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The international diversification puzzle is not as bad as you think[1]

Jonathan Heathcote

Federal Reserve Bank of Minneapolis and CEPR
heathcote@minneapolisfed.org

Fabrizio Perri

University of Minnesota, Federal Reserve Bank of Minneapolis, NBER and CEPR
fperri@umn.edu

Abstract

In simple one-good international macro models, the presence of non-diversifiable labor income risk means that country portfolios should be heavily biased toward foreign assets. The fact that the opposite pattern of diversification is observed empirically constitutes the international diversification puzzle. This paper embeds a portfolio choice decision in a two-country, two-good version of the stochastic growth model. In this environment, which is a workhorse for international business cycle research, equilibrium country portfolios can be characterized in closed form. Portfolios are biased towards domestic assets, as in the data. Home bias arises because openness to trade, through endogenous international relative price fluctuations, reduces co-movement between relative income from labor and relative income from capital, making domestic assets a good hedge against non-diversifiable labor income risk. Evidence from developed economies in recent years is qualitatively and quantitatively consistent with the mechanisms highlighted by the theory.

KEYWORDS: Country portfolios, International business cycles, Home bias

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1 Introduction

Although there has been rapid growth in international portfolio diversification in recent years, portfolios in many countries remain heavily biased towards domestic assets. For example, foreign assets accounted, on average, for only around 25% of the total value of the assets owned by U.S. residents over the period 1990-2004. There is a large theoretical literature that explores whether observed low diversification should be interpreted as evidence of incomplete insurance against country-specific risk (see, for example, Baxter and Jermann, 1997, and Lewis, 1999). These papers share a common conclusion: frictionless models, especially those with non diversifiable labor income risk, predict way too much diversification relative to the levels observed in the data. In response, recent theoretical work on diversification has focused on introducing frictions that can rationalize observed portfolios. The set of candidate frictions is long and includes proportional or fixed costs on foreign equity holdings (Lewis, 1996; Amadi and Bergin, 2006; Coeurdacier and Guibaud, 2006), costs in goods trade (Uppal, 1993; Obstfeld and Rogoff, 2000; Coeurdacier, 2006), liquidity or short sales constraints (Michaelides, 2003; DeMarzo, Kaniel and Kremer, 2004; Julliard, 2004), price stickiness in product markets (Engel and Matsumoto, 2009), weak investor rights concentrating ownership among insiders (Kho et. al., 2006), non-tradability of nontraded-good equities (Stockman and Dellas, 1989; Tesar, 1993; Pesenti and van Wincoop, 2002; Hnatkovska, 2005) and asymmetric information in financial markets (Gehrig, 1993; Jeske, 2001; Hatchondo, 2005; and van Nieuwerburgh and Veldkamp, 2009).

In this paper, we consider the two-country, two-good extension of the stochastic growth model developed by Backus, Kehoe and Kydland (1994 and 1995, henceforth BKK), which is a workhorse model for quantitative international macroeconomics. This model is frictionless, but allows for preferences to differ across countries to capture observed patterns of trade. While BKK allow for a complete set of Arrow securities to be traded between countries, we instead follow the tradition in the international diversification literature and assume that households only trade shares in domestic and foreign firms. BKK and others have shown that the international stochastic growth model is broadly consistent with a large set of international business cycle facts. We show that the same model is wholly consistent with observed levels of international diversification.

Our *theoretical contribution* is to characterize and explain portfolio choice in the BKK model. We first show that in this economy the extent of home bias in equilibrium portfolios can be expressed in terms of the covariance between relative domestic-foreign labor income, and relative asset income (dividends). This portfolio expression relates directly to the large literature that takes the perspective of an individual investor trying to smooth consumption, and notes that the optimal mix between domestic and foreign stocks depends on the covariance between returns to non-diversifiable human capital and (relative) returns to traded equity (see, for example, Baxter and Jermann, 1997). One minor difference is that our expression relates to income to labor and capital, rather than returns. This makes the task of measuring the key covariance in the data particularly simple, since one does not need a model for unobserved returns to human capital. Our more important innovation is that we embed the portfolio choice decision within a general equilibrium production economy, and this puts a lot of useful structure on the covariance pattern: wages, hours, dividends, and international relative price movements are all jointly determined in general equilibrium. We exploit this structure to derive an alternative expression for equilibrium portfolios in terms of two structural parameters: capital's share in the production, and the share of foreign goods used for consumption-investment. This structural expression indicates that countries that are more open to trade should exhibit a greater degree of international equity diversification. We develop intuition for this result by tracing out how country-specific productivity shocks drive, at the same time, investment and movements in international relative prices that complement explicit insurance via direct holdings of foreign assets.

Our *empirical contribution* is to argue that our theoretical framework can help to quantitatively explain the observed patterns of diversification within OECD countries in recent years. First, we examine the key prediction from our structural expression for equilibrium home bias, which is that countries that are more open to trade should be more diversified. We find strong confirmation for this prediction. Second, we move to examine the mechanism underlying this relationship by exploiting the dual expressions for home bias: one involving the observable covariance between relative labor income and relative capital income, and the second involving trade shares. In particular, our theory predicts that countries that are relatively closed (and thus ought to be home-biased according to the structural home bias expression) should also exhibit a particularly strong negative covariance between relative earnings and relative dividends. This strong negative covariance should then rationalize strong home bias as the optimal portfolio choice for an atomistic investor who takes wages, dividends and relative prices as given. We test this mechanism in two steps. First, we explore whether greater openness to trade is associated with a more negative empirical covariance between relative earnings and relative dividends. Second, we explore whether a stronger negative covariance is associated with stronger empirical home bias. In both cases we find the evidence is consistent with the theory. This set of tests is important because it suggests that the positive relationship between trade and diversification we document is not coincidental, but instead reflects the fact that the volume of trade has systematic implications for the joint dynamics of earnings and dividends that are at the heart of optimal portfolio choice.

To better understand the predictions of our model for portfolio choice we compare and contrast our economy to those considered by Lucas (1982), Baxter and Jermann (1997), and Cole and Obstfeld (1991). Lucas (1982) studies a two-country world in which residents of each country share common preferences and are endowed with a tree yielding stochastic fruits. He shows that perfect risk pooling, in general, involves agents of each country owning half the claims to the home endowment and half the claims to the foreign endowment. Baxter and Jermann (1997) extend Lucas' model in one direction by introducing non diversifiable labor income. They show that if asset returns and labor income are highly correlated within a country, then agents can hedge non-diversifiable labor income risk with a large short position in domestic assets i.e. aggressive diversification.

Cole and Obstfeld (1991) instead argue that in a special case of the Lucas model, diversification is not required to achieve risk-sharing. Their insight is that if the fruits yielded by the two trees are imperfect substitutes, then changes in relative endowments induce off-setting changes in the terms of trade. When preferences are log-separable between the two goods, the terms of trade responds one-for-one to changes in relative income, effectively delivering perfect risk-sharing. Thus, in sharp contrast to the results of Lucas or Baxter and Jermann, any level of diversification is consistent with complete risk-pooling, including portfolio autarky.[2]

One important difference in our analysis relative to Baxter and Jermann (1997) is that we allow for imperfect substitutability between domestic and foreign-produced traded goods. Thus, in our model, changes in international relative prices provide some insurance against country-specific shocks and, in the flavor of the Cole and Obstfeld indeterminacy result, portfolio choice does not have to do all the heavy-lifting when it comes to delivering perfect risk-sharing. This mechanism is consistent with a large body of empirical evidence which studies the response of international relative prices to productivity shocks.[3] In contrast to Cole and Obstfeld, however, the presence of

[2]Kollmann (2006) considers a two-good endowment economy with more general preferences. He finds that equilibrium diversification is sensitive to both the intra-temporal elasticity of substitution between traded goods, and the inter-temporal elasticity of substitution for the aggregate consumption bundle.

[3]See, for example, Acemoglu and Ventura (2002), Debaere and Lee (2004), and Pavlova and Rigobon (2007). These papers use different methodologies to identify productivity shocks and find support for this mechanism in a cross section of countries. For the United States the evidence is more mixed: Corsetti, Dedola and Leduc (2006) find no evidence of this mechanism, while Basu, Fernald and Kimball (2006) find that in response to US productivity growth, the US real exchange rate depreciates strongly.

production and particularly investment in our model means that returns to domestic and foreign stocks are not automatically equated, and thus agents face an interesting portfolio choice problem. Home bias arises because relative returns to domestic stocks move inversely with relative labor income in response to productivity shocks. The mechanism through which this covariation arises is novel and is due jointly to international relative price movements and to the presence of capital.

Although portfolios can be characterized analytically for one set of parameter values, for generic parameterizations this is not possible. One contribution of this paper is to adapt existing numerical methods (second order approximations of equilibrium conditions) so that they can be used to characterize equilibria across the entire parameter space. This allows us to consider the implications for diversification of varying two key parameters: the elasticity of substitution between domestic and foreign-produced goods, and the inter-temporal elasticity of substitution for the composite consumption good. We show that home bias is a robust prediction of the model for a large range of plausible values for these parameters.

In the next section we describe the basic model and derive equilibrium portfolios while section 3 offers some intuition for those portfolios. Section 4 contains the empirical analysis and Section 5 discusses some extensions of the basic model. Section 6 concludes. Proofs, details about numerical methods, and a description of the data are in the appendix.

2 The model

The modeling framework is the one developed by Backus, Kehoe and Kydland (1994,1995). There are two countries, each of which is populated by the same measure of identical, infinitely-lived households. Firms in each country use country-specific capital and labor to produce an intermediate good. The intermediate good produced in the domestic country is labeled a, while the good produced in the foreign country is labeled b. These are the only traded goods in the world economy. Intermediate-goods-producing firms are subject to country-specific productivity shocks. Within each country the intermediate goods a and b are combined to produce country-specific final consumption and investment goods. The final goods production technologies are asymmetric across countries, in that they are biased towards using a larger fraction of the locally-produced intermediate good. This bias allows the model to replicate empirical measures for the volume of trade relative to GDP.

We assume that the assets that are traded internationally are shares in the domestic and foreign representative intermediate-goods-producing firms. These firms make investment and employment decisions, and distribute any non-reinvested earnings to shareholders.

2.1 Preferences and technologies

In each period t the economy experiences one event $s_t \in S$. We denote by $s^t = (s_0, s_1, ..., s_t) \in S^t$ the history of events from date 0 to date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t)$.

Period utility for a household in the domestic country after history s^t is given by[4]

$$(1) \quad U\left(c(s^t), n(s^t)\right) = \log c(s^t) - V\left(n(s^t)\right)$$

where $c(s^t)$ denotes consumption at date t given history s^t, and $n(s^t)$ denotes labor supply. Disutility from labor is given by the positive, increasing and convex function $V(.)$. The assumption that utility is log-separable in consumption will play a role in deriving a closed-form expression for equilibrium portfolios in our baseline calibration of the model. In contrast, the equilibrium portfolio in this case will not depend on the particular functional form for $V(.)$.

Households supply labor to domestically-located perfectly-competitive intermediate-goods-producing firms. Intermediate goods firms in the domestic country produce good a, while those in the foreign country produce good b. These firms hold the capital in the economy and operate a Cobb-Douglas production technology:

$$(2) \quad F\left(z(s^t), k(s^{t-1}), n(s^t)\right) = e^{z(s^t)} k(s^{t-1})^{\theta} n(s^t)^{1-\theta},$$

where $z(s^t)$ is an exogenous productivity shock. The vector of shocks $[z(s^t), z^*(s^t)]$ evolves stochastically. For now, the only assumption we make about this process is that it is symmetric. In the baseline version of the model, productivity shocks are the only source of uncertainty.

Each period, households receive dividends from their stock holdings in the domestic and foreign intermediate-goods firms, and buy and sell shares to adjust their portfolios. After completing asset trade, households sell their holdings of intermediate goods to final-goods-producing firms. These firms are perfectly competitive and produce final goods using intermediate goods a and b as inputs

[4] The equations describing the foreign country are largely identical to those for the domestic country. We use star superscripts to denote foreign variables.

to a Cobb-Douglas technology:

$$G\left(a(s^t), b(s^t)\right) = a(s^t)^{\omega} b(s^t)^{(1-\omega)}, \qquad G^*\left(a^*(s^t), b^*(s^t)\right) = a^*(s^t)^{(1-\omega)} b^*(s^t)^{\omega}, \tag{3}$$

where $\omega > 0.5$ determines the size of the local input bias in the composition of domestically produced final goods.

Note that the Cobb-Douglas assumption implies a unitary elasticity of substitution between domestically-produced goods and imports. The Cobb-Douglas assumption, in conjunction with the assumption that utility is logarithmic in consumption, will allow us to derive a closed-form expression for equilibrium portfolios. Note, however, that a unitary elasticity is within the range of existing estimates: BKK (1994) set this elasticity to 1.5 in their benchmark calibration, while Heathcote and Perri (2002) estimate the elasticity to be 0.9. In a sensitivity analysis we will explore numerically the implications of deviating from the logarithmic utility, unitary elasticity baseline.

The terms of trade is the price of good b relative to good a. Because the law of one price applies to traded intermediate goods, this relative price is the same in both countries:

$$\frac{q_b(s^t)}{q_a(s^t)} = \frac{q_b^*(s^t)}{q_a^*(s^t)} \tag{4}$$

Let $e(s^t)$ denote the real exchange rate, defined as the price of foreign relative to domestic consumption. By the law of one price, $e(s^t)$ can be expressed as the foreign price of good a (or good b) relative to foreign consumption divided by the domestic price of good a (or b) relative to domestic consumption:

$$e(s^t) = \frac{q_a(s^t)}{q_a^*(s^t)} = \frac{q_b(s^t)}{q_b^*(s^t)} \tag{5}$$

2.2 Households' problem

The budget constraint for the domestic household is given by

$$\begin{aligned} & c(s^t) + P(s^t)\left(\lambda_H(s^t) - \lambda_H(s^{t-1})\right) + e(s^t)P^*(s^t)\left(\lambda_F(s^t) - \lambda_F(s^{t-1})\right) \\ = \; & l(s^t) + \lambda_H(s^{t-1})d(s^t) + \lambda_F(s^{t-1})e(s^t)d^*(s^t) \qquad \forall t \geq 0, s^t \end{aligned} \tag{6}$$

Here $P(s^t)$ is the price at s^t of (ex dividend) shares in the domestic firm in units of domestic consumption, $P^*(s^t)$ is the price of shares in the foreign firm in units of foreign consumption, $\lambda_H(s^t)$

$(\lambda_H^*(s^t))$ denotes the fraction of the domestic firm purchased by the domestic (foreign) agent, $\lambda_F(s^t)$ $(\lambda_F^*(s^t))$ denotes the fraction of the foreign firm bought by the domestic (foreign) agent, $d(s^t)$ and $d^*(s^t)$ denote domestic and foreign dividend payments per share, and $l(s^t) = q_a(s^t)w(s^t)n(s^t)$ denotes domestic labor earnings, where $w(s^t)$ is the wage in units of the domestically-produced intermediate good. The budget constraint for the foreign household is

$$
\begin{aligned}
(7) \qquad & c^*(s^t) + P^*(s^t)\left(\lambda_F^*(s^t) - \lambda_F^*(s^{t-1})\right) + (1/e(s^t))P(s^t)\left(\lambda_H^*(s^t) - \lambda_H^*(s^{t-1})\right) \\
= \; & l^*(s^t) + \lambda_F^*(s^{t-1})d^*(s^t) + \lambda_H^*(s^{t-1})(1/e(s^t))d(s^t) \qquad \forall t \geq 0, s^t
\end{aligned}
$$

We assume that at the start of period 0, the domestic (foreign) household owns the entire domestic (foreign) firm: thus $\lambda_H(s^{-1}) = 1$, $\lambda_F(s^{-1}) = 0$, $\lambda_F^*(s^{-1}) = 1$ and $\lambda_H^*(s^{-1}) = 0$.

At date 0, domestic households choose $\lambda_H(s^t)$, $\lambda_F(s^t)$, $c(s^t) \geq 0$ and $n(s^t) \in [0, 1]$ for all s^t and for all $t \geq 0$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t U\left(c(s^t), n(s^t)\right)$$

subject to (6) and a no Ponzi game condition.

The domestic households' first-order condition for domestic and foreign stock purchases are, respectively,

$$
\begin{aligned}
(8) \qquad U_c(s^t)P(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t)U_c(s^t, s_{t+1})\left[d(s^t, s_{t+1}) + P(s^t, s_{t+1})\right] \\
U_c(s^t)e(s^t)P^*(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t)U_c(s^t, s_{t+1})e(s^t, s_{t+1})\left[d^*(s^t, s_{t+1}) + P^*(s^t, s_{t+1})\right]
\end{aligned}
$$

where we use $U_c(s^t)$ for $\frac{\partial U\left(c(s^t), n(s^t)\right)}{\partial c(s^t)}$ and (s^t, s_{t+1}) denotes the $t + 1$ length history s^t followed by s_{t+1}

The domestic household's first-order condition for hours is

$$
\begin{aligned}
(9) \qquad U_c(s^t)q_a(s^t)w(s^t) + U_n(s^t) &\geq 0 \\
&= \text{ if } n(s^t) > 0
\end{aligned}
$$

Analogously, the foreign households' first-order condition for domestic and foreign stock purchases and hours are, respectively,

$$(10)\quad \begin{aligned} U_c^*(s^t)\frac{P(s^t)}{e(s^t)} &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[\frac{d\left(s^t,s_{t+1}\right)+P(s^t,s_{t+1})}{e\left(s^t,s_{t+1}\right)}\right] \\ U_c^*(s^t)P^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[d^*(s^t,s_{t+1})+P^*(s^t,s_{t+1})\right] \end{aligned}$$

and

$$(11)\quad \begin{aligned} U_c^*(s^t)q_b^*(s^t)w^*(s^t)+U_n^*(s^t) &\geq 0 \\ &= \text{ if } n^*(s^t)>0. \end{aligned}$$

2.3 Intermediate firms' problem

The domestic intermediate-goods firm's maximization problem is to choose $k(s^t)\geq 0$, $n(s^t)\geq 0$ for all s^t and for all $t\geq 0$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t} Q(s^t)d(s^t)$$

taking as given $k(s^{-1})$, where $Q(s^t)$ is the price the firm uses to value dividends at s^t relative to consumption at date 0, and dividends (in units of the final good) are given by

$$(12)\quad d(s^t)=q_a(s^t)\left[F\left(z(s^t),k(s^{t-1}),n(s^t)\right)-w(s^t)n(s^t)\right]-\left[k(s^t)-(1-\delta)k(s^{t-1})\right].$$

In this expression δ is the depreciation rate for capital. Analogously, foreign firms use prices $Q^*(s^t)$ to price dividends in state s^t, where foreign dividends are given by

$$(13)\quad d^*(s^t)=q_b^*(s^t)\left[F\left(z^*(s^t),k^*(s^{t-1}),n^*(s^t)\right)-w^*(s^t)n^*(s^t)\right]-\left[k^*(s^t)-(1-\delta)k^*(s^{t-1})\right].$$

The domestic and foreign firms' first order conditions for $n(s^t)$ and $n^*(s^t)$ are

$$(14)\quad w(s^t)=(1-\theta)F\left(z(s^t),k(s^{t-1}),n(s^t)\right)/n(s^t)$$

$$(15)\quad w^*(s^t)=(1-\theta)F\left(z^*(s^t),k^*(s^{t-1}),n^*(s^t)\right)/n^*(s^t).$$

The corresponding first order conditions for $k(s^t)$ and $k^*(s^t)$ are

$$(16)\quad Q(s^t) = \sum_{s_{t+1}\in S} Q(s^t, s_{t+1}) \left[q_a(s^t, s_{t+1})\theta F\left(z(s^t, s_{t+1}), k(s^t), n(s^t, s_{t+1})\right)/k(s^t) + (1-\delta)\right]$$

$$(17)\quad Q^*(s^t) = \sum_{s_{t+1}\in S} Q^*(s^t, s_{t+1}) \left[q_b^*(s^t, s_{t+1})\theta F\left(z^*(s^t, s_{t+1}), k^*(s^t), n^*(s^t, s_{t+1})\right)/k^*(s^t) + (1-\delta)\right]$$

The state-contingent consumption prices $Q(s^t)$ and $Q^*(s^t)$ obviously play a role in intermediate–goods firms' state-contingent decisions regarding how to divide earnings between investment and dividend payments. We assume that domestic firms use the discount factor of the representative domestic household to price the marginal cost of foregoing current dividends in favor of extra investment.[5] Thus

$$(18)\quad Q(s^t) = \frac{\pi(s^t)\beta^t U_c(s^t)}{U_c(s^0)}, \qquad Q^*(s^t) = \frac{\pi(s^t)\beta^t U_c^*(s^t)}{U_c^*(s^0)}.$$

2.4 Final goods firms' problem

The final goods firm's static maximization problem in the domestic country after history s^t is

$$\max_{a(s^t), b(s^t)} \left\{G(a(s^t), b(s^t)) - q_a(s^t)a(s^t) - q_b(s^t)b(s^t)\right\}$$

subject to $a(s^t)$, $b(s^t) \geq 0$.

The first order conditions for domestic and foreign firms may be written as

$$(19)\quad \begin{array}{ll} q_a(s^t) = \omega G(a(s^t), b(s^t))/a(s^t), & q_b(s^t) = (1-\omega)G(a(s^t), b(s^t))/b(s^t), \\ q_b^*(s^t) = \omega G^*\left(a^*(s^t), b^*(s^t)\right)/b^*(s^t), & q_a^*(s^t) = (1-\omega)G^*\left(a^*(s^t), b^*(s^t)\right)/a^*(s^t). \end{array}$$

2.5 Definition of equilibrium

An equilibrium is a set of quantities $c(s^t)$, $c^*(s^t)$, $k(s^t)$, $k^*(s^t)$, $n(s^t)$, $n^*(s^t)$, $a(s^t)$, $a^*(s^t)$, $b(s^t)$, $b^*(s^t)$, $\lambda_H(s^t)$, $\lambda_H^*(s^t)$, $\lambda_F(s^t)$, $\lambda_F^*(s^t)$, prices $P(s^t)$, $P^*(s^t)$, $r(s^t)$, $r^*(s^t)$, $w(s^t)$, $w^*(s^t)$, $Q(s^t)$, $Q^*(s^t)$, $q_a(s^t)$, $q_a^*(s^t)$, $q_b(s^t)$, $q_b^*(s^t)$, productivity shocks $z(s^t)$, $z^*(s^t)$ and probabilities $\pi(s^t)$ for all s^t and

[5] Under the baseline calibration of the model, the solution to the firm's problem will turn out to be the same for any set of state-contingent prices that are weighted averages of the discount factors of the representative domestic and foreign households. Note that each agent takes $Q(s^t)$ as given, understanding that their individual atomistic portfolio choices will not affect aggregate investment decisions.

for all $t \geq 0$ which satisfy the following conditions:

1. The first order conditions for intermediate-goods purchases by final-goods firms (equation 19)
2. The first-order conditions for labor demand by intermediate-goods firms (equations 14 & 15)
3. The first-order conditions for labor supply by households (equations 9 & 11)
4. The first-order conditions for capital accumulation (equations 16 & 17)
5. The market clearing conditions for intermediate goods a and b :

$$
\begin{aligned}
(20) \quad a(s^t) + a^*(s^t) &= F\left(z(s^t), k(s^{t-1}), n(s^t)\right) \\
b(s^t) + b^*(s^t) &= F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right).
\end{aligned}
$$

6. The market-clearing conditions for final goods:

$$
\begin{aligned}
(21) \qquad c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) &= G\left(a(s^t), b(s^t)\right) \\
c^*(s^t) + k^*(s^t) - (1-\delta)k^*(s^{t-1}) &= G^*\left(a^*(s^t), b^*(s^t)\right).
\end{aligned}
$$

7. The market-clearing condition for stocks:

$$
(22) \quad \lambda_H(s^t) + \lambda_H^*(s^t) = 1 \qquad \lambda_F(s^t) + \lambda_F^*(s^t) = 1.
$$

8. The households' budget constraints (equations 6 & 7)
9. The households' first-order conditions for stock purchases (equations 8 & 10)
10. The probabilities $\pi(s^t)$ are consistent with the stochastic processes for $\left[z(s^t), z^*(s^t)\right]$

2.6 Equilibrium portfolios

PROPOSITION 1: Suppose that at time zero, productivity is equal to its unconditional mean value in both countries ($z(s^0) = z^*(s^0) = 0$) and that initial capital is equalized across countries, $k(s^{-1}) = k^*(s^{-1}) > 0$. Then there is an equilibrium in this economy with the property that portfolios in both countries exhibit a constant level of diversification given by

$$
\begin{aligned}
(23)\quad \lambda_F(s^t) &= \lambda_H^*(s^t) = 1 - \lambda_H(s^t) = 1 - \lambda_F^*(s^t) = 1 - \lambda \\
&= \left(\frac{1-\theta}{1-\omega} + 2\theta\right)^{-1} \qquad \forall t, s^t
\end{aligned}
$$

Moreover, in this equilibrium stock prices are given by

$$
(24)\quad P(s^t) = k(s^t),\ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.
$$

and equilibrium allocations are efficient (i.e. perfect risk sharing is achieved).

PROOF: See the appendix

COROLLARY 1: Let $\Delta\widehat{l}(s^t) = \log\big(l(s^t)\big) - \log(e(s^t)) - \log\big(l^*(s^t)\big)$ denote relative log labor earnings in units of the domestic final good. Similarly, let $\Delta\hat{d}(s^t)$ denote relative log dividends. Let $M(s^t) = cov(\Delta\hat{l}(s^t), \Delta\hat{d}(s^t))/var(\Delta\hat{d}(s^t))$ denote the ratio of the equilibrium conditional covariance between relative log earnings and relative log dividends at $t+1$ to the variance of relative log dividends. Then

$$
(25)\quad M(s^t) \approx M = \left[\left(\frac{1-\theta}{1-\omega} + 2\theta\right)^{-1} - 1\right]\left(\frac{\theta}{1-\theta}\frac{\rho}{\rho+\delta}\right) \qquad \forall t, s^t
$$

where $\rho = (1-\beta)/\beta$. Moreover equilibrium diversification can be expressed as a function of this covariance ratio:

$$
(26)\quad 1 - \lambda \approx \frac{1}{2}\left(1 + \left(\frac{1-\theta}{\theta}\frac{\rho+\delta}{\rho}\right)M\right)
$$

PROOF: See the appendix

These theoretical results summarize our theory of international diversification, by establishing links between diversification $(1-\lambda)$, trade $(1-\omega)$, and comovement between relative labor income and relative dividends (M). In the next two sections we first provide some intuition for these results and then apply them to show that our theory can explain observed diversification in developed economies in recent years.[6]

[6]The expression for diversification in terms of structural parameters was first reported in Heathcote and Perri

3 Intuition for the result

First, we take a general equilibrium perspective, and combine a set of equilibrium conditions that link differences between domestic and foreign aggregate demand and aggregate supply in this economy. These equations shed light on how changes in relative prices coupled with modest levels of international portfolio diversification allow agents to achieve perfect risk-sharing.

We then take a more micro agent-based perspective, and explore why, from a price-taking individual's point of view, there are no incentives to trade stocks after date 0, why the covariance between relative (domestic to foreign) earnings and relative dividends is the key driver of portfolio choice, and why portfolios are home-biased on average.

3.1 Risk-sharing intuition

We now develop three equations that elucidate how the portfolio in eq. (23) delivers perfect risk-sharing.

The first equation is the hallmark condition for complete international risk-sharing, relating relative marginal utilities from consumption to the international relative price of consumption. Since the utility function is log-separable in consumption, this condition is simply

$$(27) \quad c(s^t) = e(s^t)c^*(s^t) \ \forall s^t,$$

which we can write more compactly as $\Delta c(s^t) = 0$, where $\Delta c(s^t)$ denotes the difference between domestic and foreign consumption in units of the domestic final good.

The second equation uses budget constraints to express the difference between domestic and foreign consumption as a function of relative investment and relative GDP.

Let $y(s^t) = q_a(s^t)F\left(z(s^t), k(s^{t-1}), n(s^t)\right)$ denote domestic GDP, and let $x(s^t) = k(s^t) - (1 - \delta)k(s^{t-1})$ denote investment, both in units of the domestic final good. Assuming constant portfolios, where λ denotes the fraction of the domestic (foreign) firm owned by domestic (foreign) households, domestic consumption is given by

$$\begin{aligned} c(s^t) &= l(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t) \\ (28) \qquad &= (1-\theta)y(s^t) + \lambda\left(\theta y(s^t) - x(s^t)\right) + (1-\lambda)e(s^t)\left(\theta y^*(s^t) - x^*(s^t)\right) \end{aligned}$$

(2004). The objective of that paper was to understand, in an environment with financial frictions, recent changes in international business cycle comovement between the U.S. and the rest of the world.

where the second line follows from the definitions for dividends, and the assumption that the intermediate-goods production technology is Cobb-Douglas in capital and labor. Then

$$(29) \quad \Delta c(s^t) = (1 - 2(1-\lambda)\theta)\Delta y(s^t) + (1 - 2\lambda)\Delta x(s^t)$$

where $\Delta y(s^t)$ and $\Delta x(s^t)$ are the differences between domestic and foreign GDP and investment, in units of the domestic final good. Note that in the case of complete home bias ($\lambda = 1$), the relative value of consumption across countries would simply be the difference between relative output and relative investment. For $\lambda < 1$, financial flows mean that some fraction of changes in relative output and investment are financed by foreigners.

Equations (27) and (29) do not depend on the elasticity of substitution between traded goods, and can therefore be applied unchanged to the one-good models that have been the focus of much of the previous work on portfolio diversification (in a one-good model $e(s^t) = 1$). For example, Baxter and Jermann (1997) study a one-good economy with production. They argue that since the Cobb-Douglas technology implies correlated returns to capital and labor, agents can effectively diversify non-diversifiable country-specific labor income risk by aggressively diversifying claims to capital. Assuming no investment, so that $\Delta x(s^t) = 0$, achieving perfect risk-sharing (*i.e.* $\Delta c(s^t) = 0$) means picking a value for λ such that the coefficient on $\Delta y(s^t)$ in eq. (29) is zero. The implied value for diversification is $1 - \lambda = 1/(2\theta)$, which is the portfolio described by eq.(2) in Baxter and Jermann. If capital's share θ is set to one-third, the value for $1-\lambda$ that delivers equal consumption in the two countries is 1.5. Thus, as Baxter and Jermann emphasize, a diversified portfolio involves a negative position in domestic assets.[7]

Our model enriches the Baxter and Jermann analysis along two dimensions. First, we explicitly endogenize investment. With stochastic investment, equation (29) indicates that, in general, no constant value for λ will deliver $\Delta c(s^t) = 0$. Thus, in a one-good model, perfect risk-sharing is not achievable with constant portfolios. However, our second extension relative to Baxter and Jermann is to assume that the two countries produce different traded goods that are imperfect substitutes when it comes to producing the final consumption-investment good. As we now explain, the Cobb-

[7]Note that equation (29) suggests that there will always exist a portfolio that delivers perfect risk sharing as long as $\Delta x(s^t)$ is strictly proportional to $\Delta y(s^t)$. Thus, as an alternative to assuming $\Delta x(s^t) = 0$, we could assume, for example, that firms invest a fixed fraction of output, so that $x(s^t) = \kappa y(s^t)$. In this case, in a one-good world, $\Delta x(s^t) = \kappa \Delta y(s^t)$. Now consumption equalization requires that $\Delta c(s^t) = [(1 - 2(1-\lambda)\theta) + (1-2\lambda)\kappa]\,\Delta y(s^t) = 0$ which implies $1 - \lambda = (1-\kappa)/(2\,(\theta - \kappa))$.

As an example, if the investment rate κ is equal to 0.2 and capital's share is 1/3, the value for $1-\lambda$ that delivers consumption equalization is 3.0, implying an even larger short position in domestic assets than the one predicted by Baxter and Jermann.

Douglas technology we assume for combining these traded goods implies an additional equilibrium linear relationship between $\Delta y(s^t)$, $\Delta c(s^t)$ and $\Delta x(s^t)$—our third key equation—such that perfect risk-sharing can be resurrected given appropriate constant portfolios.

From equations (5), (19) and (20), domestic GDP (in units of the final good) is given by

$$
\begin{aligned}
(30)\quad y(s^t) &= q_a(s^t)\left(a(s^t) + a^*(s^t)\right) = q_a(s^t)a(s^t) + e(s^t)q_a^*(s^t)a^*(s^t) \\
&= \omega G(s^t) + e(s^t)(1-\omega)G^*(s^t)
\end{aligned}
$$

Similarly, foreign GDP is given by

$$
(31)\quad y^*(s^t) = (1/e(s^t))(1-\omega)G(s^t) + \omega G^*(s^t)
$$

Combining the two expressions above, $\Delta y(s^t)$, the difference between the value of domestic and foreign GDP, is a linear function of relative absorption:

$$
\begin{aligned}
(32)\quad \Delta y(s^t) &= (2\omega - 1)\left(G(s^t) - e(s^t)G^*(s^t)\right) \\
&= (2\omega - 1)\left(\Delta c(s^t) + \Delta x(s^t)\right)
\end{aligned}
$$

This equation indicates that changes to relative domestic versus foreign demand for consumption or investment automatically change the terms of trade and thus, holding supply constant, the relative value of output. The fact that countries devote a constant fraction of total final expenditure to each of the two intermediate goods means that the size of the effect is proportional to the change in demand, where the constant of proportionality is $(2\omega - 1)$. When the technologies for producing domestic and foreign final goods are the same ($\omega = 0.5$), changes to relative demand do not impact the relative value of the outputs of goods a and b. When final goods are produced only with local intermediates ($\omega = 1$), an increase in domestic demand translates into an equal-sized increase in the relative price of good a . For intermediate values for ω, the stronger the preference for home-produced goods, the larger the impact on the relative value of domestic output.

Note that this equation is independent of preferences and the asset market structure, and follows solely from our Cobb-Douglas assumption, implying a unitary elasticity of substitution between the two traded goods.

We can now combine our three key equations, (27), (29) and (32) to explore the relationship between portfolio choice, relative price movements, and international risk-sharing. We start by substituting (32) into (29) to express the difference in consumption as a function solely of the difference in investment, yielding

$$(33)\quad \Delta c(s^t) \propto \underbrace{(1-2\lambda)}_{direct\ foreign\ financing} \Delta x(s^t) + \underbrace{(2\omega-1)\,(1-2(1-\lambda)\theta)}_{indirect\ foreign\ financing}\Delta x(s^t)$$

There is a unique value for λ such that the right hand side of (33) is always equal to zero. In particular, simple algebra confirms that this value is defined in Proposition 1 (eq. 23).[8]

As a first step towards understanding what eq. (33) implies for the link between diversification and risk-sharing, we first revisit some existing results for two-good models.

Lucas considers a two-good endowment economy in which domestic and foreign agents have identical preferences. In this case it is immediate that perfect risk pooling is achieved when agents hold 50 percent of both domestic and foreign shares in each period, *i.e.* $1-\lambda = 0.5$.[9] We get the same result from eq. (29) when $\theta = 1$ and $\Delta x(s^t) = 0$ for all s^t.

Cole and Obstfeld (1991) show that if domestic and foreign agents have symmetric log-separable preferences (like ours) for the two goods, then a regime of portfolio autarky (100 percent home bias or $1-\lambda = 0$) delivers the same allocations as a world with complete markets. In the context of our model, considering an endowment economy effectively implies $\Delta x(s^t) = 0$, in which case eqs. (29) and (32) become two independent equations in two unknowns, $\Delta c(s^t)$ and $\Delta y(s^t)$. The only possible equilibrium is then $\Delta c(s^t) = \Delta y(s^t) = 0$, regardless of $1-\lambda$. Thus, any value for $1-\lambda$ delivers perfect-risking sharing, including the portfolio autarky value $1-\lambda = 0$ emphasized by Cole and Obstfeld. The reason is simply that differences in relative quantities of output are automatically offset one-for-one by differences in the real exchange rate, so $y(s^t) = e(s^t)y^*(s^t)$. Thus movements in the terms of trade provide automatic and perfect insurance against fluctuations in the relative quantities of intermediate goods supplied.[10]

[8] In our model, ω is the share of the domestic intermediate good in both consumption and investment. The reader might wonder how the expression for equilibrium portfolios would differ if one allowed this parameter to take different values in separate aggregators for consumption versus investment goods. It is easy to extend the model in this fashion to allow for differential trade intensity. The relevant parameter for portfolio choice turns out to be the relative share of domestic versus imported intermediates in the production of investment goods.

[9] Cantor and Mark (1988) extend Lucas' analysis to a simple environment with production. However, they make several assumptions that ensure that their economy inherits the properties of Lucas'. In particular, (i) domestic and foreign agents have the same log-separable preferences over consumption and leisure, (ii) productivity shocks are assumed to be iid through time, (iii) firms must purchase capital and rent labor one period before production takes place, and (iv) there is 100% depreciation. When their two economies are the same size, assumptions (ii) and (iii) ensure that in an efficient allocation capital and labor are always equalized across countries. Thus to deliver perfect risk-sharing, the optimal portfolio choice simply has to ensure an equal division of next period output, which is ensured with Lucas' 50-50 portfolio split.

[10] Cole and Obstfeld also consider a version of the model with production. In this version the two goods may be consumed or used as capital inputs to produce in the next period. Like Cantor and Mark (1988) they assume

In contrast to the Cole and Obstfeld result, only one portfolio delivers perfect risk-pooling in our economy. Furthermore, portfolio autarky is only efficient in the case when there is complete specialization in tastes, so that $\omega = 1$. The reason for these differences relative to their results is that with partial depreciation and persistent productivity shocks, efficient investment will not be either constant or a constant fraction of output; rather, as in a standard growth model, positive persistent productivity shocks will be associated with a surge in investment. Thus dividends are not automatically equated across domestic and foreign stocks, and asset income is sensitive to portfolio choice. Moreover, these investment responses mediate relative price movements, so that relative earnings also fluctuate in response to productivity shocks.

We can use equation (33) to understand the effect of a change in relative investment $\Delta x(s^t)$ on relative consumption, $\Delta c(s^t)$. Absent any diversification, an increase in $\Delta x(s^t)$ would reduce $\Delta c(s^t)$ proportionately. For $1-\lambda > 0$ some of the cost of additional domestic investment is paid for by foreign shareholders directly (the first term on the right hand side) or indirectly through changes in relative prices (the second term). The value for $1-\lambda$ that delivers perfect risk-sharing is the one for which the direct and indirect effects exactly offset, so that changes in relative investment have no effect on relative consumption.

When preferences are biased towards domestically-produced goods $(\omega > 0.5)$, an increase in $\Delta x(s^t)$ increases the relative value of domestic output in proportion to the factor $(2\omega - 1)$ (see eq. 32). This captures the fact that increased relative demand for domestic final goods improves the terms of trade for the domestic economy. The fraction of this additional output that accrues as income to domestic shareholders is given by the term $(1 - 2(1-\lambda)\theta)$, which in turn amounts to labor's share of income, $(1-\theta)$, plus the difference between domestic and foreign shareholder's claims to domestic capital income, $(\lambda\theta - (1-\lambda)\theta)$. This indirect effect is positive as long as $1-\lambda < 1/(2\theta)$, reflecting the fact that an increase in domestic investment increases the relative value of domestic earnings. Risking-pooling portfolios are home-biased precisely because the indirect effect of an increase in relative domestic investment generally favors domestic residents. Thus these agents need to pay most of the direct costs of additional investment (by holding most of domestic equity) in order to equalize income and consumption across countries.

100 percent capital depreciation. When production technologies are Cobb-Douglas in the quantities of the two goods allocated for investment, portfolio autarky once again delivers perfect risk-sharing. The reason is that the assumptions of log separable preferences and full depreciation imply that consumption, investment and dividends are all fixed fractions of output, so that $\Delta x(s^t) = \kappa \Delta y(s^t)$. Given this relationship, equations 29 and 32 reduce to two independent equations in two unknowns, $\Delta c(s^t)$ and $\Delta y(s^t)$. Thus total dividend income in any given period is again independent of the portfolio split.

3.2 Hedging intuition

We now offer some intuition for why home-biased portfolios are optimal from the perspective of an atomistic investor. We do so in three steps.

First, we will argue that because equilibrium portfolios equate the value of income across countries state-by-state, agents have no incentives to actively trade assets after date zero. This no-trade result is important for understanding why the extent of diversification, as expressed in eq. 26, depends on the covariance between relative labor income and relative dividend income. Second, we will explain this covariance is determined in general equilibrium. Third, we will note that a negative covariance rationalizes a portfolio bias towards domestic assets, and close the circle of the argument by showing how home-biased equilibrium portfolios imply dynamics for relative incomes and returns that equate relative incomes state-by-state across countries.

The result that there is no trade in equilibrium reflects the fact that the equilibrium portfolio split equates the common-currency value of total labor plus financial income across countries in every date and state, as described in the previous section. Thus, given this split, maintaining passive portfolios implies equal consumption values across countries, from which it follows that domestic and foreign agents share the same inter-temporal marginal rate of substitution, and have no incentives to actively retrade. Given passive portfolios, consumption is always equal to income, and the optimal portfolio choice problem is effectively static. In a static problem, the optimal split depends on the covariance between different components of income. If relative dividends decline and the same time that relative earnings rise, then domestic stocks offer a good hedge against labor income risk. Since there is a linear relationship between relative dividends and relative earnings, by choosing just the right degree of home bias, agents can perfectly hedge relative labor income risk, so that relative total income is zero in every date and state. It is important to emphasize that optimal portfolios can be defined in terms of the covariance between relative *incomes* to labor and capital because of the no-trade result. In alternative dynamic models featuring active retrading, one could only define optimal portfolios in terms of relative *returns* to labor and capital.

Why is the covariance between relative dividends and relative earnings negative in our model? The difference between the value of domestic and foreign earnings (in units of the domestic final good) is

$$
\begin{aligned}
(34)\quad \Delta l(s^t) &= (1-\theta)\Delta y(s^t) \\
&= (1-\theta) q_a(s^t) \left(F\left(z(s^t), k(s^{t-1}), n(s^t)\right) - \frac{q_b(s^t)}{q_a(s^t)} F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right) \right).
\end{aligned}
$$

Thus the relative value of domestic earnings rises in response to an increase in relative productivity if and only if the increase in the production of good a relative to good b exceeds the increase in the terms of trade (i.e. the price of good b relative to good a). As discussed in the previous section, this condition is satisfied in our economy. At the same time, relative dividends decline, because higher relative productivity raises relative investment, and thus reduces relative dividends.

It is important to emphasize that both the increase in relative earnings and the decline in relative dividends reflect the dynamics of investment. Absent investment, capital and labor income would co-move perfectly, given our Cobb-Douglas technology. Moreover, productivity shocks would have no impact on *relative* income, because movements in the terms of trade would exactly offset changes in relative productivity, as in Cole and Obstfeld (1991). Introducing investment weakens the response of the terms of trade to a productivity shock, because following a positive domestic shock, relative investment rises. Since domestic investment is biased towards the domestic good, this implies an increase in the relative demand for domestic intermediates that partially offsets the effect of greater relative supply on the terms of trade.

Given a diversify-once-and-hold-thereafter trading strategy, whether domestic or foreign stocks offer a better hedge against labor income risk depends on the covariance between relative labor earnings and relative dividends. In our model, this covariance is negative. This rationalizes equilibrium portfolio bias towards domestic stocks. In fact, given exactly the right portfolio bias, changes in relative dividend income will always exactly offset changes in relative earnings, for any sequence of productivity shocks. How much bias is optimal depends on the magnitude of labor income risk to hedge, and the strength of the negative covariance between relative dividends and relative earnings, which determines the effectiveness of domestic shocks as a hedge. As we now explain, these factors depend on labor's share of income, $1-\theta$, and the trade share, $1-\omega$.

Equation 23 indicates that equilibrium diversification, $1-\lambda$, is increasing in the trade share, $1-\omega$. This is because more trade reduces the variance of relative earnings, and thus the demand for a hedge against earnings risk. As the mix of domestic and foreign goods in final goods production becomes increasingly symmetric, the effect of an increase in relative investment on the terms of trade becomes weaker, since domestic investment is composed of a more equal mix of the two intermediate goods. Thus, as the import share is increased, relative non-diversifiable labor income becomes less variable, since offsetting movements in the terms of trade provide ever more insurance against fluctuations in relative productivity. This pushes agents towards more symmetric portfolios, which continue to favor the asset (domestic stocks) whose income co-moves negatively with earnings, as long as trade share is less than 50%. Note that when the trade share exceeds 50% investment is biased toward foreign intermediates and hence the terms of trade response to a productivity shock is so strong that both relative earnings and relative dividends fall, inducing foreign bias in asset holding.

Equation 23 indicates that equilibrium diversification is decreasing in labor's share of income, $1 - \theta$. This is the opposite of the Baxter and Jermann (1997) result, who found that introducing labor supply made observed home bias even more puzzling from a theoretical standpoint. Both results are easy to rationalize. The larger is labor's share, the larger is the decline in relative domestic earnings following a negative productivity shock, and thus the greater is the demand for the asset which offers a hedge against labor income risk. In our economy, that asset is the domestic stock. In the Baxter and Jermann one-good world, it is the foreign stock.

The dynamics described above can be visualized by plotting some impulse responses. This requires fully parameterizing the model. Most parameters are straightforward to pick, since variations on this model have been widely studied. Here we mostly follow Heathcote and Perri (2004), who show that a similar model economy can successfully replicate a set of key international business cycle statistics for the U.S. versus an aggregate of industrial countries over the period 1986-2001. Table 1 below reports the values we use.

Figure 1 plots impulse responses to a persistent (but mean reverting) positive productivity shock in the domestic country. The path for productivity in the two countries is depicted in panel (a), while the real exchange rate is plotted in panel (d). The remaining panels show stock returns (b), labor earnings (c), stock prices (e), and dividends (f), all of which plotted in units of the domestic final consumption good.

In the period of the shock, relative domestic earnings increases, and the gap between relative earnings persists through time. The differential can persist because labor is immobile internationally.

In the period of the shock, returns to both domestic and foreign stocks increase, but the increase is larger for foreign stocks (panel b). In subsequent periods, returns to domestic and foreign stocks are equalized. The reason for this result is simply that stocks are freely traded and thus equilibrium stock prices must adjust to equalize expected returns.

Why does the relative return to foreign stocks increase in response to a positive domestic productivity shock? As panel (e) indicates, this reflects a decline in the relative price of domestic stocks. We can rationalize this response as follows.

Figure 1: Impulse responses to a domestic productivity shock

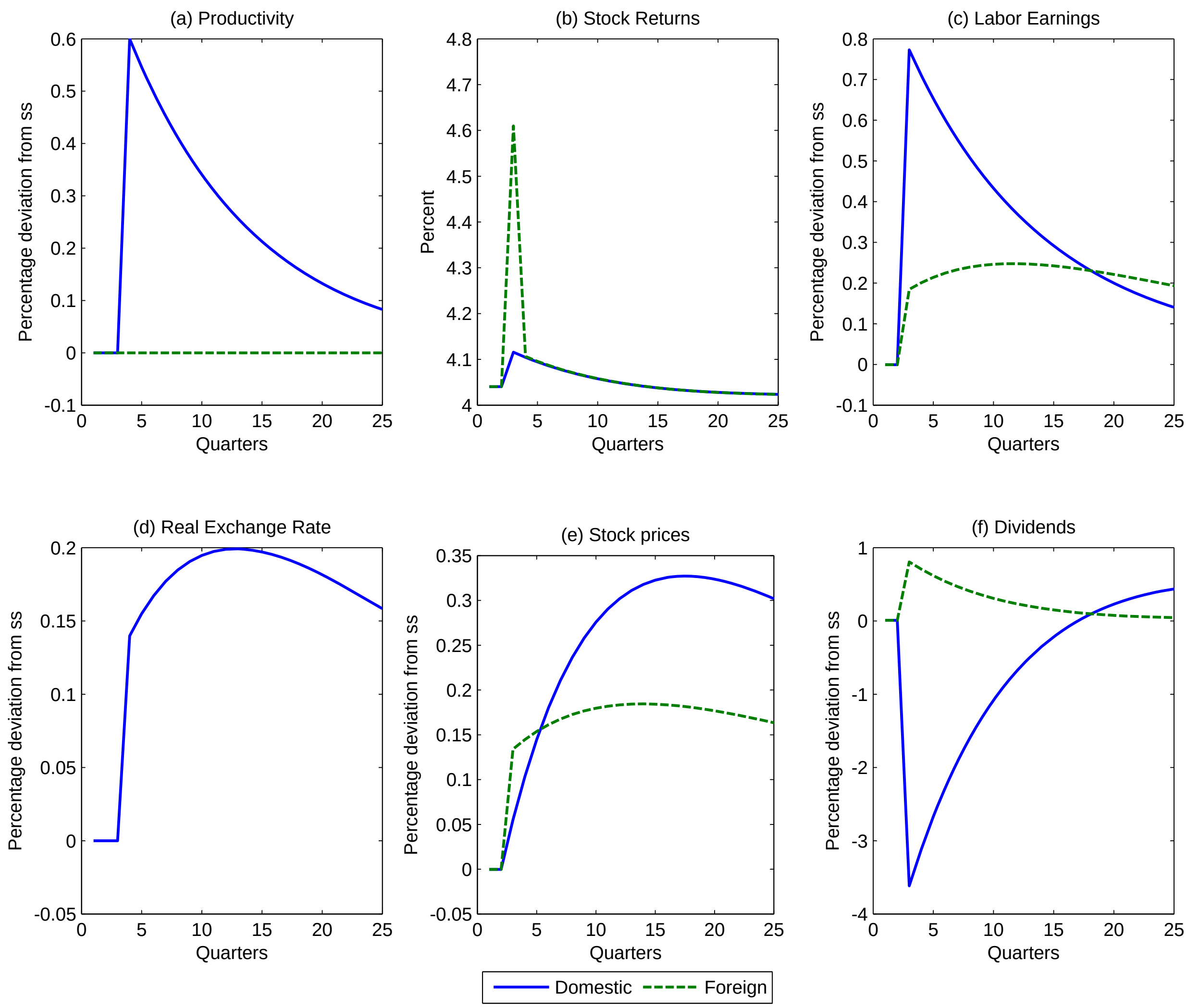


Table 1: Parameter values

Preferences	
Discount factor	$\beta = 0.99$
Disutility from labor	$V(.) = v\frac{n^{1+\phi}}{1+\phi}$
	$v = 9.06, \quad \phi = 1$
Technology	
Capital's share	$\theta = 0.36$
Depreciation rate	$\delta = 0.015$
Import share	$1 - \omega = 0.15$
Productivity Process	
$\begin{bmatrix} z(s^t) \\ z^*(s^t) \end{bmatrix} = \begin{pmatrix} 0.91 & 0.00 \\ 0.00 & 0.91 \end{pmatrix} \begin{bmatrix} z(s^{t-1}) \\ z^*(s^{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{bmatrix}$	
$\begin{bmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{bmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.006^2 & 0.00 \\ 0.00 & 0.006^2 \end{pmatrix}\right)$	

Suppose the positive domestic productivity shock occurs at date t given history s^t. Given a constant portfolio split defined by λ, the difference between the lifetime present values of domestic and foreign income, in units of domestic consumption, is

$$(35) \qquad \sum_{j=t}^{\infty} \sum_{s^j \succeq s^t} \frac{Q(s^j)\left(\Delta l(s^j) + (2\lambda - 1)\Delta d(s^j)\right)}{Q(s^t)}$$

$$(36) \qquad = \sum_{j=t}^{\infty} \sum_{s^j \succeq s^t} \frac{Q(s^j)\Delta l(s^j)}{Q(s^t)} + (2\lambda - 1)\left(\Delta d(s^t) + \Delta P(s^t)\right) = 0$$

where the first equality reflects the fact that the present value of future domestic (foreign) dividend payments is equal to the equilibrium ex-dividend price of domestic (foreign) stocks, and the second equality holds because equilibrium values of domestic and foreign income (and consumption) are the same in every date and state.

At date t, the positive domestic shock increases the present value of relative earnings, the first term on the right-hand side of eq. 35 (see panel b). Given portfolio home bias ($1 - \lambda < 1/2$) the relative present value of income can remain equal to zero only if the relative pre-dividend price of stocks of stocks, the second term in eq. 35, goes down. Using the results $P(s^t) = k(s^t)$ and $P^*(s^t) = k^*(s^t)$ and the definitions for dividends (eqs. 12 and 13), the relative pre-dividend price of stocks can be expressed as

$$(37) \quad \Delta d(s^t) + \Delta P(s^t) = \theta\left(y(s^t) - e(s^t)y^*(s^t)\right) + (1-\delta)\left(k(s^{t-1}) - e(s^t)k^*(s^{t-1})\right)$$

The first term in this expression captures the change in relative rental income in the period of the shock, and, like relative earnings (eq. 34), will increase following a positive domestic productivity shock. The second term captures the change in the relative value of undepreciated capital. A positive domestic productivity shock drives up the real exchange rate $e(s^t)$ and thus drives down this relative value (since final consumption and investment are perfectly substitutable in production, the relative price of capital is equal to the relative price of consumption). In our model, the second term in eq. 37 dominates, meaning that when faced with a positive shock, owners of domestic stocks lose more from the ensuing devaluation of domestic capital than they gain from a higher rental rate. Thus domestic stocks offer a good hedge against non-diversifiable labor income risk, rationalizing, in return space, home bias in portfolios.

Because agents do not adjust their portfolios in response to the shock, the decline in the relative price of domestic stocks on impact means that financial wealth for home-biased domestic agents declines relative to the wealth of foreigners. This means that in the periods immediately following the shock, even though returns are equalized, the total asset income accruing to foreign agents is larger, because they hold more financial wealth in total. This additional asset income exactly offsets foreigners' lower labor income, and the relative value of consumption is equalized.

Over time, the domestic productivity shocks decays, while the real exchange rate remains above its steady state level. As a consequence, foreign labor income eventually rises above domestic labor income. But notice that now, because of capital accumulation in country 1, domestic stocks are now worth more than foreign stocks, and this compensates domestic residents for the fact that they expect relatively low earnings during the remainder of the transition back to steady state.

Related Literature: We are not the first to relate portfolio choice to the pattern of co-movement between labor income and domestic and foreign stock returns. Cole (1988), Brainard and Tobin (1992), and Baxter and Jermann (1997) argued that in models driven entirely by productivity shocks, one should expect labor income to co-move more strongly with domestic rather than foreign stock returns, thereby indicating strong incentives to aggressively diversify. Bottazzi, Pesenti and van Wincoop (1996) argued that this prediction could be over-turned by extending models to incorporate additional sources of risk that redistribute income between capital and labor, and thereby lower the correlation between returns on human and physical capital. They suggested terms of trade shocks as a possible candidate. We have shown that in fact it is not necessary to introduce a second source of risk: the endogenous response of the terms of trade to productivity shocks is all that is required to generate realistic levels of home bias. The existing empirical evidence on correlations between *returns* to labor and domestic versus foreign stocks is, for the most part, qualitatively consistent with the pattern required to generate home bias. Important papers on this topic are Bottazzi et. al. (1996), Palacios-Huerta (2001), and Julliard (2002). In the next section, we will investigate the covariance between relative *income* from labor and capital, exploiting the expression for portfolio diversification in eq. 26.

Van Wincoop and Warnock (2008) emphasize a different force that can also deliver home bias in two-good models: negative covariance between the real exchange rate and the return differential between domestic and foreign stocks. If domestic stocks pay a relatively high return in states of the world in which domestic goods are expensive (i.e., the real exchange rate is low) then, since domestic residents mostly consume domestic goods, they may prefer to mostly hold domestic stocks. Note that this effect is not the driver of home bias in our basic set-up. In fact van Wincoop and Warnock show that this mechanism generates home bias only when the coefficient of relative risk aversion exceeds one. By contrast, our model generates substantial home bias even with risk aversion equal to one.[11] The most important difference between our environment and theirs is that they abstract from labor income. In the presence of non-diversifiable labor income, portfolio choice is driven primarily by the covariance between relative incomes (or returns) from capital and labor, and not by the covariance between relative equity returns and the exchange rate.

4 Explaining diversification in developed countries

The key message of our model, as summarized by Proposition 1, is that the patterns of international diversification are driven by patterns of trade. Corollary 1 suggests a two-part economic mechanism underlying this link. The first part (summarized by eq. 25) is the "general equilibrium" connection linking trade to the covariance between relative labor earnings and relative dividends. The extent of trade determines the equilibrium dynamics of investment and the terms of trade, which in turn determine the covariance between relative earnings and dividends.

[11] We experiment with alternative values for risk aversion in Section 5

The second part of the link (summarized by eq. 26) is the "partial equilibrium" connection linking the covariance between relative labor earnings and relative dividends to international portfolio diversification. We call this a "partial equilibrium" link because it reflects individual portfolio optimization, taking as given relative prices and dividends.

In this section we provide evidence for the central prediction of the theory: the link between trade and diversification, which we will show holds both in the cross section and in the time series. We also provide evidence for the two sub-links between trade and covariances and between covariances and diversification, which provide a more direct test of the mechanism highlighted by our theory.

As usual, there are a few decisions to be made as to how best to compare our simple theoretical model to the data.

4.1 Model

Evaluating quantitative predictions for the relationships between diversification $(1-\lambda)$, trade $(1-\omega)$ and covariances (M) in our model is straightforward. The relationships established in Proposition 1 and Corollary 1 involve only three structural parameters: the capital share θ, the depreciation rate δ and the discount factor β. We will assume that these parameters are the same across countries[12] and set them to the commonly used values of $\theta = 0.36$, $\delta = 0.015$, $\beta = 0.99$.

4.2 Data

The first issue we need to confront when comparing the model's prediction to data is that ours is a model with two symmetric countries and no restrictions on international financial flows, while international diversification data are drawn from countries which are heterogenous in many dimensions, including size, the level of development, and the extent of financial liberalization. One possible way to deal with this issue would be to enrich our basic model to include many heterogenous countries and to then bring such a model to the data; we view that as an interesting project, but one that is beyond the scope of this paper.[13] Here, instead, we address the issue in two ways. First, we

[12] Gollin (2002) shows that the share of income going to capital exhibits remarkably little variation across countries, given a careful accounting of cross-country variation in the size of self-employment.

[13] We did experiment with one dimension of heterogeneity. In particular we considered an extension of our main model in which the two countries differ in terms of population. We then solve this version of the model numerically, given the parameter values described in Table 1, and compare the average equilibrium level of diversification to the level predicted by equation 23. We find that, for the smaller economy, the equilibrium level of diversification exceeds that which would be observed in the corresponding symmetric-size economy, while for the larger economy, the equilibrium level of diversification is below that which would be prediction by (23), given the country's import share. However, these differences are generally small (less than 1%), unless the smaller country is both very open and very small.

restrict our empirical analysis to a group of relatively homogenous and financially liberalized countries: members of the OECD throughout the period 1990-2004.[14] Second, within this group, we assess whether factors omitted from the model, such as size or level of development, are important empirical factors in explaining diversification patterns.

A second issue is how to measure the three key variables in our theory: diversification, trade intensity and covariance between relative labor earnings and relative dividends.

Our measure of international diversification in the model, $1-\lambda$, is both the ratio of gross foreign assets to wealth and the ratio of gross foreign liabilities to wealth. Thus to construct empirical measures of diversification we need data on gross foreign assets, gross foreign liabilities, and total country wealth. We obtain data on total gross foreign assets (FA) and total gross foreign liabilities (FL) from the exhaustive dataset collected by Lane and Milesi-Ferretti (2006). The empirical counterpart of model equity is not just corporate traded equity, but any asset that represents a claim to country output. Thus we include all assets that constitute such claims: portfolio equity investment, foreign direct investment, debt (including loans or trade credit), financial derivatives and reserve assets (excluding gold). We identify total country wealth as the value of the entire domestic capital stock plus gross foreign assets less gross foreign liabilities: $K + FA - FL$. We start from the initial capital stock figures in Dhareshwar and Nehru (1993), and then construct time series by cumulating investments from the Penn World Tables 6.2 (as, for example, in Kraay et. al. 2005).[15] We measure international diversification for country i in period t as

$$DIV_{it} = \frac{FA_{it} + FL_{it}}{2\left(K_{it} + FA_{it} - FL_{it}\right)}.$$

The intensity of trade in the model is driven by the preference parameter $1-\omega$, which can be measured using either steady state import or export to GDP ratios. We measure the trade intensity for country i, $(1-\omega)_i$, using national income data from the Penn World Tables 6.2, as the time

[14] The exact set of countries we included in our sample is described in Appendix C.

[15] One important issue regarding the capital stock is whether it should be measured at book value (i.e. by cumulating investment) or at market value (as reflected, for example, in stock prices). Ideally, one would like to construct a measure of capital that is consistent with the valuation of foreign assets and foreign liabilities. Unfortunately, values for some asset categories (such as foreign direct investment) are constructed using book values, while others (such as portfolio equity investment) are constructed using market values. We computed an alternative measure of the capital stock, using information on stock market growth to revalue the publicly-traded component of the capital stock. In practice the particular measure of capital used makes little difference for our results (for details, see Heathcote and Perri, 2008).

average of import and export shares i.e.

$$(1-\omega)_i = \frac{1}{15}\sum_{t=1990}^{2004} \frac{Imports_{it} + Exports_{it}}{2GDP_{it}}.$$

The final variable we need to test our theory is M, the covariance between relative earnings and relative dividends divided by the variance of relative dividends. We therefore need to estimate M for each country in our sample.

To do so we use quarterly series which are comparable across countries and that are sufficiently long so that comovements between components can be measured without too much sampling error. We use OECD quarterly national accounts from the period 1980 to 2007. Consistently with our broad interpretation of wealth, we measure dividends as aggregate capital income less aggregate investment, where capital income is a constant fraction θ of GDP. Note that this corresponds directly to model dividends as defined in eq. 12. Labor earnings are then equal to fraction $(1-\theta)$ of GDP. For each country, we construct a measure of foreign dividends and foreign earnings by taking a weighted average of log dividends and log earnings of all the other countries in the sample, where weights are given by relative shares in world GDP. When aggregating across trading partners and then computing domestic relative to foreign earnings and dividends, we measure everything in a common currency.

Both the variance of relative log earnings and relative log dividends, and the covariance between the two, are largely driven by volatility in bilateral nominal exchange rates. However, given the existence of liquid forward foreign exchange markets, it has been argued that variations in relative returns associated with nominal exchange rate movements should not determine portfolio choice. For example, van Wincoop and Warnock (2008), Coeurdacier, Kollmann and Martin (2009), and Coeurdacier and Gourinchas (2009), Engel and Matsumoto (2009) show that given either a forward foreign exchange market or trade in nominal bonds, portfolio choice depends on the covariance properties of the component of relative returns that is orthogonal to fluctuations in nominal exchange rates. Thus, for each country, we first regress relative log earnings and relative log dividends on bilateral nominal exchange rates, and compute our empirical measures of the covariance ratio M using the residuals.[16]

[16] Notice that this procedure is equivalent to defining the empirical counterpart of the model real exchange rate to be the component of the observed real exchange rate that is orthogonal to the nominal exchange rate. Interestingly, Hess and Shin (2009) show that this measure of the real exchange rate exhibits a much higher correlation with relative consumption than the standard measure, and thus narrows the discrepancy between the predictions of our baseline model (in which the correlation is 1, see Backus and Smith, 1993) and data with respect to this moment.

4.3 Findings

Figure 2 confronts the theoretical link between diversification and trade with cross sectional evidence. The dots represent the time averages for diversification $(DIV)_i$ and the trade intensity $(1-\omega)_i$ for each country in our dataset. Note that there is a great deal of heterogeneity in both the trade intensity and the diversification, with both ranging from around 10% to around 100%. The curve is the theoretical relationship between trade and diversification implied by the model (eq. 23). The figure suggests that the cross-country diversification evidence is broadly consistent with the predictions from our model, and that the trade share is an important factor in explaining observed variation in international diversification.[17] Of course, a significant fraction of heterogeneity in diversification is not explained by our model, reflecting the reality that countries differ in multiple dimensions in addition to openness. For example, Great Britain is excessively diversified from the standpoint of the theory, perhaps reflecting its special position as an international financial center.

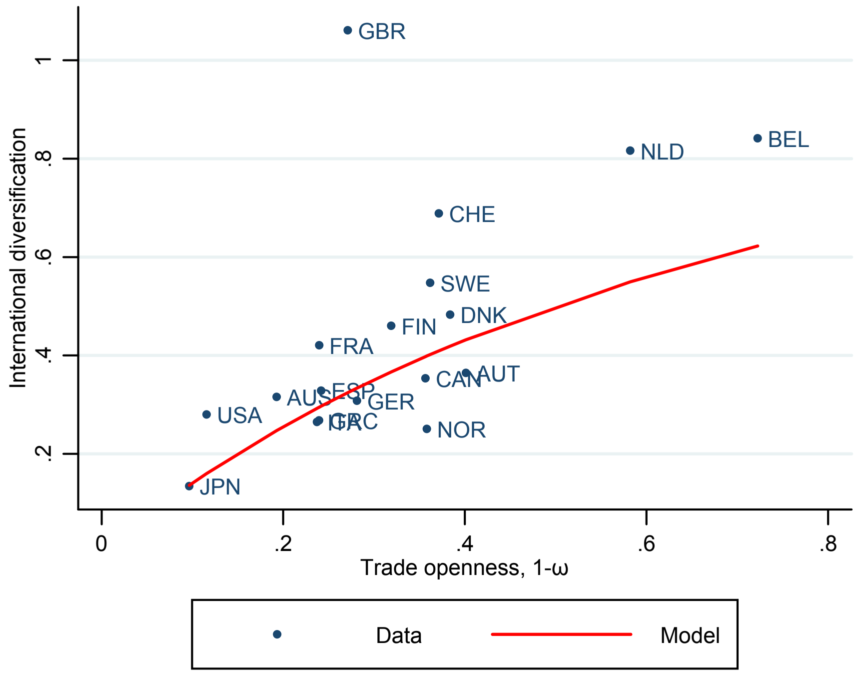
In Table 2 we make the comparison between model and data more precise using linear regression analysis.[18] Column (1) of the table reports results from the baseline regression. The line labeled "Trade openness" reports the coefficient obtained regressing (using OLS) diversification on trade openness. It shows that diversification and trade are significantly related, with a coefficient of about 1, suggesting that a 10% difference in trade openness between two countries is reflected in about a 10% difference in diversification. The line labeled "Predicted median div." reports a statistic meant to capture the level of diversification in the data and in the model. The statistic is diversification for a hypothetical country with the median trade share (and median values for other independent variables), as predicted by regressions on real-world or model-generated variables. The line shows that predicted diversification for such a country in the data is around 44%.

In column (2) we add to the basic regression GDP per capita and size (measured by population). Our symmetric model is silent about the effects of those variables, but it is interesting to assess i) whether these variables are indeed statistically correlated with diversification, and ii) whether

[17] Using bilateral data on trade and cross-border asset holding, Aviat and Coeurdacier (2007) explore the relationship between trade and diversification within a simultaneous gravity equations framework. They estimate that a 10% increase in bilateral trade raises bilateral asset holdings by 6% to 7%, that causality runs primarily from trade to diversification rather than from diversification to trade, and that controlling for trade greatly reduces the explanatory power of distance for cross-border asset holdings. Portes and Rey (2005) and Collard et al. (2007) also highlight a strong empirical relation between trade in assets and trade in goods.

[18] In order to estimate a linear relation in the model we use data on trade intensity for the countries in our sample together with equations (23) ,(25) and our chosen parameter values to generate artificial data on diversification and on covariances, on which we run the same regression we run on the actual data.

Figure 2: Diversification and trade, data and model



the relation between trade and diversification is affected by the inclusion of these variables. In particular, since small countries and rich countries tend to trade more, one might wonder whether trade matters for diversification only to the extent that trade proxies for size or GDP per capita. The numbers in the table do not support this conjecture. Size and GDP per capita are not statistically related to diversification, as long as the openness variable is retained. Furthermore, the statistical and economic significance of the relationship between diversification and trade is largely unaffected by whether or not these additional controls are included. Finally, column (3) redoes the baseline regression applying the least absolute deviations (LAD) metric. The fact that statistics for OLS and LAD are similar suggest that our results are not driven by a very small subset of countries. Finally, the R^2 figures in the last row suggest that differences in openness to trade can alone explain between 30 and 40 percent of cross-country variation in portfolio diversification.[19]

[19]The reader might wonder what is the role of sample size in our results. We have replicated the exercise of Table 2 on two different sets of countries. The first is an extended set of developed economies, i.e. the set of rich countries, as defined by the world bank. This gives a sample of 28 countries. Results for this set of countries are available in Heathcote and Perri (2008) and are very similar to the ones presented here. The reason why we use a smaller sample here is that we could not get comparable national accounts data, needed to compute covariance ratios, for the larger set of countries. We have also repeated the analysis for a even larger group of countries, including developing economies. We found that, although the strong link between trade and diversification remains, income per capita

Table 2: Diversification and trade

Dependent variable is diversification

	Data			Model
	OLS		LAD	
	(1)	(2)	(3)	(4)
Trade openness	$1.02^{\dagger}$	$1.21^{\dagger}$	$1.06^{\dagger}$	0.78
	(0.16)	(0.20)	(0.26)	(0.06)
Log GDP per capita		0.04		
		(0.05)		
Log population		0.02		
		(0.16)		
Predicted median div.	$0.44^{\dagger}$	$0.42^{\dagger}$	$0.39^{\dagger}$	0.34
	(0.05)	(0.05)	(0.05)	(0.01)
R^2	0.39	0.42	0.35	0.97
Observations	18	18	18	18

Note: Robust (for the OLS specifications) standard errors are in parentheses. A † next to a data statistic indicates that the corresponding model statistic lies within two standard deviation bands around the data.

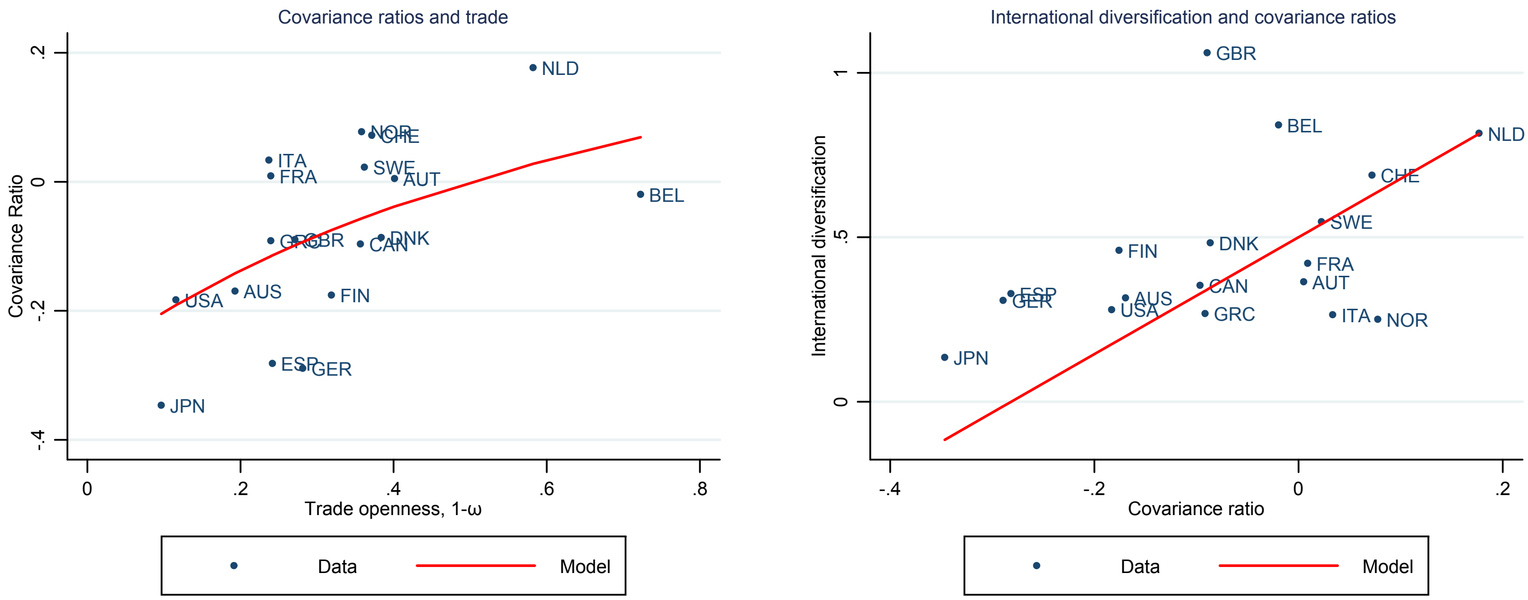
Comparing columns (1) through (3) with column (4) indicates that the model's predictions both for the the *level of diversification* and for the *relation between diversification and trade* are close, in a statistical sense, to the data.

Figure 3 delves deeper into the relation between trade and diversification by examining whether the model's predictions on the two sub-links highlighted in Corollary 1 are consistent with the data. In the left panel we document the general equilibrium link, connecting the covariance ratio between relative log earnings and relative log dividends to trade openness (eq. 25). The dots represents the covariance ratio and trade intensity in our sample of countries, while the solid curve is the relationship between the two implied by the model. Note that for a majority of countries, the covariance ratio is negative in both data and model, suggesting that a majority of countries should hold portfolios biased toward domestic assets. Note also that in the data, as in the model, countries which trade less tend to have a more negative covariance ratio.

The right panel of Figure 3 provides evidence for the partial equilibrium link (eq. 26) between international diversification and covariance ratios. The dots represent diversification and covariance

becomes an important determinant of diversification, with richer countries being more diversified.

Figure 3: Diversification, Covariance and Trade



ratios in the data while the solid line represents the relationship in the model. The panel shows that, consistently with the model, countries in which the covariance ratio is smaller in absolute value tend to exhibit less home bias.

Table 3 offers a more quantitative comparison between model and data with respect to the relationships shown in Figure 3. Panel (a) reports the results of OLS and LAD regressions of the covariance ratio on trade openness. Comparing data and model values for the "Trade openness" coefficient indicates that the magnitude of the empirical link between openness and the covariance ratio is comparable to the one predicted by the model. The line labeled "Pred. median cov" reports the covariance ratio for a hypothetical country with the median trade share, as predicted by regressions on real-world data and model-generated data, and shows that the covariance ratio for this median country in the data is of the same order of magnitude as the one predicted by the model.

Panel (b) reports corresponding results from regressing diversification on covariance ratios. Note that both data and model imply that countries which exhibit a more negative covariance should be less diversified. However, the effect of covariance on diversification in the model (1.78) is stronger than what we measure in the data using OLS or LAD (0.78 or 1.12). One possible explanation is that the true covariance ratios are measured with (sampling) error, which would bias downwards

Table 3: Diversification, Covariance and Trade

	Dependent variable:								
	a. Covariance Ratio				b. Diversification				
	Data		Model		Data			Model	
	(1)	(2)	(3)		(4)	(5)	(6)	(7)	
	OLS	LAD	OLS		OLS	LAD	IV	OLS	
Trade openness	0.56†	0.73†	0.43	Covariance ratio	0.78	1.12†	1.82†	1.78	
	(0.23)	(0.36)	(0.03)		(0.27)	(0.46)	(0.74)	(0.00)	
Pred. median cov.	-0.09†	-0.07†	-0.09	Pred. median div.	0.45†	0.42†	0.44†	0.34	
	(0.03)	(0.05)	(0.02)		(0.05)	(0.07)	(0.06)	(0.00)	
R^2	0.36	0.18	0.97		0.20	0.13	0.39	1.00	
Observations	18	18	18		18	18	18	18	

Note: Robust (for the OLS specifications) standard errors are in parentheses. A † next to a data statistic indicates that the corresponding model statistic lies within two standard deviation bands around the data.

the estimated relation between diversification and the covariance ratio. In order to explore this possibility we instrument covariance ratios using trade shares (which are precisely measured, and should be correlated with the true covariance ratios but not with the measurement error). Column (6) reports the result of this exercise, and shows that measurement error in the covariance ratio is potentially important. In particular, instrumenting for the covariance ratio increases the estimated coefficient by more than a factor of two, and significantly narrows the gap between model and data.

The evidence discussed in this section leads us to conclude that the predictions of the model regarding the level of diversification and the relationships between diversification, trade and co-movements between aggregate labor earnings and dividends are qualitatively and quantitatively helpful for understanding the cross-section of country portfolios in developed economies.

4.4 Diversification over time

In this section we explore the panel dimension of our dataset to assess whether our framework can also be used to understand the evolution of international diversification in recent years. Strictly speaking, diversification does not change over time in equilibria of our baseline model. However, suppose that between period t and period $t + k$ two countries experience an unexpected and permanent change in ω, the parameter which determines their trade share. Then the model predicts a corresponding change in international diversification.

In Table 4 we explore this prediction by regressing changes in diversification on changes in the trade share for our panel of countries. In order to focus on changes in the trade share in the data that are (possibly) persistent and unanticipated, we examine changes over five year intervals.[20] Columns (1) through (3) reports the results of running the regression on the data, adding various controls, while column (4) reports the results from running the same regression on changes in diversification generated by the model. Even after including growth in GDP per capita and population and country

Table 4: Changes in diversification and changes in trade

Dependent variable is five year change in diversification

	Data			Model
	(1)	(2)	(3)	(4)
Change in openness	1.14†	1.21†	1.06†	0.72
	(0.47)	(0.42)	(0.47)	(0.08)
Change in log GDP pc		1.34	1.15	
		(0.48)	(0.37)	
Change in log pop.		-2.83	1.77	
		(1.19)	(1.05)	
Country & period dummies	No	No	Yes	No
Observations	180	180	180	180
R^2	0.07	0.22	0.77	0.90

Note: numbers in parentheses are robust standard errors clustered at the country level
A data statistic with † indicates that the corresponding model statistic lies within
two standard deviation bands around the data.

and period dummies—and thus allowing for a variety of factors to affect changes in diversification—the link between trade and diversification remains significant and quantitatively similar to the link predicted by the model and to the link estimated on cross sectional data. Thus, to our previous finding that countries which trade more are more diversified, we can add the finding that countries which exhibit faster growth in trade also experience faster growth in diversification. An important qualification is that the median five year change in diversification in our sample is around 15%

[20]Specifically, our data points include changes of the relevant variables over all possible five year intervals within the period 1990-2004, for all 18 countries of our sample described above. This gives a total of 180 observations. We have also conducted the analysis focusing on changes over three and seven year intervals, and on a larger sample of countries (28 including all World Bank rich countries) and found that results are not significantly affected. These results are available on the authors' web pages.

while the median change in openness is only 2.8%. Thus our estimate of the effect of trade on diversification of around 1 suggests that observed growth in trade over the period 1990-2004 can only explain about one-fifth of the increase in international diversification over the same period. Investigating the causes of the residual growth in diversification is an interesting direction for future research.

5 Sensitivity Analysis

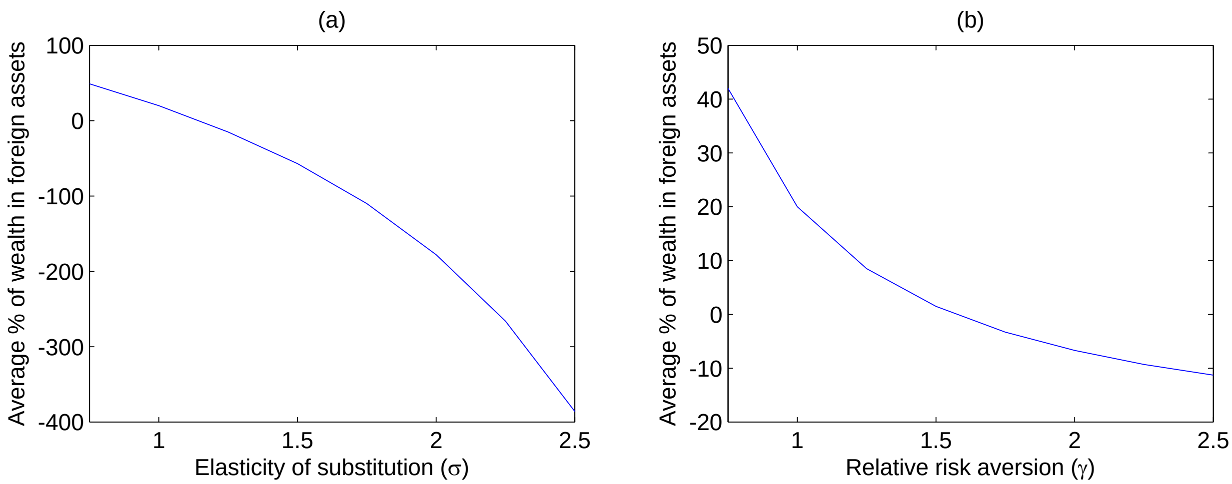
Two key assumptions are required to deliver our closed-form expression for portfolio choice: first, that the elasticity of substitution between traded intermediate goods is unity (so that the G functions are Cobb-Douglas); second, that utility is logarithmic in consumption. We now experiment with relaxing these assumptions. The main finding from these experiments is that a strong bias toward domestic assets is a robust feature of this model: the only case in which home bias disappears is when the elasticity of substitution between domestic and foreign goods is very high, in which case portfolios resemble those in the one-good model.

In order to compute equilibrium country portfolios in a general set-up we use parameters specified in table 1, solve the model numerically, and compute average values for diversification in simulations.[21] Solving for equilibria numerically requires a non-standard numerical method, since standard linearization techniques cannot handle the consumers' portfolio problem. The numerical technique we employ is described in detail in Appendix B.

The Cobb-Douglas aggregator for producing final goods implies a unitary elasticity of substitution between the traded goods a and b. This elasticity is towards the low end of estimates used in the business cycle literature. Panel (a) of figure 2 shows how the average equilibrium level of diversification changes as the elasticity of substitution, σ, is varied from 0.8 to 2.5, given a CES aggregator of the form $G(a,b) = (\omega a^{\frac{\sigma-1}{\sigma}} + (1-\omega) b^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$. The main message of the picture is that for commonly-used elasticities, theory predicts strong (even too strong) home bias. Notice also that increasing substitutability strengthens home bias within this range of values for σ. The logic for this result is that the more substitutable are a and b, the less relative prices change in response to shocks. This means that following a positive domestic shock, the increase in the relative value of domestic labor earnings becomes larger and, at the same time, the decline in relative domestic stock returns becomes smaller. Thus agents must overweight domestic stocks to an even greater

[21] Note that, in general, the share of foreign assets in wealth need not be constant.

Figure 4: Sensitivity analysis



extent in order to hedge such risks.

For very high elasticities (values for σ exceeding 4), price movements become so small that, following a positive domestic shock, returns to domestic stocks exceed returns to foreign stocks, and the correlation between relative labor income and relative domestic stock returns turns positive. For such high elasticities, the two-good model is sufficiently close to the one-good model that its portfolio implications are similar. In particular it is optimal for the individual to hedge against shocks to relative labor income by shorting domestic assets. Thus the average portfolio displays a very strong—and counter-factual—foreign bias.

Panel (b) of figure 4 shows how diversification changes as we relax the log utility assumption, assume utility from consumption of the form $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and vary the coefficient of relative risk aversion, γ. Notice that higher risk aversion leads to higher home-bias. Changing the risk aversion coefficient does not impact two of the equilibrium relationships (equations 29 and 32) developed in Section 3.1. Changing γ does, however, change the pattern of co-movement between domestic and foreign consumption consistent with perfect risk-sharing. In particular, since higher risk aversion corresponds to a lower inter-temporal elasticity of substitution for consumption, γ^{-1}, desired consumption becomes less sensitive to changes in relative prices. Thus in choosing portfolios, agents want to ensure that their total income does not decline too much in periods when domestic productivity falls and the relative price of domestic consumption increases ($e(s^t)$ declines). This pushes agents further towards domestic stocks, whose relative return rises in periods when domestic productivity and earnings decline.

6 Conclusion

In this paper we have shown that standard macroeconomic theory predicts patterns for international portfolio diversification that are broadly consistent with those observed empirically in recent years. The economic model we used to generate theoretical predictions for portfolio choice was a standard two-country two-good version of the stochastic growth model that has been widely used in business cycle research. We conclude that, from the perspective of standard macroeconomic theory, the observed bias towards domestic assets is not a puzzle. We have explored the economics underlying this result, and argued that the dynamics of investment and international relative price movements are central to understanding portfolio choice. We have also provided evidence from developed nations in recent years which is quantitatively consistent with these dynamics that lead to home bias in our model.

Important questions remain. In our analysis we have focussed primarily on the predictions of our model for portfolio diversification. However, it is well known that it is difficult to reconcile many features of asset prices with the predictions of stochastic general equilibrium production economies. A general equilibrium theoretical resolution of these pricing puzzles is required to bridge the gap between the macroeconomic theory and empirical finance literatures on portfolio choice. A somewhat less ambitious task for future work is to build a multi-country version of the model which allows for multi-lateral trade in goods, and multi-lateral diversification in assets. Such a model would lead to a better understanding of the separate roles of size and openness in understanding portfolio diversification. It would also generate richer predictions linking trading patterns to distributions of foreign assets and liabilities by country.

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Appendix A. Proofs

Proof of proposition 1

We prove this result by showing that these portfolios decentralize the solution to an equal-weighted planner's problem in the same environment. In particular, we consider the problem of a planner who seeks to maximize the equally-weighted expected utilities of the domestic and foreign agents, subject only to resource constraints of the form (20) and (21). We then describe a set of candidate prices such that if the conditions that define a solution to the planner's problem are satisfied, then the conditions that define a competitive equilibrium in the stock trade economy are also satisfied when portfolios are given by equation (23). Let $G(s^t)$ and $F(s^t)$ be compact notations for $G(a(s^t), b(s^t))$ and $F\left(z(s^t), k(s^{t-1}), n(s^t)\right)$.

The equations that characterize a solution to the planner's problem are:

1. First order conditions for hours:

$$\begin{aligned} U_c(s^t)\frac{\omega G(s^t)}{a(s^t)}\frac{(1-\theta)F(s^t)}{n(s^t)} + U_n(s^t) &\geq 0 \\ U_c^*(s^t)\frac{\omega G^*(s^t)}{b^*(s^t)}\frac{(1-\theta)F^*(s^t)}{n^*(s^t)} + U_n^*(s^t) &\geq 0 \end{aligned}$$

2. First order conditions for allocating intermediate goods across countries:

$$\begin{aligned} U_c(s^t)\omega G(s^t)/a(s^t) &= U_c^*(s^t)(1-\omega)G^*(s^t)/a^*(s^t) \\ U_c(s^t)(1-\omega)G(s^t)/b(s^t) &= U_c^*(s^t)\omega G^*(s^t)/b^*(s^t) \end{aligned}$$

3. First order conditions for investment:

$$\begin{aligned} \widetilde{Q}(s^t) &= \sum_{s_{t+1}\in S}\widetilde{Q}(s^t,s_{t+1})\left[\frac{\omega G(s^t,s_{t+1})}{a(s^t,s_{t+1})}\frac{\theta F(s^t,s_{t+1})}{k(s^t)} + (1-\delta)\right] \\ \widetilde{Q}^*(s^t) &= \sum_{s_{t+1}\in S}\widetilde{Q}^*(s^t,s_{t+1})\left[\frac{\omega G^*(s^t,s_{t+1})}{b^*(s^t,s_{t+1})}\frac{\theta F^*(s^t,s_{t+1})}{k^*(s^t)} + (1-\delta)\right] \end{aligned}$$

where

$$\begin{aligned} \widetilde{Q}(s^t) &= \frac{1}{2}\pi(s^t)\beta^t U_c(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t)\frac{(1-\omega)}{\omega}\frac{G^*(s^t)}{G(s^t)}\frac{a(s^t)}{a^*(s^t)} \\ \widetilde{Q}^*(s^t) &= \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c(s^t)\frac{\omega}{(1-\omega)}\frac{G(s^t)}{G^*(s^t)}\frac{a^*(s^t)}{a(s^t)} \end{aligned}$$

4. Resource constraints of the form (20) and (21).

Consider the set of allocations that satisfies this set of equations, *i.e.* the solution to the planner's problem. We now show there exists a set of prices at which these same allocations also satisfy the set of equations defining equilibrium in the stock trade economy (see Section 2.5), given the portfolios described in equation (23). In other words, we can decentralize the complete markets allocations with asset trade limited to two stocks and constant portfolios.

Let intermediate-goods prices be given by equation (19). Then condition (1) for the stock trade economy is satisfied. Let wages be given by equations (14) and (15). Then condition (2) for the stock trade economy is satisfied. Substituting these prices into condition (1) from the planner's problem gives condition (3) for the stock trade economy. Let the real exchange rate by given by equation (5). Then combining conditions (2) and (3) from the planner's problem gives condition (4) for the stock trade economy. Condition (4) from the planner's problem translates directly into conditions (5) and (6) for the stock trade economy. Condition (7)—stock market clearing—follows immediately from the symmetry of the candidate stock purchase rules.

Condition (8) is that households' budget constraints are satisfied. Given constant portfolios, the domestic household's budget constraint simplifies to

$$c(s^t) = q_a(s^t)w(s^t)n(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t)$$

Substituting in the candidate function for $w(s^t)$, the resource constraint for intermediate goods, and the definitions for dividends (and suppressing the state-contingent notation) gives

$$c = q_a(1-\theta)(a+a^*) + \lambda\left(q_a\theta(a+a^*) - x\right) + (1-\lambda)e\left(q_b^*\theta(b+b^*) - x^*\right)$$

Using the candidate expression for the real exchange rate gives

$$c = (1-\theta+\lambda\theta)\left(q_a a + e q_a^* a^*\right) - \lambda x + (1-\lambda)\theta\left(q_b b + e q_b^* b^*\right) - (1-\lambda)e x^*$$

Now using the candidate expressions for intermediate goods prices and collecting terms gives

$$c = \left[\omega + (1-\lambda)(\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(\theta - 2\omega\theta\right)\right]G^* - \lambda x - (1-\lambda)e x^*$$

Using the resource constraint for final goods firms gives

$$\begin{aligned} G &= \left[\omega + (1-\lambda)(\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(\theta - 2\omega\theta\right)\right]G^* \\ &\quad + (1-\lambda)(G-c) - (1-\lambda)e(G^* - c^*) \end{aligned}$$

Given the candidate expression for the real exchange rate, and exploiting the assumption that utility is logarithmic in consumption, condition (2) for the planners problem implies

$$c = ec^*.$$

Thus the budget constraint can be rewritten as

$$G = \left[\omega + (1-\lambda)(1+\theta-2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(1+\theta-2\omega\theta\right)\right]G^*$$

Finally substituting in the candidate expression for λ confirms that the domestic consumer's budget constraint is satisfied. The foreign consumer's budget constraint is satisfied by Walras' Law.

Condition (9) is the households' inter-temporal first order conditions for stock purchases. Substituting condition (2) from the planner's problem into condition (3), the planner's first order conditions for investment may be rewritten as

$$\begin{aligned} U_c(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta) \right] \\ U_c^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta) \right] \end{aligned}$$

Multiplying both sides of the first (second) of these two equations by $k(s^t)$ $(k^*(s^t))$ gives

$$\begin{aligned} U_c(s^t)k(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \theta F(s^t, s_{t+1}) + (1-\delta)k(s^t) \right] \\ U_c^*(s^t)k^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \theta F^*(s^t, s_{t+1}) + (1-\delta)k^*(s^t) \right] \end{aligned}$$

Let stock prices by given by

(38) $P(s^t) = k(s^t),\ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.$

Substituting these candidate prices for stocks, the prices for intermediate goods, the wage, and the expressions for dividends into the planner's first order conditions for investment gives the domestic household's first order condition for domestic stock purchases, and the foreign household's first order condition for foreign stock purchases. The remaining two first-order conditions for stock purchases follow immediately by substituting condition (2) from the planner's problem into these two conditions.

PROOF OF COROLLARY 1

Equilibrium portfolios deliver perfect risk-sharing, which implies that the value of consumption is equated across countries, eq. 27. Moreover, since there is no asset trade in equilibrium, the value of income is also equated across countries. Log linearizing,

$$\bar{l}\widehat{l}(s^t) + \lambda \bar{d}\hat{d}(s^t) + (1-\lambda)\bar{d}\hat{d}^*(s^t) = e(s^t) \left(\bar{l}\widehat{l}^*(s^t) + (1-\lambda)\bar{d}\hat{d}(s^t) + \lambda \bar{d}\hat{d}^*(s^t) \right)$$

which implies

$$\bar{l}\Delta\widehat{l}(s^t) = -(2\lambda - 1)\bar{d}\Delta\hat{d}(s^t)$$

which implies

$$cov\left(\Delta\widehat{l}(s^t), \Delta\widehat{l}(s^t)\right) = -\frac{\bar{d}}{\bar{l}}(2\lambda-1)var(\Delta d(s^t))$$

$$1-\lambda \approx \frac{1}{2}\left(1+\frac{\bar{l}}{\bar{d}}\frac{cov\left(\hat{l}(s^t),\Delta\hat{d}(s^t)\right)}{var\left(\Delta\hat{d}(s^t)\right)}\right)$$
$$= \frac{1}{2}\left(1+\left(\frac{1-\theta}{\theta}\frac{r+\delta}{r}\right)\frac{cov\left(\hat{l}(s^t),\Delta\hat{d}(s^t)\right)}{var\left(\Delta\hat{d}(s^t)\right)}\right)$$

Appendix B. Computational algorithm

Here we describe the algorithm that allows us to solve for equilibrium portfolio holdings in the generalized version of the model described in Section 4. By generalized, we mean parameterizations for which Proposition 1 does not apply, and for which portfolios must be characterized numerically. Our algorithm can be used to solve for equilibria in more general international macro models with portfolio choice, and thus it complements the recent work of Devereux and Sutherland (2006), Tille and van Wincoop (2007), and Evans and Hnatkovska (2007). Matlab programs that implement this algorithm are available on the authors' websites. We now outline the steps of the algorithm.

Step 1. Pick a non-stochastic symmetric steady state equilibrium (i.e. an equilibrium in which agents know that productivities $z(s^t), z^*(s^t)$ are constant and equal to 0). We denote such a steady state with the vector $[\lambda_H, \lambda_F^*, X, Y]$, where $\lambda_H, \lambda_F^* \in R$ are the fractions of local stocks held by home and foreign residents, respectively, $X \in R^n$ is the vector of non portfolio state variables (i.e. productivities and capital stock), while $Y \in R^m$ is the vector of non portfolio control variables (i.e. consumption, investment, terms of trade etc.). Notice that first order conditions plus symmetry uniquely pin down X and Y, while any value $\lambda_0 = \lambda_H = \lambda_F^*$ is a non-stochastic symmetric steady state equilibrium.

Step 2. Compute decision rules $\lambda_{H,t+1} = g_1(\lambda_{H,t}, \lambda_{F,t}^*, X_t)$, $\lambda_{H^*,t+1} = g_2(\lambda_{H,t}, \lambda_{F,t}^*, X_t)$, $X_{t+1} = g_3(\lambda_{H,t}, \lambda_{F,t}^*, X_t, \varepsilon_{t+1})$, $Y_t = g_4(\lambda_{H,t}, \lambda_{F,t}^*, X_t)$ that characterize the solution to a second-order approximation of the stochastic economy around the steady state. The functions g_1, g_2 , g_3, and g_4 are quadratic forms in their arguments and can be computed using the methods described by Schmitt-Grohe and Uribe (2004) or Gomme and Klein (2006) among others. Note that in order to apply those methods here it is necessary to slightly modify the model by adding a small adjustment cost for changing the portfolio from its steady state value. This step yields decision rules for all variables (including portfolio decisions) that are correct up to a second-order approximation, in a neighborhood of the steady state around which the economy is linearized. However, we do not yet know whether the steady state portfolio λ_0 we started with is equal to the average equilibrium portfolio in the true stochastic economy.

Step 3: Starting from our guess for the steady state, simulate the model for a large number of periods using the decision rules from Step 2, and compute the average share of wealth held by domestic agents along the simulation. If this average share is different from the initial steady state share, we set the new guess for the steady state portfolio, λ_1, equal to the average simulated share and return to Step 1. If the simulated average is equal (up to a small tolerance error) to the initial steady state λ_0, then λ_0 constitutes a good approximation of the long run portfolio holdings and we take it as the solution to our portfolio problem.

As a test, we apply this method to our benchmark parameterization and to the one-good model of Baxter and Jermann (for both these cases we know the true portfolio solution). In both cases our algorithm converges very rapidly to the true solution, regardless of the initial guess. We also find that the portfolio adjustment costs can be set to an arbitrarily small (but positive) number, such that changing the size of these costs locally (e.g. doubling their size) does not affect the solution.

Appendix C. Data

The countries we use in our empirical analysis are the set of countries which have been in the OECD throughout the period 1990-2004 and for which we could get consistent and comparable series for quarterly GDP, investment (needed to construct the covariance ratios M), for foreign assets, trade and capital stock. This group includes the following 18 countries (the codes used in Figures 2 and 3 are in parentheses): Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Japan (JPN), Netherlands (NLD), Norway (NOR), Spain (ESP), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR), United States (USA). The data on gross international diversification positions (total foreign assets and foreign liabilities) are in US dollars and are from Lane and Milesi-Ferretti (2006).

We denote by $K_{i,t}$ our measure (in US dollars) for the capital stock in country i in period t. We construct $K_{i,t}$ by multiplying GDP in US dollars (as reported by Lane and Milesi Ferretti, 2006) by the capital-output ratio. The capital-output ratio is computed as follows: in 1989 we take it directly from Dhareshwar and Nehru (1993), who report both physical capital stock and GDP figures. After 1989 we construct the capital-output ratio in period $t+1$ for country i, $\left(\frac{k}{y}\right)_{i,t+1}$ using the following recursion:

$$\left(\frac{k}{y}\right)_{i,t+1}\left(\frac{y_{i,t+1}}{y_{i,t}}\right) = (1-\delta)\left(\frac{k}{y}\right)_{i,t} + \left(\frac{x}{y}\right)_{i,t}$$

where $\left(\frac{y_{i,t+1}}{y_{i,t}}\right)$ is the growth rate of GDP for country i (PPP adjusted, in constant prices (chain method) from the Penn World Tables 6.2, see Heston and al. (2006)), $\left(\frac{x}{y}\right)_{i,t}$ is the ratio between investment and GDP (both PPP adjusted, in current prices, from Penn World Tables 6.2), δ is the depreciation rate which, in the absence of better information, we set equal to 6% (this value is also used by Kraay et al. 2005) for all countries and for all years. The complete dataset is available online on the authors' websites.