus the Mirrlees problem is

$$\begin{aligned} & \max_{\{c_i, y_i\}} \sum_i W_i \pi_i \left\{ u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \right\} \\ & \text{s.t.} \\ & u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \geq u(c_j) - v\left(\frac{y_j}{\theta_j}\right) \text{ for all } i, j \\ & \sum_i \pi_i c_i + G = \sum_i \pi_i y_i \end{aligned}$$

There are lots of incentive constraints!

Fortunately most of them will not be binding

Suppose planner wants to redistribute downwards => lower taxes on less productive agents => possible incentive to pretend to be less productive, but no incentive to pretend to be more productive => only downward IC constraints will bind

In fact, only local downward constraints will bind.

So we can simplify the problem to

$$\begin{aligned} & \max_{\{c_i, y_i\}} \sum_i W_i \pi_i \left\{ u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \right\} \\ \pi_i \mu_i & : u(c_i) - v\left(\frac{y_i}{\theta_i}\right) \geq u(c_{i-1}) - v\left(\frac{y_{i-1}}{\theta_{i-1}}\right) \text{ for } i = 2, \dots, I \\ \lambda & : \sum_i \pi_i c_i + G \leq \sum_i \pi_i y_i \end{aligned}$$

$2I + I - 1 + 1$  unknowns

$2I$  FOCs +  $I$  constraints

This problem will have a solution. How can we decentralize it? We need to come up with a tax system such that taxes depend on earnings, and all agents are on their FOC

$$u'(c_i) \theta_i (1 - T'(y_i)) = v'\left(\frac{y_i}{\theta_i}\right)$$

and

$$c_i = y_i - T(y_i)$$

Note that marginal and average tax rates are only exactly pinned down at grid points.

FOCs

$$\begin{aligned} c_i &: W_i \pi_i u'(c_i) + \pi_i \mu_i u'(c_i) - \pi_{i+1} \mu_{i+1} u'(c_i) - \lambda \pi_i = 0 \\ y_i &: -W_i \pi_i v' \left( \frac{y_i}{\theta_i} \right) \frac{1}{\theta_i} - \pi_i \mu_i v' \left( \frac{y_i}{\theta_i} \right) \frac{1}{\theta_i} + \pi_{i+1} \mu_{i+1} v' \left( \frac{y}{\theta_{i+1}} \right) \frac{1}{\theta_{i+1}} + \lambda \pi_i = 0 \end{aligned}$$

Note that for  $i = I$  the  $i + 1$  terms are absent, so can combine the two to give

$$\begin{aligned} W_i \pi_i u'(c_i) + \pi_i \mu_i u'(c_i) &= W_i \pi_i v' \left( \frac{y_i}{\theta_i} \right) \frac{1}{\theta_i} + \pi_i \mu_i v' \left( \frac{y_i}{\theta_i} \right) \frac{1}{\theta_i} \\ (W_i \pi_i + \pi_i \mu_i) \theta_i u'(c_i) &= (W_i \pi_i + \pi_i \mu_i) v' \left( \frac{y_i}{\theta_i} \right) \end{aligned}$$

So no distortion at the top.

More generically, suppose  $v(x) = (1 + \sigma)^{-1} x^{1+\sigma}$ , so  $v'(x) = x^\sigma$

$$v' \left( \frac{y_i}{\theta_{i+1}} \right) = v' \left( \frac{y_i}{\theta_i} \frac{\theta_i}{\theta_{i+1}} \right) = v' \left( \frac{y_i}{\theta_i} \right) \left( \frac{\theta_i}{\theta_{i+1}} \right)^\sigma$$

$$\begin{aligned} (W_i \pi_i + \pi_i \mu_i - \pi_{i+1} \mu_{i+1}) u'(c_i) - \lambda \pi_i &= 0 \\ \left( -W_i \pi_i - \pi_i \mu_i + \pi_{i+1} \mu_{i+1} \left( \frac{\theta_i}{\theta_{i+1}} \right)^{1+\sigma} \right) \frac{1}{\theta_i} v' \left( \frac{y_i}{\theta_i} \right) + \lambda \pi_i &= 0 \end{aligned}$$

So we have a “wedge” in the intra-temporal FOC given by

$$wedge = (1 - tax) = \frac{(W_i \pi_i + \pi_i \mu_i - \pi_{i+1} \mu_{i+1})}{\left( W_i \pi_i + \pi_i \mu_i - \pi_{i+1} \mu_{i+1} \left( \frac{\theta_i}{\theta_{i+1}} \right)^{1+\sigma} \right)}$$

Note that wedge is less than one, so tax is positive => labor supply is distorted => can't achieve first best

What about at the bottom. There is no IC for  $i = 1$ , equivalently  $\mu_1 = 0$ .

How do we solve this numerically?

Guess  $\lambda$

Guess  $c_1$

Solve for  $\mu_2$  for FOC for  $c_1$

Solve for  $y_1$  for FOC for  $y_1$

3 equations (2 FOCs and IC<sub>2</sub>) to solve for  $c_2$ ,  $y_2$  and  $\mu_3$   
Iterate upwards through the grid  
At  $I - 1$  we solve for  $\mu_I$   
Then we have 2 FOCs at  $I$  to solve for  $c_I$  and  $y_I$   
Check IC <sub>$I$</sub>  and adjust  $c_1$  if not satisfied  
Finally check resource constraint and adjust  $\lambda$