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The International Diversification Puzzle Is Not as Bad as You Think*

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Abstract

The international diversification puzzle is the fact that country portfolios are on average biased toward domestic assets, while one-good international macro models with non-diversifiable labor income risk predict the opposite pattern of diversification. This paper embeds a portfolio choice decision in a two-good international business cycle model and provides a closed form solution for equilibrium country portfolios. Equilibrium portfolios are biased toward domestic assets because endogenous international relative price fluctuations make domestic assets a good hedge against labor income risk. Evidence from developed economies in recent years is qualitatively and quantitatively consistent with the mechanisms highlighted by the theory.

KEYWORDS: Country portfolios, International business cycles, Home bias

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1 Introduction

Although international portfolio diversification has increased in recent years, portfolios in many countries remain heavily biased toward domestic assets. For example, foreign assets accounted, on average, for only around 32 percent of the total value of the assets owned by US residents over the period 1990-2007. A large literature concludes that observed low levels of international diversification are puzzling from the viewpoint of standard one-good macro models (see, for example, Baxter and Jermann 1997 and Lewis 1999).

We revisit this issue in the context of a workhorse model for quantitative international macroeconomics: the two-country, two-good extension of the stochastic growth model developed by Backus, Kehoe, and Kydland (1994 and 1995, henceforth BKK). In Heathcote and Perri (2004), we developed a variant of that model with international stock trade to study the business cycle implications of increasing financial integration. Here, we use the same variant to study in detail the mechanics of international diversification and to argue that the economic mechanisms incorporated in this model help rationalize observed levels of diversification across a large set of countries.

The theoretical section of the paper relates and interprets two alternative closed-form expressions for equilibrium diversification that emerge in this economy. We first show that equilibrium country portfolios can be expressed in terms of the covariance between relative domestic-foreign labor income and relative asset income (dividends). This portfolio expression relates directly to the large literature that takes the perspective of an individual investor trying to smooth consumption, and notes that the optimal mix between domestic and foreign stocks depends on the covariance between returns to nondiversifiable human capital and (relative) returns to traded equity (see, for example, Baxter and Jermann 1997). One difference is that our expression relates to income to labor and capital, rather than returns. This simplifies the task of measuring the key covariance in the data, since one does not need a model for unobserved returns to human capital.

In our model, labor and capital income and international relative prices are all determined jointly in general equilibrium, which adds valuable structure to how they covary. Following Heathcote and Perri (2004) we exploit this structure to derive an alternative expression for equilibrium portfolios in terms of only two structural parameters: capital's share in production and the share of foreign inputs in final demand. We develop economic intuition for this result by tracing how country-specific productivity shocks drive capital investment and movements in international relative prices that complement explicit insurance via direct holdings of foreign assets.

To better understand the predictions of our model for portfolio choice, we compare and contrast our economy to those considered by Lucas (1982), Baxter and Jermann (1997), and Cole and Obstfeld (1991). Lucas (1982) studies a two-country world in which residents of each country share common preferences and are endowed with a tree yielding stochastic fruits. He shows that perfect risk pooling, in general, involves agents of each country owning half the claims to the home endowment and half the claims to the foreign endowment. Baxter and Jermann (1997) extend Lucas' model in one direction by introducing nondiversifiable labor income. They show that if asset returns and labor income are highly correlated within a country, then agents can hedge nondiversifiable labor income risk with a large short position in domestic assets (i.e., aggressive diversification).

Cole and Obstfeld (1991) instead argue that in a special case of the Lucas model, diversification is not required to achieve risk sharing. Their insight is that if the fruits yielded by the two trees are imperfect substitutes, then changes in relative endowments induce offsetting changes in the terms of trade. When preferences are log separable between the two goods, the terms of trade responds one-for-one to changes in relative income, effectively delivering perfect risk sharing. Thus, in sharp contrast to the results of Lucas (1982) or Baxter and Jermann (1997), any level of diversification is consistent with complete risk pooling, including portfolio autarky.[1]

One important difference in our analysis relative to Baxter and Jermann (1997) is that we allow for imperfect substitutability between domestic and foreign-produced traded goods. Thus, in our model, changes in international relative prices provide some insurance against country-specific shocks and, in the flavor of the Cole and Obstfeld (1991) indeterminacy result, portfolio choice does not have to do all the heavy lifting when it comes to delivering perfect risk sharing. This mechanism is consistent with a large body of empirical evidence which studies the response of international relative prices to productivity shocks.[2] In contrast to Cole and Obstfeld (1991), however, the presence of production and particularly investment in our model means that returns to domestic and foreign stocks are not automatically equated, and thus agents face an interesting portfolio choice problem. Home bias arises because relative returns to domestic stocks move inversely with relative returns to labor in response to productivity shocks. The mechanism through which this

[1]Kollmann (2006) considers a two-good endowment economy with more general preferences. He finds that equilibrium diversification is sensitive to both the intratemporal elasticity of substitution between traded goods and the intertemporal elasticity of substitution for the aggregate consumption bundle.

[2]See, for example, Acemoglu and Ventura (2002), Debaere and Lee (2004), and Pavlova and Rigobon (2007). These papers use different methodologies to identify productivity shocks in a cross section of countries, and find support for the notion that gains in relatively productivity are associated with offsetting deteriorations in the terms of trade. For the United States the evidence is more mixed: Corsetti, Dedola, and Leduc (2008a) find no evidence of this mechanism, whereas Basu, Fernald, and Kimball (2006) find that in response to US productivity growth, the US real exchange rate depreciates strongly.

covariation arises is novel and is due jointly to international relative price movements and to the presence of capital.

Our empirical application documents the quantitative success of the theory in accounting for observed patterns of diversification within OECD countries in recent years. First, we examine the prediction from our structural expression for equilibrium home bias, which is that countries that are more open to trade should be more diversified. We find empirical support for this prediction. Second, we move to examine the mechanism underlying this relationship by exploiting the dual expressions for home bias: the structural expression involving trade shares, and the risk-hedging expression involving the covariance between relative labor income and relative capital income. In particular, our theory predicts that countries that are relatively closed – and thus ought to be relatively home-biased according to the structural expression – should also exhibit a relatively large negative covariance between relative earnings and relative dividends. The more negative is this covariance, the more attractive are domestic shocks as a hedge against labor income risk, thereby rationalize stronger home bias from the perspective of individual price-taking investors. We test this mechanism in two steps. First, we explore whether greater openness to trade is in fact associated with a more negative empirical covariance between relative earnings and relative dividends. Second, we explore whether a more negative covariance is indeed associated with stronger home bias. In both cases we find the evidence is consistent with the theory. This set of tests suggests that the positive relationship between trade and diversification we document is not coincidental, but instead reflects the fact that the volume of trade has systematic implications for the joint dynamics of earnings and dividends that are at the heart of optimal portfolio choice.

Our baseline model parameterization assumes a unitary elasticity of substitution between domestic and foreign-produced goods, and preferences that are logarithmic in a composite final consumption good. Given these assumptions, equilibrium portfolios in the model can be characterized in closed-form. In a sensitivity analysis we use numerical methods to consider the implications for diversification of relaxing both these assumptions. In particular we consider alternative values for the elasticity of substitution between domestic and foreign-produced goods, and alternative values for the intertemporal elasticity of substitution for the composite consumption good.

While the paper shows that a workhorse international model can account for observed portfolio diversification, many important features of international macroeconomic data remain unexplained. In particular, this model does not provide a satisfactory explanation for the volatility and persistence of international relative prices and their comovement with relative quantities (see Backus and

Smith, 1993). In addition, in common with standard production economies, asset prices are much less volatile than the prices of traded equity.

In the next section we describe the basic model and derive equilibrium portfolios. Section 3 offers some intuition for those portfolios. Section 4 contains the empirical analysis, and Section 5 describes the sensitivity analysis. Section 6 concludes. A definition of equilibrium, proofs, and a description of the data are in the appendices.

2 The model

We begin this section by describing the environment (which follows BKK 1994, 1995) and the asset market structure. We then derive the two key theoretical results on international diversification. The first (also derived in Heathcote and Perri 2004) relates diversification to structural parameters of the economy. The second relates diversification to comovement between relative labor income and relative asset income.

There are two countries, each of which is populated by the same measure of identical, infinitely lived households. Firms in each country use country-specific capital and labor to produce an intermediate good. The intermediate good produced in the domestic country is labeled a, and the good produced in the foreign country is labeled b. These are the only traded goods in the world economy. Intermediate-goods-producing firms are subject to country-specific productivity shocks. Within each country, the intermediate goods a and b are combined to produce country-specific final consumption and investment goods. The final goods production technologies are asymmetric across countries, in that they are biased toward using a larger fraction of the locally produced intermediate good. This bias allows the model to replicate empirical measures for the volume of trade relative to GDP.

The assets that are traded internationally are shares in the domestic and foreign representative intermediate-goods-producing firms. These firms make investment and employment decisions, and distribute non-reinvested earnings to shareholders.

2.1 Preferences and technologies

In each period t, the economy experiences one event $s_t \in S$. We denote by $s^t = (s_0, s_1, ..., s_t) \in S^t$ the history of events from date 0 to date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t)$.

Period utility for a household in the domestic country after history s^t is given by[3]

$$(1) \quad U\left(c(s^t), n(s^t)\right) = \log c(s^t) - V\left(n(s^t)\right),$$

where $c(s^t)$ denotes consumption at date t given history s^t, and $n(s^t)$ denotes labor supply. Disutility from labor is given by the positive, increasing, and convex function V. The assumption that utility is log separable in consumption will play a role in deriving a closed-form expression for equilibrium portfolios in our baseline calibration of the model. In contrast, the equilibrium portfolio in this case will not depend on the particular functional form for V.

Households supply labor to domestically located, perfectly competitive intermediate-goods-producing firms. Intermediate goods firms in the domestic country produce good a, and those in the foreign country produce good b. These firms hold the capital in the economy and operate a Cobb-Douglas production technology:

$$(2) \quad F\left(z(s^t), k(s^{t-1}), n(s^t)\right) = e^{z(s^t)} k(s^{t-1})^{\theta} n(s^t)^{1-\theta},$$

where $z(s^t)$ is an exogenous productivity shock. The vector of shocks $[z(s^t), z^*(s^t)]$ evolves stochastically. For now, the only assumption we make about this process is that it is symmetric. Each period, households receive dividends from their stock holdings in the domestic and foreign intermediate goods firms, and buy and sell shares to adjust their portfolios.

Final-goods-producing firms are perfectly competitive and produce final goods using intermediate goods a and b as inputs to a Cobb-Douglas technology:

$$(3) \quad G\left(a(s^t), b(s^t)\right) = a(s^t)^{\omega} b(s^t)^{(1-\omega)}, \qquad G^*\left(a^*(s^t), b^*(s^t)\right) = a^*(s^t)^{(1-\omega)} b^*(s^t)^{\omega},$$

where $\omega > 0.5$ determines the size of the local input bias in the composition of domestically produced final goods.

Note that the Cobb-Douglas assumption implies a unitary elasticity of substitution between domestically produced goods and imports. The Cobb-Douglas assumption, in conjunction with the assumption that utility is logarithmic in consumption, will allow us to derive a closed-form expression for equilibrium portfolios. Note, however, that a unitary elasticity is within the range of existing estimates: BKK (1994) set this elasticity to 1.5 in their benchmark calibration, whereas

[3]The equations describing the foreign country are largely identical to those for the domestic country. We use asterisks to denote foreign variables.

Heathcote and Perri (2002) estimate the elasticity to be 0.9. In a sensitivity analysis, we will explore numerically the implications of deviating from the logarithmic utility, unitary elasticity baseline.

Let $q_a(s^t)$ and $q_b(s^t)$ denote the prices of goods a and b relative to the domestic final good, and let $q_a^*(s^t)$ and $q_b^*(s^t)$ denote the analogous prices relative to the foreign final good. The terms of trade, $p(s^t)$, is the price of good b relative to the price of good a. Because the law of one price applies to traded intermediate goods, this relative price is the same in both countries:

$$(4) \quad p(s^t) = \frac{q_b(s^t)}{q_a(s^t)} = \frac{q_b^*(s^t)}{q_a^*(s^t)}$$

Let $e(s^t)$ denote the real exchange rate, defined as the price of foreign consumption relative to domestic consumption. By the law of one price, $e(s^t)$ can be expressed as the domestic price of good a (or good b) relative to domestic consumption divided by the foreign price of good a (or b) relative to foreign consumption:

$$(5) \quad e(s^t) = \frac{q_a(s^t)}{q_a^*(s^t)} = \frac{q_b(s^t)}{q_b^*(s^t)}.$$

2.2 Households' problem

The budget constraint for the domestic household is given by

$$(6) \qquad \begin{aligned} & c(s^t) + P(s^t)\left(\lambda_H(s^t) - \lambda_H(s^{t-1})\right) + e(s^t)P^*(s^t)\left(\lambda_F(s^t) - \lambda_F(s^{t-1})\right) \\ = \; & l(s^t) + \lambda_H(s^{t-1})d(s^t) + e(s^t)\lambda_F(s^{t-1})d^*(s^t) \qquad \forall t \geq 0, s^t. \end{aligned}$$

Here $P(s^t)$ is the price at s^t of (ex dividend) shares in the domestic firm in units of domestic consumption, $P^*(s^t)$ is the price of shares in the foreign firm in units of foreign consumption, $\lambda_H(s^t)$ ($\lambda_H^*(s^t)$) denotes the fraction of the domestic firm purchased by the domestic (foreign) agent, $\lambda_F(s^t)$ ($\lambda_F^*(s^t)$) denotes the fraction of the foreign firm bought by the domestic (foreign) agent, $d(s^t)$ and $d^*(s^t)$ denote domestic and foreign dividend payments per share, and $l(s^t) = q_a(s^t)w(s^t)n(s^t)$ denotes domestic labor earnings, where $w(s^t)$ is the wage in units of the domestically produced intermediate good. The budget constraint for the foreign household is

$$(7) \qquad \begin{aligned} & c^*(s^t) + P^*(s^t)\left(\lambda_F^*(s^t) - \lambda_F^*(s^{t-1})\right) + (1/e(s^t))P(s^t)\left(\lambda_H^*(s^t) - \lambda_H^*(s^{t-1})\right) \\ = \; & l^*(s^t) + \lambda_F^*(s^{t-1})d^*(s^t) + (1/e(s^t))\lambda_H^*(s^{t-1})d(s^t) \qquad \forall t \geq 0, s^t. \end{aligned}$$

We assume that at the start of period 0, the domestic (foreign) household owns the entire domestic (foreign) firm: thus $\lambda_H(s^{-1}) = 1$, $\lambda_F(s^{-1}) = 0$, $\lambda_F^*(s^{-1}) = 1$ and $\lambda_H^*(s^{-1}) = 0$.

At date 0, domestic households choose $\lambda_H(s^t)$, $\lambda_F(s^t)$, $c(s^t) \geq 0$, and $n(s^t) \in [0,1]$ for all s^t and for all $t \geq 0$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t U\left(c(s^t), n(s^t)\right)$$

subject to (6) and a no-Ponzi-game condition. The problem for foreign households is analogous.

2.3 Firm's problem

The domestic intermediate goods firm's maximization problem is to choose $k(s^t) \geq 0$, $n(s^t) \geq 0$ for all s^t and for all $t \geq 0$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) d(s^t),$$

taking as given $k(s^{-1})$, where $Q(s^t)$ is the price the firm uses to value dividends at s^t relative to consumption at date 0, and dividends (in units of the final good) are given by

$$(8) \quad d(s^t) = q_a(s^t)\left[F\left(z(s^t), k(s^{t-1}), n(s^t)\right) - w(s^t)n(s^t)\right] - \left[k(s^t) - (1-\delta)k(s^{t-1})\right],$$

where δ is the depreciation rate for capital. Analogously, foreign firms use prices $Q^*(s^t)$ to price dividends in state s^t, where foreign dividends are given by

$$(9) \quad d^*(s^t) = q_b^*(s^t)\left[F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right) - w^*(s^t)n^*(s^t)\right] - \left[k^*(s^t) - (1-\delta)k^*(s^{t-1})\right].$$

The state-contingent consumption prices $Q(s^t)$ and $Q^*(s^t)$ play a role in intermediate goods firms' state-contingent decisions regarding how to divide earnings between investment and dividend payments. We assume that firms use the discount factor of the representative domestic household to price the marginal cost of foregoing current dividends in favor of extra investment:[4]

[4] We have also computed equilibrium allocations when the firm puts positive weight on the discount factor of the foreign representative household. Under the baseline calibration of the model, the stochastic discount factors of the domestic and foreign households are equalized in equilibrium, and hence the solution to the firm's problem is the same for any state-contingent prices that are weighted averages of the two discount factors. More generally, for all the parameterizations we considered, we found that the two discount factors are highly positively correlated, and thus results are quantitatively insensitive to the choice of weights.

$$(10) \quad Q(s^t) = \frac{\pi(s^t)\beta^t U_c(s^t)}{U_c\left(s^0\right)}, \qquad Q^*(s^t) = \frac{\pi(s^t)\beta^t U_c^*(s^t)}{U_c^*\left(s^0\right)}.$$

The final-goods-producing firms buy intermediate goods, and sell the final consumption-investment good. Their static maximization problem in the domestic country after history s^t is

$$\max_{a(s^t),b(s^t)} \left\{G(a(s^t), b(s^t)) - q_a(s^t)a(s^t) - q_b(s^t)b(s^t)\right\}$$

subject to $a(s^t)$, $b(s^t) \geq 0$.

In Appendix A we describe the first-order conditions for the household and firm problems, and define a competitive equilibrium.

2.4 Equilibrium portfolios

PROPOSITION 1: (Heathcote and Perri 2004) Suppose that at time zero, productivity is equal to its unconditional mean value in both countries ($z(s^0) = z^*(s^0) = 0$) and that initial capital is equalized across countries, $k(s^{-1}) = k^*(s^{-1}) > 0$. Then there is an equilibrium in this economy with the property that portfolios in both countries exhibit a constant level of diversification, denoted $1 - \lambda$, given by

$$\begin{aligned}(11) \quad \lambda_F(s^t) = \lambda_H^*(s^t) = 1 - \lambda_H(s^t) = 1 - \lambda_F^*(s^t) \quad &= \quad 1 - \lambda \\ &= \quad \left(\frac{1-\theta}{1-\omega} + 2\theta\right)^{-1} \qquad \forall t, s^t.\end{aligned}$$

Moreover, in this equilibrium stock prices are given by

$$(12) \quad P(s^t) = k(s^t), \ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t$$

and equilibrium allocations are efficient (i.e., perfect risk sharing is achieved).

PROOF: See Appendix B.

PROPOSITION 2: Let $\Delta\widehat{l}(s^t) = \log\left(l(s^t)\right) - \log(e(s^t)) - \log\left(l^*(s^t)\right)$ denote relative log labor earnings in units of the domestic final good. Similarly, let $\Delta\hat{d}(s^t)$ denote relative log dividends. Let $M(s^t) = cov(\Delta\hat{l}(s^t), \Delta\hat{d}(s^t))/var(\Delta\hat{d}(s^t))$ denote the ratio of the equilibrium conditional covariance between relative log earnings and relative log dividends to the variance of relative log dividends.

Then

$$(13)\quad M(s^t) \approx M = \left[2\left(\frac{1-\theta}{1-\omega}+2\theta\right)^{-1}-1\right]\left(\frac{\theta}{1-\theta}\frac{\rho}{\rho+\delta}\right) \qquad \forall t, s^t,$$

where $\rho = (1-\beta)/\beta$. Moreover, equilibrium diversification can be expressed as a function of this covariance ratio:

$$(14)\quad 1-\lambda \approx \frac{1}{2}\left(1+\left(\frac{1-\theta}{\theta}\frac{\rho+\delta}{\rho}\right)M\right).$$

PROOF: See Appendix B.

Propositions 1 and 2 summarize our theory of international diversification by establishing links between diversification $(1-\lambda)$, trade $(1-\omega)$, and comovement between relative labor income and relative dividends (M). In the next two sections, we first provide some intuition for these results and then apply the results to show that our theory can explain observed diversification in developed economies in recent years.

3 Intuition for the results

First, we take a general equilibrium perspective and combine a set of equilibrium conditions that link differences between domestic and foreign aggregate demand and aggregate supply in this economy. These equations shed light on how changes in relative prices coupled with modest levels of international portfolio diversification allow agents to achieve perfect risk sharing.

We then take a more micro agent-based perspective and explore why, from a price-taking individual's point of view, there are no incentives to trade stocks after date 0, why the covariance between relative (domestic to foreign) earnings and relative dividends is the key driver of portfolio choice, and why portfolios are home biased on average.

3.1 Risk-sharing intuition

We now develop three equations that elucidate how the portfolio in eq. (11) delivers perfect risk sharing.

The first equation is the hallmark condition for complete international risk sharing, relating relative marginal utilities from consumption to the international relative price of consumption. Since the utility function is log separable in consumption, this condition is simply

$$(15) \quad c(s^t) = e(s^t)c^*(s^t) \ \forall s^t,$$

which we can write more compactly as $\Delta c(s^t) = 0$, where $\Delta c(s^t)$ denotes the difference between domestic and foreign consumption in units of the domestic final good.

The second equation uses budget constraints to express the difference between domestic and foreign consumption as a function of relative investment and relative GDP.

Let $y(s^t) = q_a(s^t)F\left(z(s^t), k(s^{t-1}), n(s^t)\right)$ denote domestic GDP, and let $x(s^t) = k(s^t) - (1 - \delta)k(s^{t-1})$ denote investment, both in units of the domestic final good. Assuming constant portfolios, domestic consumption is given by

$$\begin{aligned} c(s^t) &= l(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t) \\ (16) \qquad &= (1-\theta)y(s^t) + \lambda\left(\theta y(s^t) - x(s^t)\right) + (1-\lambda)e(s^t)\left(\theta y^*(s^t) - x^*(s^t)\right) \end{aligned}$$

where the second line follows from the definitions for dividends and the assumption that the intermediate goods production technology is Cobb-Douglas in capital and labor. Analogously,

$$(17) \quad c^*(s^t) = (1-\theta)y^*(s^t) + \lambda\left(\theta y^*(s^t) - x^*(s^t)\right) + (1-\lambda)(1/e(s^t))\left(\theta y(s^t) - x(s^t)\right).$$

Then,

$$(18) \quad \Delta c(s^t) = (1 - 2(1-\lambda)\theta)\Delta y(s^t) + (1-2\lambda)\Delta x(s^t),$$

where $\Delta y(s^t)$ and $\Delta x(s^t)$ are the differences between domestic and foreign GDP and investment in units of the domestic final good. Note that in the case of complete home bias ($\lambda = 1$), the relative value of consumption across countries would simply be the difference between relative output and relative investment. For $\lambda < 1$, financial flows mean that some fraction of changes in relative output and investment are financed by foreigners.

Equations (15) and (18) do not depend on the elasticity of substitution between traded goods and can therefore be applied unchanged to the one-good models that have been the focus of much of the previous work on portfolio diversification (in a one-good model $e(s^t) = 1$). For example, Baxter and Jermann (1997) study a one-good economy with production. They argue that since the Cobb-Douglas technology implies correlated returns to capital and labor, agents can effectively diversify nondiversifiable country-specific labor income risk by aggressively diversifying claims to capital. Assuming no investment, so that $\Delta x(s^t) = 0$, achieving perfect risk sharing (i.e., $\Delta c(s^t) = 0$) means picking a value for λ such that the coefficient on $\Delta y(s^t)$ in eq. (18) is zero. The implied value for diversification is $1 - \lambda = 1/(2\theta)$, which is the portfolio described by eq. (2) in Baxter and Jermann (1997). If capital's share θ is set to one-third, the value for $1 - \lambda$ that delivers equal consumption in the two countries is 1.5. Thus, as Baxter and Jermann (1997) emphasize, the risk pooling portfolio involves a negative position in domestic assets.[5]

Our model enriches the Baxter and Jermann (1997) analysis along two dimensions. First, we explicitly endogenize investment. With stochastic investment, eq. (18) indicates that, in general, no constant value for λ will deliver $\Delta c(s^t) = 0$. Thus, in a one-good model, perfect risk sharing is not achievable with constant portfolios. However, our second extension relative to Baxter and Jermann (1997) is to assume that the two countries produce different traded goods that are imperfect substitutes when it comes to producing the final consumption-investment good. As we now explain, the Cobb-Douglas technology we assume for combining these traded goods implies an additional equilibrium linear relationship between $\Delta y(s^t)$, $\Delta c(s^t)$, and $\Delta x(s^t)$—our third key equation—such that perfect risk sharing can be resurrected given appropriate constant portfolios.

From eqs. (5), (35), and (36), domestic GDP (in units of the final good) is given by

$$\begin{aligned}(19)\quad y(s^t) &= q_a(s^t)\left(a(s^t) + a^*(s^t)\right) = q_a(s^t)a(s^t) + e(s^t)q_a^*(s^t)a^*(s^t) \\ &= \omega G(s^t) + e(s^t)(1-\omega)G^*(s^t).\end{aligned}$$

Similarly, foreign GDP is given by

$$(20)\quad y^*(s^t) = (1/e(s^t))(1-\omega)G(s^t) + \omega G^*(s^t).$$

Combining the two expressions above, $\Delta y(s^t)$, the difference between the value of domestic and

[5] Note that eq. (18) suggests that there will always exist a portfolio that delivers perfect risk sharing as long as $\Delta x(s^t)$ is strictly proportional to $\Delta y(s^t)$. Thus, as an alternative to assuming $\Delta x(s^t) = 0$, we could assume, for example, that firms invest a fixed fraction of output, so that $x(s^t) = \kappa y(s^t)$. In this case, in a one-good world, $\Delta x(s^t) = \kappa \Delta y(s^t)$. Now consumption equalization requires that $\Delta c(s^t) = [(1 - 2(1-\lambda)\theta) + (1-2\lambda)\kappa]\,\Delta y(s^t) = 0$, which implies that $1 - \lambda = (1-\kappa)/(2\,(\theta - \kappa))$.

As an example, if the investment rate κ is equal to 0.2 and capital's share is 1/3, the value for $1-\lambda$ that delivers consumption equalization is 3.0, implying an even larger short position in domestic assets than the one predicted by Baxter and Jermann (1997).

foreign GDP, is a linear function of relative absorption:

$$
\begin{aligned}
(21)\quad \Delta y(s^t) &= (2\omega-1)\left(G(s^t)-e(s^t)G^*(s^t)\right) \\
&= (2\omega-1)\left(\Delta c(s^t)+\Delta x(s^t)\right).
\end{aligned}
$$

This equation indicates that changes to relative domestic versus foreign demand for consumption or investment automatically change the terms of trade and thus, holding supply constant, the relative value of output. The fact that countries devote a constant fraction of total final expenditure to each of the two intermediate goods means that the size of the effect is proportional to the change in demand, where the constant of proportionality is $(2\omega-1)$. When the technologies for producing domestic and foreign final goods are the same ($\omega = 0.5$), changes to relative demand do not impact the relative value of the outputs of goods a and b. When final goods are produced only with local intermediates ($\omega = 1$), an increase in domestic demand translates into an equal-sized increase in the relative price of good a . For intermediate values for ω, the stronger the preference for home-produced goods, the larger the impact on the relative value of domestic output.

Note that this equation is independent of preferences and the asset market structure, and follows solely from our Cobb-Douglas assumption, implying a unitary elasticity of substitution between the two traded goods.

We can now combine our three key equations, (15), (18), and (21), to explore the relationship between portfolio choice, relative price movements, and international risk sharing. We start by substituting (21) into (18) to express the difference in consumption as a function solely of the difference in investment, yielding

$$
(22)\quad \Delta c(s^t) \propto -\Delta x + \underbrace{2(1-\lambda)\Delta x(s^t)}_{direct\ foreign\ financing} + \underbrace{(2\omega-1)\left(1-2(1-\lambda)\theta\right)\Delta x(s^t)}_{indirect\ foreign\ financing}.
$$

There is a unique value for λ such that the right-hand side of (22) is always equal to zero. In particular, simple algebra confirms that this value is defined in Proposition 1 (eq. 11).[6]

We can use equation (22) to understand who pays for a change in relative investment, $\Delta x(s^t)$, and how this varies with diversification, $1-\lambda$. Foreigners finance part of the cost directly in their

[6] In our model, ω is the share of the domestic intermediate goods in both consumption and investment. The reader might wonder how the expression for equilibrium portfolios would differ if one allowed this parameter to take different values in separate aggregators for consumption versus investment goods. It is easy to extend the model in this fashion to allow for differential trade intensity. The only relevant parameter for portfolio choice turns out to be the relative share of domestic versus imported intermediates in the production of investment goods.

role as part-owners of domestic firms (the term labelled "direct foreign financing", which is positive as long as $1-\lambda > 0$). They also finance part of the cost indirectly in equilibrium, because a change in relative investment induces a change in relative prices in favor of domestic households (the term labelled "indirect foreign financing"). The value of $1-\lambda$ that delivers perfect risk sharing is the one for which direct and indirect foreign financing completely cover the cost of investment, so that relative consumption is unchanged.

What is the intuition for the indirect foreign financing term? When preferences are biased towards domestically produced goods $(\omega > 0.5)$, an increase in $\Delta x(s^t)$ increases the relative value of domestic output in proportion to the factor $(2\omega - 1)$ (see eq. 21). This captures the fact that increased relative demand for domestic final goods improves the terms of trade for the domestic economy. The fraction of this additional output that accrues as income to domestic shareholders is given by the term $(1 - 2(1-\lambda)\theta)$, which in turn amounts to labor's share of income, $(1-\theta)$, plus the difference between domestic and foreign shareholders' claims to domestic capital income, $(\lambda\theta - (1-\lambda)\theta)$. This indirect effect is positive as long as $1-\lambda < 1/(2\theta)$, reflecting the fact that an increase in domestic investment increases the relative value of domestic earnings. Risking-pooling portfolios are home biased precisely because the indirect effect of an increase in relative domestic investment generally favors domestic residents. Thus these agents need to pay most of the direct costs of additional investment (by holding most of domestic equity) in order to equalize income and consumption across countries.

Equations (18) and (21) can also be readily applied to understand diversification and risk sharing in other environments.

Lucas (1982) considers a two-good endowment economy in which domestic and foreign agents have identical preferences. In this case, it is immediate that perfect risk pooling is achieved when agents hold 50 percent of both domestic and foreign shares in each period, i.e., $1-\lambda = 0.5$.[7] We get the same result from eq. (18) when $\theta = 1$ and $\Delta x(s^t) = 0$ for all s^t.

Cole and Obstfeld (1991) show that if domestic and foreign agents have symmetric log-separable preferences (like ours) for the two goods, then a regime of portfolio autarky (100 percent home bias

[7] Cantor and Mark (1988) extend Lucas' analysis to a simple environment with production. However, they make several assumptions that ensure that their economy inherits the properties of Lucas' economy. In particular, (i) domestic and foreign agents have the same log-separable preferences over consumption and leisure, (ii) productivity shocks are assumed to be i.i.d. through time, (iii) firms must purchase capital and rent labor one period before production takes place, and (iv) there is 100 percent depreciation. When their two economies are the same size, assumptions (ii) and (iii) ensure that in an efficient allocation, capital and labor are always equalized across countries. Thus, to deliver perfect risk sharing, the optimal portfolio choice simply has to ensure an equal division of next-period output, which is ensured with Lucas' 50-50 portfolio split.

or $1 - \lambda = 0$) delivers the same allocations as a world with complete markets. In the context of our model, considering an endowment economy effectively implies $\Delta x(s^t) = 0$, in which case eqs. (18) and (21) become two independent equations in two unknowns, $\Delta c(s^t)$ and $\Delta y(s^t)$. The only possible equilibrium, then, is $\Delta c(s^t) = \Delta y(s^t) = 0$, regardless of $1 - \lambda$. Thus, any value for $1 - \lambda$ delivers perfect risk sharing, including the portfolio autarky value $1 - \lambda = 0$ emphasized by Cole and Obstfeld (1991). The reason is simply that differences in relative quantities of output are automatically offset one-for-one by differences in the real exchange rate, so $y(s^t) = e(s^t)y^*(s^t)$. Thus movements in the terms of trade provide automatic and perfect insurance against fluctuations in the relative quantities of intermediate goods supplied.[8]

In contrast to the Cole and Obstfeld (1991) result, only one portfolio delivers perfect risk pooling in our economy. Furthermore, portfolio autarky is efficient only in the case when there is complete specialization in tastes, so that $\omega = 1$. The reason for these differences relative to their results is that with partial depreciation and persistent productivity shocks, efficient investment will not be either constant or a constant fraction of output; rather, as in a standard growth model, positive persistent productivity shocks will be associated with a surge in investment. Thus, dividends are not automatically equated across domestic and foreign stocks, and asset income is sensitive to portfolio choice. Moreover, these investment responses mediate relative price movements, so that relative earnings also fluctuate in response to productivity shocks.

3.2 Hedging intuition

We now offer some intuition for why home-biased portfolios are optimal from the perspective of an atomistic investor. We do so in two steps.

First, because equilibrium portfolios equate the common-currency value of income across countries state-by-state, agents have no incentives to actively trade assets after date zero: maintaining passive portfolios implies equal consumption levels and equal inter-temporal marginal rates of substitution across countries. This no-trade result means that the optimal portfolio choice problem is effectively static. That is why the extent of diversification, as expressed in eq. (14), depends

[8] Cole and Obstfeld (1991) also consider a version of the model with production. In this version, the two goods may be consumed or used as capital inputs to produce in the next period. Like Cantor and Mark (1988), they assume 100 percent capital depreciation. When production technologies are Cobb-Douglas in the quantities of the two goods allocated for investment, portfolio autarky once again delivers perfect risk sharing. The reason is that the assumptions of log-separable preferences and full depreciation imply that consumption, investment, and dividends are all fixed fractions of output, so that $\Delta x(s^t) = \kappa \Delta y(s^t)$. Given this relationship, eqs. (18) and (21) reduce to two independent equations in two unknowns, $\Delta c(s^t)$ and $\Delta y(s^t)$. Thus, total dividend income in any given period is again independent of the portfolio split.

on the covariance ratio between relative labor *income* and relative dividend *income*. In alternative dynamic models featuring active retrading, one could define optimal portfolios only in terms of relative *returns* to labor and capital.

Second, if relative dividends decline at the same time that relative earnings rise, then domestic stocks offer a good hedge against labor income risk. We explain in detail below why the relative dividends and relative earnings comove negatively in this way. Given an inverse and linear relationship between relative dividends and relative earnings, by choosing just the right degree of home bias, agents perfectly hedge labor income risk, so that relative total income is zero in every date and state.

Note that these two steps are inter-linked. Assuming no asset trade after date zero, agents choose portfolios to equate relative income state-by-state. Given equal income state-by-state, agents have no incentives to trade after date zero.

Why is the covariance between relative dividends and relative earnings negative in our model? The difference between the value of domestic and foreign earnings (in units of the domestic final good) is

$$
\begin{aligned}
(23)\quad \Delta l(s^t) &= (1-\theta)\Delta y(s^t) \\
&= (1-\theta)q_a(s^t)\left(F\left(z(s^t),k(s^{t-1}),n(s^t)\right)-p(s^t)F\left(z^*(s^t),k^*(s^{t-1}),n^*(s^t)\right)\right).
\end{aligned}
$$

Thus, the relative value of domestic earnings rises in response to an increase in relative productivity if and only if the increase in the production of good a relative to good b exceeds the increase in the terms of trade, $p(s^t)$. As discussed in the previous section, this condition is satisfied in our economy. At the same time, relative dividends decline, because higher relative productivity raises relative investment and thus reduces relative dividends. Note that both the increase in relative earnings and the decline in relative dividends reflect the dynamics of investment.[9]

A negative covariance ratio, together with the right portfolio home bias, implies that changes in relative dividend income will always exactly offset changes in relative earnings, for any sequence of productivity shocks. How much bias is optimal depends on the magnitude of labor income risk to

[9] In Engel and Matsumoto (2009), firms must set prices prior to the realization of productivity. Because firms cannot cut prices following a positive domestic productivity shock they must instead economize on labor. Thus, relative domestic earnings decline following a positive shock, while they rise in our flexible-price model. At the same time, relative domestic stock returns rise following a positive domestic shock in their model, while in ours a fall in the relative price of domestic capital reduces relative domestic returns (Engel and Matsumoto abstract from physical capital). Thus while both their model and ours ultimately generate a negative covariance between relative earnings and relative stock returns, the two mechanisms are quite different.

hedge and the strength of the negative covariance between relative dividends and relative earnings, which determines the effectiveness of domestic shocks as a hedge. As we now explain, these factors depend on labor's share of income, $1 - \theta$, and the trade share, $1 - \omega$.

Equation 11 indicates that equilibrium diversification, $1 - \lambda$, is increasing in the trade share, $1 - \omega$. This is because more trade reduces the variance of relative earnings, and thus the demand for a hedge against earnings risk. The logic is as follows. Following an increase in relative domestic productivity, the increase in the relative supply of the domestic intermediate will lead to an offsetting deterioration in the domestic country's terms of trade. However, domestic investment will increase in response to the shock, and because the composition of the final investment good has a domestic bias, this increase in investment will raise the relative demand for domestic goods, weakening the terms of trade response. The larger is the trade share, the more similar is the mix of domestic and foreign goods in final goods production, and the weaker effect of an increase in relative investment on the terms of trade. Thus, as the import share is increased, relative labor income becomes less variable, since offsetting movements in the terms of trade provide ever more insurance against fluctuations in relative productivity. This pushes agents toward more symmetric portfolios, which continue to favor the asset (domestic stocks) whose income comoves negatively with earnings.[10]

Equation 11 indicates that equilibrium diversification is decreasing in labor's share of income, $1 - \theta$. This is the opposite of the result in Baxter and Jermann (1997), who found that introducing labor supply made observed home bias even more puzzling from a theoretical standpoint. Both results are easy to rationalize. The larger is labor's share, the larger is the decline in relative domestic earnings following a negative productivity shock, and thus the greater is the demand for the asset that offers a hedge against labor income risk. In our economy, that asset is the domestic stock. In the Baxter and Jermann (1997) one-good world, it is the foreign stock.[11]

The dynamics described above can be visualized by plotting some impulse responses. This requires fully parameterizing the model. Because this model has been widely explored in business cycles studies, and because the dynamics described here are qualitatively consistent with a wide range of parameter values, we relegate the calibration details to Appendix F. In that appendix we

[10] When the trade share exceeds 50 percent, investment is biased toward foreign intermediates, and the terms of trade response to a productivity shock is so strong that both relative earnings and relative dividends fall in response to a positive shock, inducing a foreign bias in asset holding.

[11] Suppose that instead of trading stocks, agents were only able to trade two real bonds, denominated in units of the domestic and foreign final goods respectively. In this alternative market structure, agents would bias portfolios towards domestic bonds, because the relative price of domestic goods – and hence the relative return on domestic bonds – covaries negatively with relative domestic labor earnings.

also report standard business cycle statistics for the model, alongside their empirical counterparts.

Figure 1 plots impulse responses to a persistent (but mean-reverting) positive productivity shock in the domestic country. The path for productivity in the two countries is depicted in panel (a), and the real exchange rate is plotted in panel (d). The remaining panels show stock returns (b), labor earnings (c), stock prices (e), and dividends (f), all of which are plotted in units of the domestic final consumption good.

In the period of the shock, relative domestic earnings increases, and the gap between relative earnings persists through time. The differential can persist because labor is immobile internationally.

In the period of the shock, returns to both domestic and foreign stocks increase, but the increase is larger for foreign stocks (panel b). In subsequent periods, returns to domestic and foreign stocks are equalized. The reason for this result is simply that stocks are freely traded, and thus equilibrium stock prices must adjust to equalize expected returns.

Why does the relative return to foreign stocks increase in response to a positive domestic productivity shock? As panel (e) indicates, this reflects a decline in the relative price of domestic stocks. We can rationalize this response as follows.

Suppose the positive domestic productivity shock occurs at date t given history s^t. Given a constant portfolio split defined by λ, the difference between the lifetime present values of domestic and foreign income, in units of domestic consumption, is

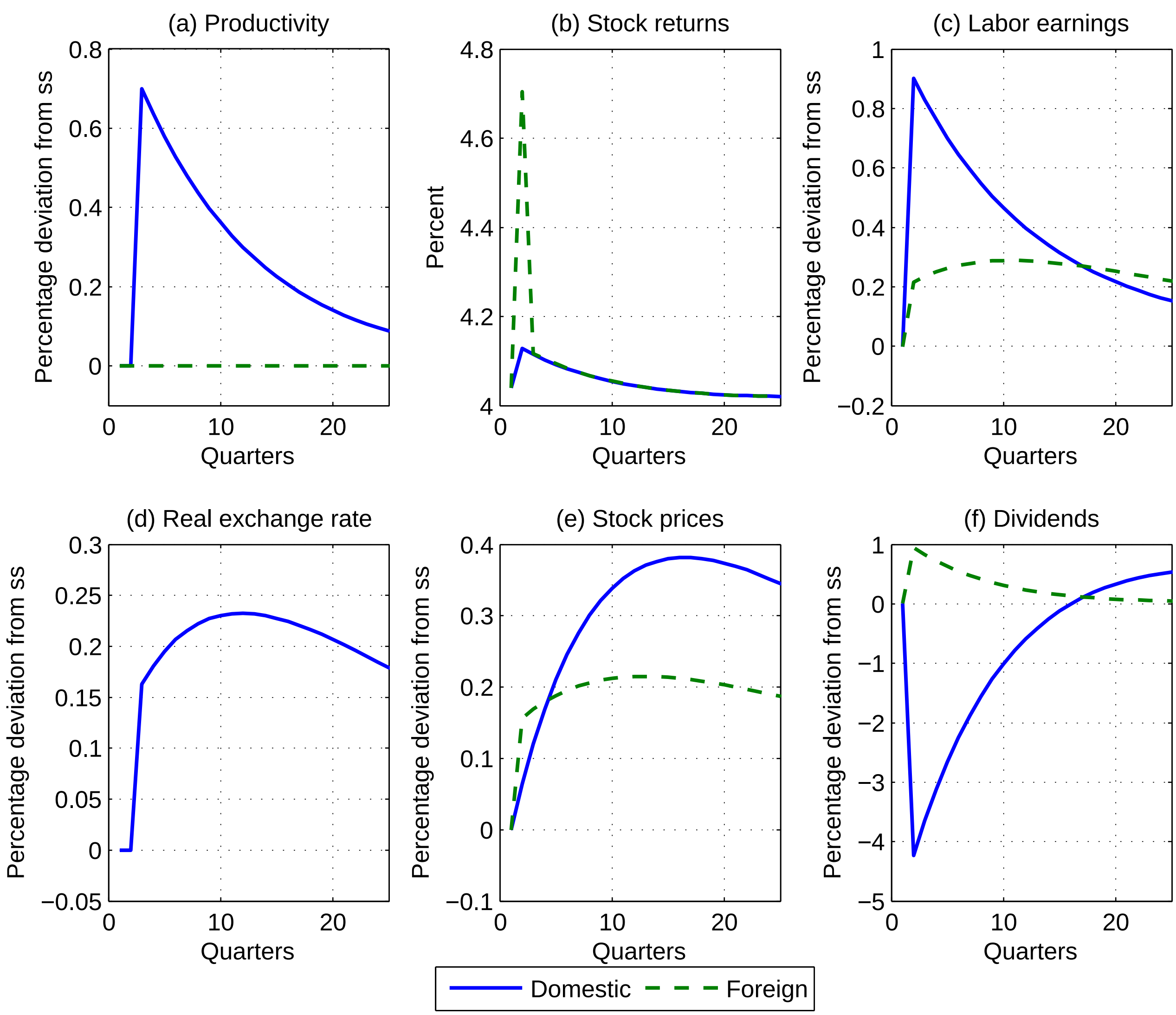
$$\sum_{j=t}^{\infty} \sum_{s^j \succeq s^t} \frac{Q(s^j)\left(\Delta l(s^j) + (2\lambda - 1)\Delta d(s^j)\right)}{Q(s^t)} \tag{24}$$

$$= \sum_{j=t}^{\infty} \sum_{s^j \succeq s^t} \frac{Q(s^j)\Delta l(s^j)}{Q(s^t)} + (2\lambda - 1)\left(\Delta d(s^t) + \Delta P(s^t)\right) = 0, \tag{25}$$

where the first equality reflects the fact that the present value of future domestic (foreign) dividend payments is equal to the equilibrium ex-dividend price of domestic (foreign) stocks, and the second equality holds because equilibrium values of domestic and foreign income (and consumption) are the same in every date and state.

At date t, the positive domestic shock increases the present value of relative earnings, the first term on the right-hand side of eq. 24 (see panel b). Given portfolio home bias ($1 - \lambda < 1/2$), the relative present value of income can remain equal to zero only if the relative pre-dividend price of stocks, the term $\Delta d(s^t) + \Delta P(s^t)$, goes down. Using the results $P(s^t) = k(s^t)$ and $P^*(s^t) = k^*(s^t)$

Figure 1: Impulse responses to a domestic productivity shock



and the definitions for dividends (eqs. 8 and 9), the relative pre-dividend price of stocks can be expressed as

$$\Delta d(s^t) + \Delta P(s^t) = \theta \left(y(s^t) - e(s^t) y^*(s^t) \right) + (1-\delta) \left(k(s^{t-1}) - e(s^t) k^*(s^{t-1}) \right). \tag{26}$$

The first term in this expression captures the change in relative rental income in the period of the shock and, like relative earnings (eq. 23), will increase following a positive domestic productivity shock. The second term captures the change in the relative value of undepreciated capital. A positive domestic productivity shock drives up the real exchange rate $e(s^t)$ and thus drives down this relative value (since final consumption and investment are perfectly substitutable in production, the relative price of capital is equal to the relative price of consumption). In our model, the second term in eq. (26) dominates, meaning that when faced with a positive shock, owners of domestic stocks lose more from the ensuing devaluation of domestic capital than they gain from a higher rental rate. Thus, domestic stocks offer a good hedge against nondiversifiable labor income risk, rationalizing, in return space, home bias in portfolios.

Because agents do not adjust their portfolios in response to the shock, the decline in the relative price of domestic stocks on impact means that financial wealth for home-biased domestic agents declines relative to the wealth of foreigners. This means that in the periods immediately following the shock, even though returns are equalized, the total asset income accruing to foreign agents is larger, because they hold more financial wealth in total. This additional asset income exactly offsets foreigners' lower labor income, and the relative value of consumption is equalized.

Over time, the domestic productivity shocks decays, while the real exchange rate remains above its steady-state level. As a consequence, foreign labor income eventually rises above domestic labor income. But notice that now, because of capital accumulation in country 1, domestic stocks are now worth more than foreign stocks, and this compensates domestic residents for the fact that they expect relatively low earnings during the remainder of the transition back to steady state.

Related Literature: Cole (1988), Brainard and Tobin (1992), and Baxter and Jermann (1997) argued that in models driven entirely by productivity shocks, one should expect labor income to comove more strongly with domestic rather than foreign stock returns, thereby indicating strong incentives to aggressively diversify. Bottazzi, Pesenti, and van Wincoop (1996) argued that this prediction could be overturned by extending models to incorporate additional sources of risk that redistribute income between capital and labor, and thereby lower the correlation between returns on human and physical capital. They suggested terms of trade shocks as a possible candidate. We have shown that in fact it is not necessary to introduce a second source of risk: the endogenous response of the terms of trade to productivity shocks is all that is required to generate realistic levels of home bias. The existing empirical evidence on correlations between *returns* to labor and domestic versus foreign stocks is, for the most part, qualitatively consistent with the pattern required to generate home bias. Important papers on this topic are Bottazzi, Pesenti, and Van Wincoop (1996), Palacios-Huerta (2001), and Julliard (2002). In the next section, we will investigate the covariance between relative *income* from labor and capital, exploiting the expression for portfolio diversification in eq. (14).

Adler and Dumas (1983) and Van Wincoop and Warnock (2010) emphasize a different force that can also deliver home bias in two-good models: negative covariance between the real exchange rate and the return differential between domestic and foreign stocks. If domestic stocks pay a relatively high return in states of the world in which domestic goods are expensive (i.e., the real exchange rate is low) then, since domestic residents mostly consume domestic goods, they may prefer to mostly hold domestic stocks. Note that this effect is not the driver of home bias in our basic setup. In fact, this mechanism generates home bias only when the coefficient of relative risk aversion exceeds one. By contrast, our model generates substantial home bias even with risk aversion equal to one.[12] The reason is that our environment features nondiversifiable labor income, from which these other papers abstract. In the presence of nondiversifiable labor income, portfolio choice is driven primarily by the covariance between relative incomes (or returns) from capital and labor, and not by the covariance between relative equity returns and the exchange rate.

4 Explaining diversification in developed countries

Proposition 1 suggests that the patterns of international diversification are driven by patterns of trade. Proposition 2 suggests a two-part economic mechanism underlying this link. The first part (summarized by eq. 13) is the "general equilibrium" connection linking trade to the covariance between relative labor earnings and relative dividends. The extent of trade determines the equilibrium dynamics of investment and the terms of trade, which in turn determine the covariance between relative earnings and dividends.

[12] We experiment with alternative values for risk aversion in Section 5.

The second part (summarized by eq. 14) is the "partial equilibrium" connection linking the covariance between relative labor earnings and relative dividends to international portfolio diversification. We label this a "partial equilibrium" link because it reflects individual portfolio optimization, taking as given relative prices and dividends.

In this section we first provide evidence for one prediction of the theory: the link between trade and diversification, which we will show holds both in the cross section and in the time series. We also provide evidence for the two sub-links between trade and covariances and between covariances and diversification, which provide a more direct test of the mechanism highlighted by our theory.

We now describe our methodology for comparing the simple theoretical model to the data.

4.1 Data

To compute an empirical counterpart to the expression for diversification in Proposition 2, we need to compute relative earnings and relative dividends across countries. The dynamics of these variables are strongly influenced by fluctuations in real exchange rates. Standard international macro models like ours sharply under-predict real exchange rate volatility because they abstract from sources of nominal exchange rate risk (real exchange rates vary much less for countries with fixed exchange rates).[13] Given the existence of liquid forward foreign exchange and nominal bond markets, we will assume that people can hedge against nominal exchange rate fluctuations. One might then conjecture that the relevant covariance ratio M defining equilibrium portfolio diversification in eq. (14) should involve the components of relative earnings and relative dividends that are orthogonal to the nominal exchange rate. In Appendix D we verify this conjecture by working out a simpler version of our model in which we introduce explicit exogenous nominal exchange rate risk and a forward foreign exchange market. Guided by this theory, we construct empirical covariances by first regressing relative log earnings and relative log dividends (defined as described below) on bilateral nominal exchange rates, and then compute M using the residuals.[14]

A second issue is that ours is a model with two symmetric countries and no restrictions on international financial flows, whereas international diversification data are drawn from countries that are heterogeneous along many dimensions, including size, the level of development, and the

[13] See Appendix F for a comparison of the business cycle properties of the real exchange rate in the model versus the data.

[14] Van Wincoop and Warnock (2010), Coeurdacier, Kollmann, and Martin (2008), Coeurdacier and Gourinchas (2011), and Engel and Matsumoto (2009) follow a similar empirical strategy. Note that our procedure changes importantly the statistical properties of the covariance ratio M and thus the implications of eq. (14) for diversification. For example, focusing on the United States, the orthogonalized covariance ratio is -0.18, whereas the raw covariance ratio is 0.47, implying, respectively, equilibrium portfolio shares in foreign assets of 9.3 percent and 154 percent.

extent of financial liberalization. One possible way to deal with this issue would be to enrich our basic model to include many heterogeneous countries and to then bring such a model to the data; we view that project as interesting, but one that is also beyond the scope of this paper.[15] Here, instead, we address the issue in two ways. First, we restrict our empirical analysis to a group of relatively homogeneous and financially liberalized countries: members of the OECD throughout the period 1990-2007.[16] Second, within this group, we assess whether factors omitted from the model, such as size or level of development, are important empirical factors in explaining diversification patterns.

Finally, we need operational definitions for diversification, trade intensity, dividends, and labor income. Our theoretical measure of international diversification, $1 - \lambda$, is both the ratio of gross foreign assets to wealth and the ratio of gross foreign liabilities to wealth. Thus, to construct empirical measures of diversification, we need data on gross foreign assets, gross foreign liabilities, and total country wealth. We obtain data on total gross foreign assets (FA) and total gross foreign liabilities (FL) from the exhaustive dataset collected by Lane and Milesi-Ferretti (2007). The empirical counterpart of model equity is not just corporate traded equity, but any asset that represents a claim to country output. Thus we include all assets that constitute such claims: portfolio equity investment, foreign direct investment, debt (including loans or trade credit), financial derivatives and reserve assets (excluding gold).[17] We identify total country wealth as the value of the entire domestic capital stock plus gross foreign assets less gross foreign liabilities: $K + FA - FL$. Some foreign asset categories (such as foreign direct investment) are constructed using book values, whereas others (such as portfolio equity investment) are constructed using market values. This raises the question of the most appropriate way to measure the capital stock K. Our baseline will be a book value measure, constructed by cumulating investment using the perpetual inventory method. In addition, we will also consider a measure for K that uses stock prices to value the fraction of capital that is publicly traded. See Appendix C for a precise description of the two measures. Given K

[15] We did experiment with one dimension of heterogeneity. In particular, we considered an extension of our main model in which the two countries differ in terms of population. We then solve this version of the model numerically, given the parameter values described in table F1, and compare the average equilibrium level of diversification to the level predicted by eq. (11). We find that, for the smaller economy, the equilibrium level of diversification exceeds that which would be observed in the corresponding symmetric-size economy, whereas for the larger economy, the equilibrium level of diversification is below that which would be predicted by (11), given the country's import share. However, these differences are generally small (less than 1 percent), unless the smaller country is both very open and very small.

[16] The exact set of countries we included in our sample is described in Appendix C.

[17] We consider a model economy with bonds in Section 5.

we measure international diversification for country i in period t as

$$DIV_{it} = \frac{FA_{it} + FL_{it}}{2\left(K_{it} + FA_{it} - FL_{it}\right)}.$$

The intensity of trade in the model is driven by the preference parameter $1 - \omega$, which can be measured using either steady-state import or export to GDP ratios. We measure the trade intensity for country i, $(1 - \omega)_i$, using national income data from the OECD Quarterly National Accounts, as the time average of import and export shares; that is,

$$(1 - \omega)_i = \frac{1}{19} \sum_{t=1990}^{2007} \frac{Imports_{it} + Exports_{it}}{2GDP_{it}}.$$

The final variable we need to test our theory is M, the covariance between relative earnings and relative dividends divided by the variance of relative dividends. To estimate covariances for the countries in our sample, we use quarterly series that are comparable across countries and that are sufficiently long so that comovements between components can be measured without too much sampling error. We use OECD quarterly national accounts from the period 1980.1 to 2007.4. Consistently with our broad interpretation of wealth, we measure dividends as aggregate capital income less aggregate investment, where capital income is a constant fraction θ of GDP. Note that this corresponds directly to model dividends as defined in eq. (8). Labor earnings are then equal to fraction $(1 - \theta)$ of GDP. For each country, we construct a measure of foreign dividends and foreign earnings by taking a weighted average of log dividends and log earnings (measured in a common currency) of all the other countries in the sample, where weights are given by relative shares in world GDP. As described above, we first regress relative log earnings and relative log dividends on bilateral nominal exchange rates, and then compute the covariance M using the residuals.

4.2 Model

Evaluating quantitative predictions for the relationships between diversification $(1-\lambda)$, trade $(1-\omega)$, and covariances (M) in our model is straightforward. The relationships established in Propositions 1 and 2 involve only four structural parameters: the discount factor β, the depreciation rate of capital δ, the trade openness parameter ω, and capital's share θ. We will assume that the first two parameters are the same across countries and equal to the commonly used values of $\beta = 0.99$ and $\delta = 0.015$. Trade share estimates identify country-specific values for ω, as described above. We have experimented with three alternative measures of capital's share θ. The first is a standard value of $\theta = 0.36$, common across countries. The second and third measures are country specific and are computed following the methodologies of Gollin (2002) and Bernanke and Gürkaynak (2002). Implied diversification varies across the three alternative measures by at most two percentage points for all the countries in our sample.[18] We therefore report results only for the common θ case.

4.3 Findings

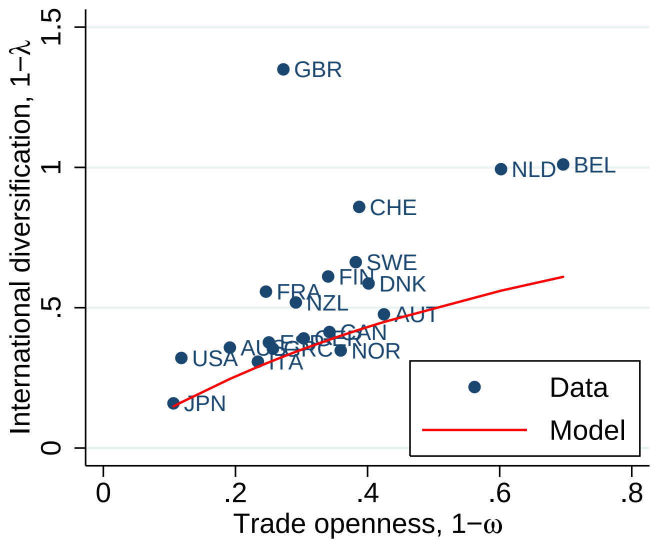
Figure 2 confronts the theoretical link between diversification and trade with cross-sectional evidence. The dots represent the time averages for diversification DIV_i and trade intensity $(1-\omega)_i$ for each country in our dataset. Note that there is a great deal of heterogeneity in both trade intensity and diversification, with both ranging from around 10 percent to around 100 percent. The solid curve is the theoretical relationship between trade and diversification implied by the model (eq. 11). The figure suggests that the cross-country diversification evidence is broadly consistent with the predictions from our model, and that the trade share is an important factor in explaining observed variation in international diversification.[19] Relative to our theory, more countries are over-diversified than under-diversified. Moreover, a significant fraction of heterogeneity in observed diversification is not explained by the model, reflecting the reality that countries differ along multiple dimensions in addition to openness. For example, Great Britain is much too diversified from the standpoint of the theory, perhaps reflecting its special position as an international financial center.

In table 1 we make the comparison between model and data more precise using linear regression analysis. Column (1) of the table reports results from the baseline regression. The line labeled "Trade openness" reports the coefficient obtained from an ordinary least squares (OLS) regression of diversification on trade openness. It shows that diversification and trade are significantly related, with a coefficient slightly above one, indicating that one percentage point more trade relative to output translates into slightly more than one percentage point more foreign assets relative to wealth.

[18] This is for two reasons. The first is that the share of income going to capital exhibits little variation across countries, given a careful accounting of cross-country variation in the size of self-employment. The second is that theoretical equilibrium portfolio shares are not very sensitive to changes in capital's share. Indeed in the special case in which the trade share is 50 percent, predicted diversification does not depend at all on capital's share.

[19] Using bilateral data on trade and cross-border asset holding, Aviat and Coeurdacier (2007) explore the relationship between trade and diversification within a simultaneous gravity equations framework. They estimate that a 10 percent increase in bilateral trade raises bilateral asset holdings by 6 percent to 7 percent, that causality runs primarily from trade to diversification rather than from diversification to trade, and that controlling for trade greatly reduces the explanatory power of distance for cross-border asset holdings. Portes and Rey (2005) and Collard et al. (2007) also highlight a strong empirical relation between trade in assets and trade in goods.

Figure 2: Diversification and trade: data and model



The line labeled "Predicted median div." reports diversification for a hypothetical country with the median trade share, as predicted by the regression. Predicted diversification for such a country is 53 percent.

In column (2) we add to the basic regression GDP per capita and population size. Our symmetric model is silent about the roles of development and size, but it is interesting to assess i) whether these variables are statistically correlated with diversification and ii) whether the relation between trade and diversification is affected by the inclusion of these variables. In particular, since small countries and rich countries tend to trade more, one might wonder whether trade matters for diversification only to the extent that trade proxies for size or level of development. The numbers in the table do not support this conjecture. Population and GDP per capita are not statistically related to diversification, as long as the openness variable is included. Furthermore, the statistical and economic significance of the relationship between diversification and trade is largely unaffected by whether or not these additional controls are included.

Column (3) applies our alternative measure of diversification, where the capital stock K is computed using stock prices (as described in Appendix C). Using stock prices to value capital implies somewhat higher estimates for wealth and a correspondingly lower value for predicted

Table 1: Diversification and trade

Dependent variable is diversification

	Data				Model
	OLS			LAD	
	(1)	(2)	(3)	(4)	(5)
Trade openness	1.26	1.52	1.08†	1.42†	0.78
	(0.19)	(0.22)	(0.15)	(0.31)	(0.05)
Log GDP per capita		-0.13			
		(0.24)			
Log population		0.05			
		(0.05)			
Predicted median div.	0.53	0.51	0.47†	0.45†	0.34
	(0.06)	(0.05)	(0.05)	(0.04)	(0.01)
R^2	0.37	0.40	0.41	0.37	0.97
Observations	19	19	19	19	19

Note: Robust (for the OLS specifications) standard errors are in parentheses.
A † next to a data statistic indicates that the hypothesis that the statistic is equal to the corresponding model statistic cannot be rejected at the 5% confidence level.

median diversification. The strong empirical relationship between trade and diversification remains. Column (4) redoes the baseline regression applying the least absolute deviations (LAD) metric. The fact that statistics for OLS and LAD are similar suggests that our results are not driven by a very small subset of countries. The R^2 values in the last row of the table indicate that differences in openness to trade can alone explain around 40 percent of cross-country variation in portfolio diversification.[20]

In order to estimate a linear relation between trade and diversification in the model (where the true relationship is nonlinear) we feed country-specific ω's corresponding to trade openness plus a common-across-countries value for θ of 0.36 into eq. (11) in order to generate model-predicted

[20]The reader might wonder about the role of sample size in our results. We have replicated the exercise of table 1 on two different sets of countries. The first is an extended set of developed economies, i.e., the set of rich countries, as defined by the World Bank. This gives a sample of 28 countries. Results for this set of countries are available in Heathcote and Perri (2008) and are very similar to the ones presented here. We use a smaller sample here because we could not obtain comparable national accounts data, needed to compute covariance ratios, for the larger set of countries. We have also repeated the analysis for a even larger group of countries, including developing economies. We found that, although the strong link between trade and diversification remains, income per capita becomes an important determinant of diversification, with richer countries being more diversified.

country-specific values for diversification. We can then run the same linear regression of diversification against openness that we run on the actual data. Comparing columns (1) through (4) with column (5) indicates once again that the theory somewhat underpredicts observed diversification: predicted median diversification is 34 percent according to the theory, compared to around 50 percent according to the empirical models. However, the model's predictions both for the *level of diversification* and for the *relation between diversification and trade* are not far, in a statistical sense, from the data.

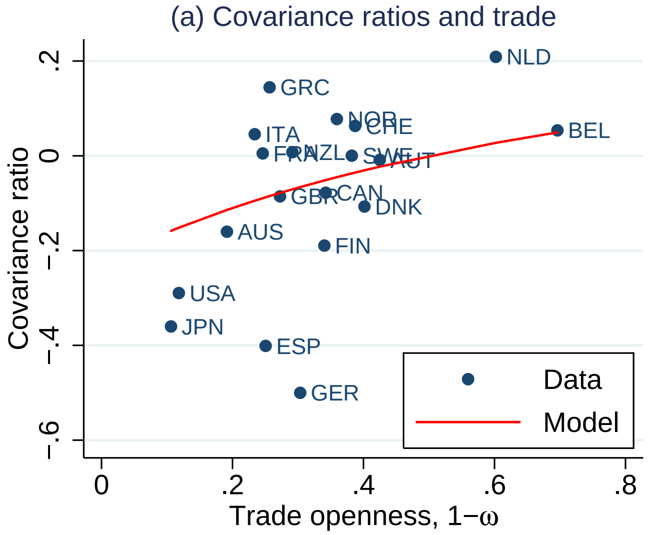
While the model replicates the observed correlation between trade and diversification, there are potentially a range of alternative models that could generate such a relationship (see, for example, Collard et al., 2007). To provide more direct evidence in favor of our mechanism, we delve deeper into the relation between trade and diversification by examining whether the model's predictions on the two sublinks highlighted in Proposition 2 are consistent with the data. In panel (a) of figure 3 we document the general equilibrium link, connecting the covariance ratio between relative log earnings and relative log dividends to trade openness. The dots represent the covariance ratio and trade intensity in our sample of countries, and the solid curve is the relationship between the two implied by eq. (13) in the model. Note that for a majority of countries, the covariance ratio is negative in both data and model, suggesting that a majority of countries should hold portfolios biased toward domestic assets. Note also that in the data, as in the model, countries that trade more tend to have a more positive covariance ratio (which points to more international diversification).

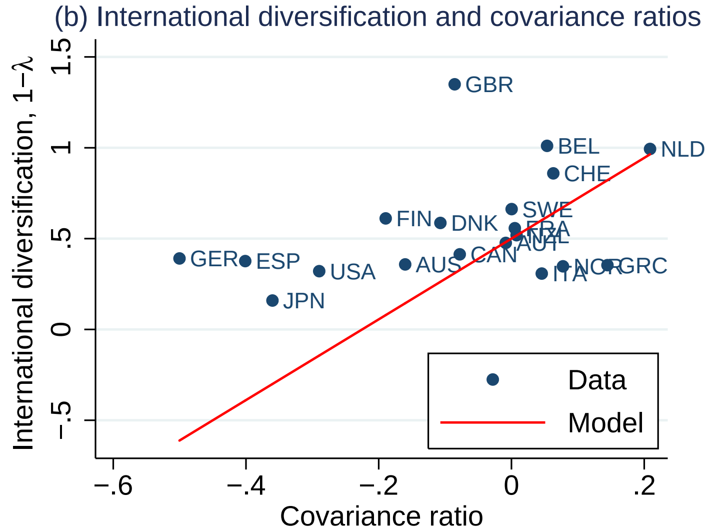
Panel (b) of figure 3 provides evidence for the partial equilibrium link between international diversification and covariance ratios. The dots represent diversification and covariance ratios in the data, and the solid line represents the relationship in the model (eq. 14). The panel shows that, consistently with the model, countries in which the covariance ratio is larger (less negative) tend to exhibit less home bias.[21]

Table 2 offers a more quantitative comparison between model and data with respect to the relationships shown in figure 3. Panel (a) reports the results of OLS and LAD regressions of the covariance ratio on trade openness. In columns (1) and (2) the dependent variable is the empirical covariance ratio, and in column (3) it is the covariance ratio predicted in theory by eq. (13). Comparing data and model values for the "Trade openness" coefficient indicates that the magnitude of the empirical link between openness and the covariance ratio is larger but not

[21]Figure 3 uses model-consistent covariances between relative earnings and relative dividends. The picture is qualitatively similar if covariances are instead constructed using relative stock returns (rather than dividends).

Figure 3: Diversification, covariance, and trade





significantly from the one predicted by the model. The line labeled "Pred. med. cov." reports the covariance ratio for a hypothetical country with the median trade share, as predicted by regressions on real-world data and model-generated data, and shows that the covariance ratio for this median country in the data is of the same order of magnitude as the one predicted by the model.

Panel (b) reports corresponding results from regressing diversification on covariance ratios. Note that in both data and theory, countries that exhibit a more negative covariance tend to be less diversified. However, the effect of covariance on diversification in the theory (2.22) is larger than the effect estimated by OLS (0.51). However, if covariance ratios are measured with (sampling) error, we would expect downward bias in the estimated relation between diversification and the covariance ratio.[22] In order to explore this possibility, we instrument covariance ratios using trade shares, which are precisely measured and should be correlated with the true covariance ratios but not with the measurement error. Column (6) reports the result of this IV exercise and shows that measurement error in the covariance ratio is potentially important. In particular, instrumenting for the covariance ratio gives a much larger estimated effect of covariance on diversification, and one that is similar to that implied by theory.

[22] Note that attenuation bias is not an issue when the covariance ratio is the dependent variable in the regressions in Panel (a).

Table 2: Diversification, covariance, and trade

	Dependent variable is:								
	a. Covariance Ratio				b. Diversification				
	Data		Model		Data			Model
	(1)	(2)	(3)		(4)	(5)	(6)	(7)
	OLS	LAD	OLS		OLS	LAD	IV	OLS
Trade openness	0.70†	0.84†	0.35	Covariance ratio	0.51	0.31	1.54†	2.22
	(0.20)	(0.32)	(0.02)		(0.21)	(0.31)	(0.68)	(0.00)
Pred. med. cov.	-0.10†	-0.07†	-0.07	Pred. med. div.	0.53†	0.49†	0.54†	0.39
	(0.04)	(0.05)	(0.02)		(0.07)	(0.08)	(0.09)	(0.00)
R^2	0.28	0.17	0.97		0.16	0.12	-	1.00
Observations	19	19	19		19	19	19	19

Note: Robust (for the OLS specifications) standard errors are in parentheses. A † next to a data statistic indicates that the hypothesis that the statistic is equal to the corresponding model statistic cannot be rejected at the 5% confidence level.

The evidence discussed in this section leads us to conclude that the predictions of the model regarding the level of diversification and the relationships between diversification, trade, and the cross-country dynamics of labor earnings and dividends are qualitatively and quantitatively helpful for understanding the cross section of country portfolios in developed economies.

4.4 Diversification over time

In this section we explore the panel dimension of our dataset to assess whether our framework can also be used to understand the evolution of international diversification in recent years. Strictly speaking, diversification does not change over time in equilibria of our baseline model. However, suppose that between period t and period $t+k$, two countries experience an unexpected and permanent change in ω, the parameter that determines their trade share. Then the model predicts a corresponding change in international diversification.

In table 3 we explore this prediction by regressing changes in diversification on changes in the trade share for our panel of countries. In order to focus on changes in the trade share in the data that are (possibly) persistent and unanticipated, we examine changes over five-year intervals.[23]

[23] Specifically, our data points include changes of the relevant variables over all possible five-year intervals within the period 1990-2007, for all 19 countries of our sample described above. This gives a total of 247 observations. We have also conducted the analysis focusing on changes over three-year and seven-year intervals and found that results

Columns (1) through (3) report the results of running the regression on the data, adding various controls, and column (4) reports the results from regressing changes in diversification predicted by eq. (11) on the observed changes in openness. Even after including growth in GDP per capita

Table 3: Changes in diversification and changes in trade

Dependent variable is five-year change in diversification

	Data			Model
	(1)	(2)	(3)	(4)
Change in openness	1.21†	0.91†	0.92†	0.77
	(0.45)	(0.53)	(0.33)	(0.04)
Change in log GDP pc		1.49	1.19	
		(0.53)	(0.52)	
Change in log pop.		-0.76	3.61	
		(1.19)	(1.83)	
Country & period dummies	No	No	Yes	No
Observations	247	247	247	247
R^2	0.05	0.16	0.55	0.93

Note: Numbers in parentheses are robust standard errors clustered at the country level. A † next to a data statistic indicates that the hypothesis that the statistic is equal to the corresponding model statistic cannot be rejected at the 5% confidence level.

and population and country and period dummies—and thus allowing for a variety of factors to affect changes in diversification—the link between trade and diversification remains significant and quantitatively similar to the link predicted by the model and to the link estimated on cross-sectional data. Thus, to our previous finding that countries that trade more are more diversified, we can add the finding that countries which exhibit faster growth in trade also experience faster growth in diversification. An important qualification is that the median five-year change in diversification in our sample is around 17 percent, whereas the median change in openness is only 2.5 percent. Thus, our earlier estimate of the effect of trade on diversification of around one suggests that observed growth in trade over the period 1990-2007 can explain only a small fraction of the increase in international diversification over the same period. Investigating the causes of the residual growth

are not significantly affected.

in diversification is an interesting direction for future research.

5 Sensitivity analysis

Three key assumptions are required to deliver our closed-form expressions for portfolio choice. First, the elasticity of substitution between traded intermediate goods is restricted to be equal to one (so that the G functions are Cobb-Douglas). Second, preferences are assumed logarithmic in consumption. Third, the only assets traded internationally are equities. We now experiment with relaxing these assumptions.

In order to compute equilibrium country portfolios in a general setup, we use parameters specified in table F1, solve the model numerically, and compute average values for diversification in simulations. We use the solution method developed by Devereux and Sutherland (2011). The method involves taking second-order approximations to all equilibrium conditions except those relating to portfolio choice, which are approximated to the third order. This approach allows us to solve simultaneously for the level and dynamics of portfolio diversification.

In our baseline model, we have shown that λ_t is constant. Once we deviate from the baseline parameterization, λ_t is potentially time varying. However, in Appendix E we show that λ_t follows a stationary stochastic process, so that it is meaningful to talk about the average level of diversification predicted by theory.

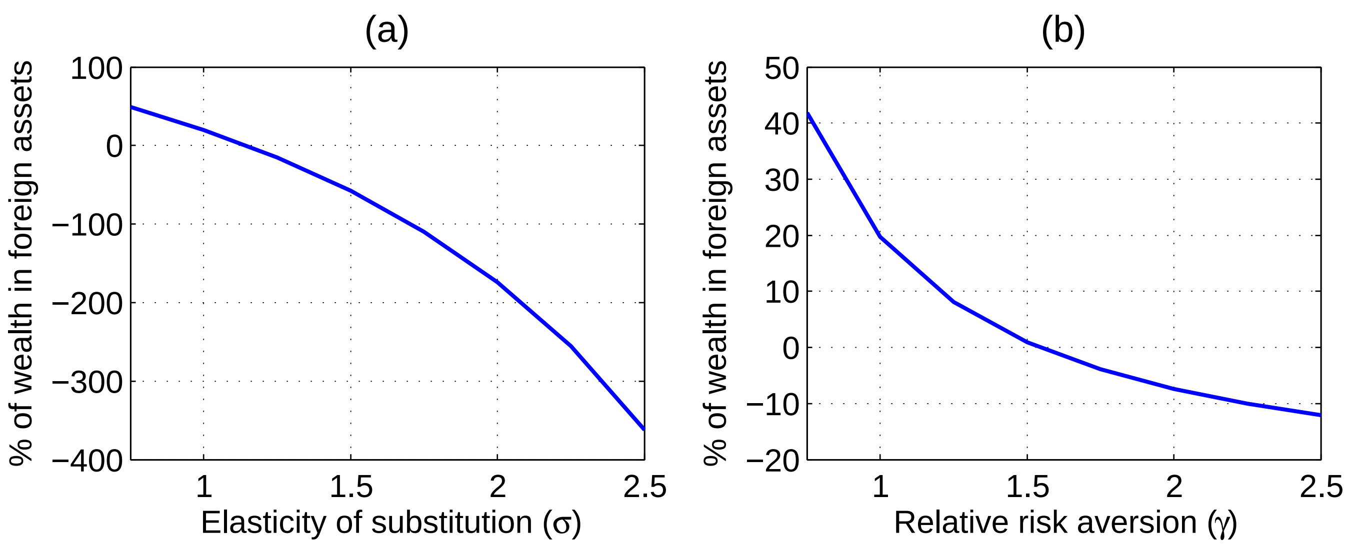
The Cobb-Douglas aggregator for producing final goods implies a unitary elasticity of substitution between the traded goods a and b. This elasticity is similar to typical estimates in the macroeconomic literature, but is lower than typical estimates in the microeconomic trade literature.[24] Panel (a) of figure 4 shows how the average equilibrium level of diversification changes as the elasticity of substitution, σ, is varied from 0.8 to 2.5, given a CES aggregator of the form $G(a,b) = (\omega a^{\frac{\sigma-1}{\sigma}} + (1-\omega) b^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$. The main message of the figure is that for elasticities typically used in the macro literature, theory predicts strong home bias. Notice also that increasing substitutability strengthens home bias within this range of values for σ. The logic for this result is that the more substitutable are a and b, the less relative prices change in response to shocks. This means that following a positive domestic shock, the increase in the relative value of domestic

[24] See Feenstra, Obstfeld, and Russ (2012) for a recent paper that differentiates explicitly between the "macro" elasticity between home and import goods, and the "micro" elasticity between alternative foreign sources of imports. They estimate a median micro elasticity of 3.1, and macro elasticities close to one. Ruhl (2008) develops a model with export-entry decisions in which lower elasticities at business cycle frequencies are consistent with much larger responses of trade quantities in response to trade liberalizations that change relative prices permanently. Corsetti, Dedola, and Leduc (2008b) make a case for an elasticity even lower than one.

labor earnings becomes larger and, at the same time, the decline in relative domestic stock returns becomes smaller. Thus, agents must overweight domestic stocks to an even greater extent in order to hedge such risks.

For high elasticities (values for σ exceeding 4), price movements become so small that, following a positive domestic shock, returns to domestic stocks exceed returns to foreign stocks, and the correlation between relative labor income and relative domestic stock returns turns positive. For such high elasticities, the two-good model is sufficiently close to the one-good model that its portfolio implications are similar. In particular, it is optimal for the individual to hedge against shocks to relative labor income by shorting domestic assets. Thus, the average portfolio displays a very strong—and counterfactual—foreign bias.

Figure 4: Sensitivity analysis



Panel (b) of figure 4 shows how diversification changes as we relax the log utility assumption, assume utility from consumption of the form $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and vary the coefficient of relative risk aversion, γ. Notice that higher risk aversion leads to higher home bias. Changing the risk aversion coefficient does not have an impact on two of the equilibrium relationships (eqs. 18 and 21) developed in Section 3.1. Changing γ does, however, change the pattern of comovement between domestic and foreign consumption consistent with perfect risk sharing. In particular, since higher risk aversion corresponds to a lower intertemporal elasticity of substitution for consumption,

γ^{-1}, desired consumption becomes less sensitive to changes in relative prices. Thus, in choosing portfolios, agents want to ensure that their total income does not decline too much in periods when domestic productivity falls and the relative price of domestic consumption increases ($e(s^t)$ declines). This pushes agents further toward domestic stocks, whose relative return rises in periods when domestic productivity and earnings decline.

In our final set of experiments, we consider an alternative asset market structure in which the assets traded internationally are two bonds, one of which is a one-period claim to the domestic final consumption good, while the other is a claim to the foreign consumption good. In this economy we assume that all equity is locally owned, and that bonds are in zero net supply. All parameter values are equal to their baseline values. We define diversification as the average market value of foreign bond holdings relative to total domestically-owned wealth, analogously with the baseline market structure. There is no closed-form expression for diversification in this economy, and we must solve for the average value for diversification numerically.

Figure 5: Bond model versus stock model

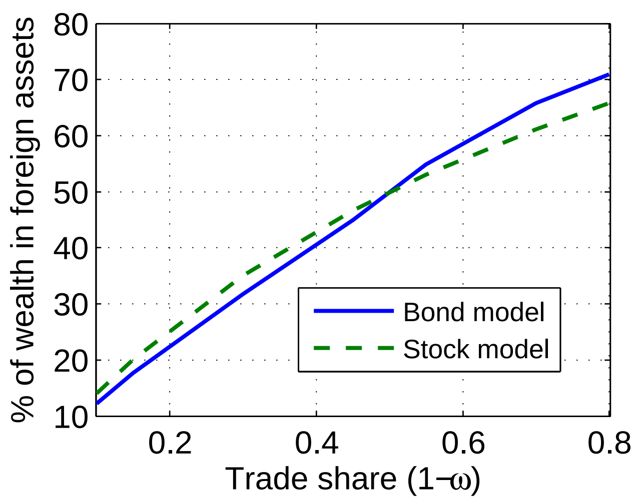


Figure 5 shows how diversification varies with the trade share in the model with bonds, along with the corresponding plot for diversification given our baseline market structure with trade in equities (i.e., eq. 11). Quantitatively the bond model predicts similar levels of diversification relative to the baseline equity model, but a slightly larger sensitivity of diversification to trade openness. In both models home bias in portfolio composition mirrors home bias in trade and the reason is the same for both market structures: the pay-off from domestic assets is tied to the price of the domestic consumption good, the value of which declines following a shock that increases domestic earnings.

Why is diversification slightly lower in the bond model than the stock model for trade shares below 50 percent? The intuition is that bonds offer a slightly better hedge than stocks against shocks to relative labor earnings since the relative return to bonds entirely reflects movements in the real exchange rate – which depreciates following a positive productivity shock – while part of the return to stocks (relative rental income, the first term in eq. 26) co-moves positively with labor earnings.

Why does portfolio bias switch from home to foreign when the trade share rises above 50 percent? The intuition is that agents always want to over-weight the asset whose return co-moves negatively with domestic labor earnings, but the identity of that asset changes as the composition of final goods changes from mostly domestic to mostly foreign goods. In particular, for trade shares below 50 percent, the real exchange rate depreciates following a positive domestic shock, while for trade shares above 50 percent the real exchange rate appreciates – because $1 - \omega > 0.5$ implies that domestic consumption and capital are disproportionately made up of relatively scarce foreign produced inputs.[25]

From this exercise we conclude that our key theoretical predictions – that portfolios should typically be home-biased, and that diversification should be increasing with openness to trade – are not specific to the particular equities-only asset market structure we assumed in our baseline model. The finding that theory predicts quantitatively similar diversification levels for the two different market structures considered also offers some support for the broad empirical measure of diversification encompassing foreign holdings of both equity-like and bond-like assets that we adopted in Section 4.

6 Conclusion

In this paper, we have shown that standard macroeconomic theory predicts patterns for international portfolio diversification that are broadly consistent with those observed empirically in recent years. The economic model we used to generate theoretical predictions for portfolio choice was a

[25] This discussion suggests that our assumption on the denominations of traded bonds is important for the portfolio predictions plotted in figure 5 In fact, if bonds were denominated in units of intermediate goods rather than final consumption goods, equilibrium bond portfolios would be home-biased for any trade share, since the terms of trade always deteriorates following a positive domestic shock.

standard two-country two-good version of the stochastic growth model that has been widely used in business cycle research. We conclude that, from the perspective of standard macroeconomic theory, the observed bias toward domestic assets is not a puzzle. We have explored the economics underlying this result and argued that the dynamics of investment and international relative price movements are central to understanding portfolio choice. We have also provided empirical evidence on these dynamics from developed economies, and shown that the patterns observed empirically are quantitatively consistent with those predicted by theory.

Important questions remain. Some readers will find it surprising that we address portfolio diversification in a model that does not replicate some important features of asset prices, especially the high volatility of stock returns. It is well known that it is difficult to reconcile many features of asset prices within stochastic general equilibrium production economies. A general equilibrium theoretical resolution of these pricing puzzles is required to bridge the gap between the macro-theory and empirical finance literatures on portfolio choice.

Another important open issue in international macroeconomics is to understand the volatility and persistence of exchange rate movements and the apparent disconnect between real exchange rate movements and fundamentals (e.g., Backus and Smith 1993). In Heathcote and Perri (2008) we show that introducing taste shocks in this class of models increases real exchange rate volatility and lowers the correlation between the real exchange rate and relative consumption, while preserving the home bias result. Thus exploring the origins of taste-like shocks may prove a fruitful avenue for future research. At the same time, in an accounting sense real exchange rate volatility mostly reflects fluctuations in nominal exchange rates. Thus a satisfactory analysis of the joint dynamics of real exchange rates and standard macro aggregates should include a theory of nominal exchange rate determination.

In the meantime, this paper teaches us that international price movements and investment dynamics can help account for observed portfolio choice, just as they help explain business cycle and current account dynamics. The importance of these model elements in accounting for all these features of the data suggests that they should be incorporated in future work to address the remaining open puzzles in international finance.

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Appendix A. First-Order Conditions and Definition of Equilibrium

First-Order Conditions

The domestic households' first-order condition for domestic and foreign stock purchases are, respectively,

$$
\begin{aligned}
(27)\qquad U_c(s^t)P(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c(s^t,s_{t+1})\left[d(s^t,s_{t+1})+P(s^t,s_{t+1})\right] \\
U_c(s^t)e(s^t)P^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c(s^t,s_{t+1})e(s^t,s_{t+1})\left[d^*(s^t,s_{t+1})+P^*(s^t,s_{t+1})\right],
\end{aligned}
$$

where we use $U_c(s^t)$ for $\frac{\partial U\left(c(s^t),n(s^t)\right)}{\partial c(s^t)}$ and (s^t, s_{t+1}) denotes the $t+1$ length history s^t followed by s_{t+1}.

The foreign households' first-order condition for domestic and foreign stock purchases are

$$
\begin{aligned}
(28)\qquad U_c^*(s^t)\frac{P(s^t)}{e(s^t)} &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[\frac{d\left(s^t,s_{t+1}\right)+P(s^t,s_{t+1})}{e\left(s^t,s_{t+1}\right)}\right] \\
U_c^*(s^t)P^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[d^*(s^t,s_{t+1})+P^*(s^t,s_{t+1})\right].
\end{aligned}
$$

The domestic and foreign household's first-order conditions for labor supply are

$$
\begin{aligned}
(29)\qquad U_c(s^t)q_a(s^t)w(s^t)+U_n(s^t) &\geq 0 \\
&= \text{ if } n(s^t)>0
\end{aligned}
$$

$$
\begin{aligned}
(30)\qquad U_c^*(s^t)q_b^*(s^t)w^*(s^t)+U_n^*(s^t) &\geq 0 \\
&= \text{ if } n^*(s^t)>0.
\end{aligned}
$$

The domestic and foreign intermediate firms' first-order conditions for labor demand are

$$(31)\qquad w(s^t)=(1-\theta)F\left(z(s^t),k(s^{t-1}),n(s^t)\right)/n(s^t)$$

$$(32)\qquad w^*(s^t)=(1-\theta)F\left(z^*(s^t),k^*(s^{t-1}),n^*(s^t)\right)/n^*(s^t).$$

The corresponding first-order conditions for $k(s^t)$ and $k^*(s^t)$ are

$$(33)\qquad Q(s^t)=\sum_{s_{t+1}\in S} Q(s^t,s_{t+1})\left[q_a(s^t,s_{t+1})\theta F\left(z(s^t,s_{t+1}),k(s^t),n(s^t,s_{t+1})\right)/k(s^t)+(1-\delta)\right]$$

(34)
$$Q^*(s^t)=\sum_{s_{t+1}\in S} Q^*(s^t,s_{t+1})\left[q_b^*(s^t,s_{t+1})\theta F\left(z^*(s^t,s_{t+1}),k^*(s^t),n^*(s^t,s_{t+1})\right)/k^*(s^t)+(1-\delta)\right].$$

The first-order conditions for domestic and foreign final goods firms are

$$\begin{array}{llr} (35) & q_a(s^t) = \omega G(a(s^t), b(s^t))/a(s^t), & q_b(s^t) = (1-\omega)G(a(s^t), b(s^t))/b(s^t), \\ & q_b^*(s^t) = \omega G^* \left(a^*(s^t), b^*(s^t)\right)/b^*(s^t), & q_a^*(s^t) = (1-\omega)G^* \left(a^*(s^t), b^*(s^t)\right)/a^*(s^t). \end{array}$$

Definition of Equilibrium

An equilibrium is a set of quantities $c(s^t)$, $c^*(s^t)$, $k(s^t)$, $k^*(s^t)$, $n(s^t)$, $n^*(s^t)$, $a(s^t)$, $a^*(s^t)$, $b(s^t)$, $b^*(s^t)$, $\lambda_H(s^t)$, $\lambda_H^*(s^t)$, $\lambda_F(s^t)$, $\lambda_F^*(s^t)$, prices $P(s^t)$, $P^*(s^t)$, $r(s^t)$, $r^*(s^t)$, $w(s^t)$, $w^*(s^t)$, $Q(s^t)$, $Q^*(s^t)$, $q_a(s^t)$, $q_a^*(s^t)$, $q_b(s^t)$, $q_b^*(s^t)$, productivity shocks $z(s^t)$, $z^*(s^t)$ and probabilities $\pi(s^t)$ for all s^t and for all $t \geq 0$,which satisfy the following conditions:

1. The first-order conditions for intermediate goods purchases by final goods firms (eq. 35)
2. The first-order conditions for labor demand by intermediate-goods firms (eqs. 31 and 32)
3. The first-order conditions for labor supply by households (eqs. 29 and 30)
4. The first-order conditions for capital accumulation (eqs. 33 and 34)
5. The market clearing conditions for intermediate goods a and b :

$$\begin{array}{lrcl} (36) & a(s^t) + a^*(s^t) & = & F\left(z(s^t), k(s^{t-1}), n(s^t)\right) \\ & b(s^t) + b^*(s^t) & = & F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right). \end{array}$$

6. The market-clearing conditions for final goods:

$$\begin{array}{lrcl} (37) & c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) & = & G\left(a(s^t), b(s^t)\right) \\ & c^*(s^t) + k^*(s^t) - (1-\delta)k^*(s^{t-1}) & = & G^*\left(a^*(s^t), b^*(s^t)\right). \end{array}$$

7. The market-clearing condition for stocks:

$$(38) \quad \lambda_H(s^t) + \lambda_H^*(s^t) = 1 \qquad \lambda_F(s^t) + \lambda_F^*(s^t) = 1.$$

8. The households' budget constraints (eqs. 6 and 7)
9. The households' first-order conditions for stock purchases (eqs. 27 and 28)
10. The probabilities $\pi(s^t)$ are consistent with the stochastic processes for $\left[z(s^t), z^*(s^t)\right]$

Appendix B: Proofs

Proof of proposition 1

We prove that there is a competitive equilibrium in which portfolios are described by eq. (11) and that allocations in this equilibrium deliver perfect risk sharing by showing that these portfolios decentralize the solution to an equal-weighted planner's problem in the same environment. In particular, we consider the problem of a planner who seeks to maximize the equally weighted expected utilities of the domestic and foreign agents, subject only to resource constraints of the form (36) and (37). We then describe a set of candidate prices such that if the conditions that define a solution to the planner's problem are satisfied, then the conditions that define a competitive equilibrium in the stock trade economy are also satisfied at these prices when portfolios are given by eq. (11).

Let $G(s^t)$ and $F(s^t)$ be compact notations for $G(a(s^t), b(s^t))$ and $F\left(z(s^t), k(s^{t-1}), n(s^t)\right)$.

The equations that characterize a solution to the planner's problem are

1. First-order conditions for hours:

$$
\begin{aligned}
U_c(s^t)\frac{\omega G(s^t)}{a(s^t)}\frac{(1-\theta)F(s^t)}{n(s^t)} + U_n(s^t) &\geq 0 \\
U_c^*(s^t)\frac{\omega G^*(s^t)}{b^*(s^t)}\frac{(1-\theta)F^*(s^t)}{n^*(s^t)} + U_n^*(s^t) &\geq 0.
\end{aligned}
$$

2. First-order conditions for allocating intermediate goods across countries:

$$
\begin{aligned}
U_c(s^t)\omega G(s^t)/a(s^t) &= U_c^*(s^t)(1-\omega)G^*(s^t)/a^*(s^t) \\
U_c(s^t)(1-\omega)G(s^t)/b(s^t) &= U_c^*(s^t)\omega G^*(s^t)/b^*(s^t).
\end{aligned}
$$

3. First-order conditions for investment:

$$
\begin{aligned}
\widetilde{Q}(s^t) &= \sum_{s_{t+1}\in S}\widetilde{Q}(s^t, s_{t+1})\left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})}\frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta)\right] \\
\widetilde{Q}^*(s^t) &= \sum_{s_{t+1}\in S}\widetilde{Q}^*(s^t, s_{t+1})\left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})}\frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta)\right],
\end{aligned}
$$

   where

$$
\begin{aligned}
\widetilde{Q}(s^t) &= \frac{1}{2}\pi(s^t)\beta^t U_c(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t)\frac{(1-\omega)}{\omega}\frac{G^*(s^t)}{G(s^t)}\frac{a(s^t)}{a^*(s^t)} \\
\widetilde{Q}^*(s^t) &= \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c(s^t)\frac{\omega}{(1-\omega)}\frac{G(s^t)}{G^*(s^t)}\frac{a^*(s^t)}{a(s^t)}.
\end{aligned}
$$

4. Resource constraints of the form (36) and (37).

Consider the set of allocations that satisfies this set of equations, i.e., the solution to the planner's problem. We now show there exists a set of prices at which these same allocations also satisfy the set of equations defining equilibrium in the stock trade economy (see Appendix A), given the portfolios described in eq. (11). In other words, we can decentralize the complete markets allocations with asset trade limited to two stocks and constant portfolios.

Let intermediate-goods prices be given by eq. (35). Then condition (1) for the stock trade economy is satisfied. Let wages be given by eqs. (31) and (32). Then condition (2) for the stock trade economy is satisfied. Substituting these prices into condition (1) from the planner's problem gives condition (3) for the stock trade economy. Let the real exchange rate be given by eq. (5). Then combining conditions (2) and (3) from the planner's problem gives condition (4) for the stock trade economy. Condition (4) from the planner's problem translates directly into conditions (5) and (6) for the stock trade economy. Condition (7)—stock market clearing—follows immediately from the symmetry of the candidate stock purchase rules.

Condition (8) is that households' budget constraints are satisfied. Given constant portfolios, the domestic household's budget constraint simplifies to

$$c(s^t) = q_a(s^t)w(s^t)n(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t).$$

Substituting in the candidate function for $w(s^t)$, the resource constraint for intermediate goods, and the definitions for dividends (and suppressing the state-contingent notation) gives

$$c = q_a(1-\theta)(a+a^*) + \lambda\left(q_a\theta(a+a^*) - x\right) + (1-\lambda)e\left(q_b^*\theta(b+b^*) - x^*\right).$$

Using the candidate expression for the real exchange rate gives

$$c = (1-\theta+\lambda\theta)\left(q_a a + e q_a^* a^*\right) - \lambda x + (1-\lambda)\theta\left(q_b b + e q_b^* b^*\right) - (1-\lambda)ex^*.$$

Now using the candidate expressions for intermediate goods prices and collecting terms gives

$$c = [\omega + (1-\lambda)(\theta - 2\omega\theta)]\, G + e\,[(1-\omega) - (1-\lambda)\,(\theta-2\omega\theta)]\, G^* - \lambda x - (1-\lambda)ex^*.$$

Using the resource constraint for final goods firms gives

$$\begin{aligned} G \;&=\; [\omega + (1-\lambda)(\theta - 2\omega\theta)]\, G + e\,[(1-\omega) - (1-\lambda)\,(\theta - 2\omega\theta)]\, G^* \\ &\quad +(1-\lambda)(G-c) - (1-\lambda)e(G^*-c^*). \end{aligned}$$

Given the candidate expression for the real exchange rate, and exploiting the assumption that utility is logarithmic in consumption, condition (2) for the planner's problem implies

$$c = ec^*.$$

Thus, the budget constraint can be rewritten as

$$G = [\omega + (1-\lambda)(1+\theta-2\omega\theta)]\, G + e\,[(1-\omega) - (1-\lambda)\,(1+\theta-2\omega\theta)]\, G^*.$$

Finally, substituting in the candidate expression for λ (eq. 11) confirms that the domestic consumer's budget constraint is satisfied. The foreign consumer's budget constraint is satisfied by Walras' law.

Condition (9) is the households' intertemporal first-order conditions for stock purchases. Substituting condition (2) from the planner's problem into condition (3), the planner's first-order conditions for investment may be rewritten as

$$
\begin{aligned}
U_c(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta) \right] \\
U_c^*(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta) \right].
\end{aligned}
$$

Multiplying both sides of the first (second) of these two equations by $k(s^t)$ ($k^*(s^t)$) gives

$$
\begin{aligned}
U_c(s^t) k(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \theta F(s^t, s_{t+1}) + (1-\delta) k(s^t) \right] \\
U_c^*(s^t) k^*(s^t) &= \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \theta F^*(s^t, s_{t+1}) + (1-\delta) k^*(s^t) \right].
\end{aligned}
$$

Let stock prices be given by

(39) $P(s^t) = k(s^t),\ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.$

Substituting these candidate prices for stocks, the prices for intermediate goods, the wage, and the expressions for dividends into the planner's first-order conditions for investment gives the domestic household's first-order condition for domestic stock purchases and the foreign household's first-order condition for foreign stock purchases. The remaining two first-order conditions for stock purchases follow immediately by substituting condition (2) from the planner's problem into these two conditions.

Proof of proposition 2

Equilibrium portfolios deliver perfect risk sharing, which implies that the value of consumption is equated across countries, eq. (15). Moreover, since there is no asset trade in equilibrium, the value of income is also equated across countries. Log linearizing,

$$
\begin{aligned}
& \bar{l}\widehat{l}(s^t) + \lambda \bar{d}\hat{d}(s^t) + (1-\lambda)\bar{d}\hat{d}^*(s^t) + (1-\lambda)\hat{e}(s^t)\bar{d}(s^t) \\
\approx\ & \bar{l}\hat{l}^*(s^t) + \bar{l}\hat{e}(s^t) + (1-\lambda)\bar{d}\hat{d}(s^t) + \lambda \bar{d}\hat{d}^*(s^t) + \lambda \bar{d}\hat{e}(s^t),
\end{aligned}
$$

which implies

$$
\bar{l}\Delta\widehat{l}(s^t) \approx -(2\lambda - 1)\bar{d}\Delta\hat{d}(s^t),
$$

which implies

$$
cov\left(\Delta\widehat{l}(s^t), \Delta\hat{d}(s^t)\right) \approx -\frac{\bar{d}}{\bar{l}}(2\lambda - 1) var(\Delta\hat{d}(s^t))
$$

and thus

$$
\begin{aligned}
1-\lambda &\approx \frac{1}{2}\left(1+\frac{\bar{l}}{\bar{d}}\frac{cov\left(\Delta\hat{l}(s^t),\Delta\hat{d}(s^t)\right)}{var\left(\Delta\hat{d}(s^t)\right)}\right) \\
&= \frac{1}{2}\left(1+\left(\frac{1-\theta}{\theta}\frac{\rho+\delta}{\rho}\right)M\right),
\end{aligned}
$$

where the last line is the expression for diversification in eq. (14). The expression for M in eq. (13) follows from combining the two alternative expressions for diversification in eqs. (11) and (14).

Appendix C: Data

The countries we use in our empirical analysis are the set of countries that have been in the OECD throughout the period 1990-2007 and for which we could get consistent and comparable series for quarterly GDP, investment (needed to construct the covariance ratios M), for foreign assets, trade and capital stock. This group includes the following 19 countries (the codes used in Figures 2 and 3 are in parentheses): Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Japan (JPN), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Spain (ESP), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR), United States (USA). The data on gross international diversification positions (total foreign assets and foreign liabilities) are in US dollars and are from Lane and Milesi-Ferretti (2007).

We now describe how we construct our two measures of $K_{i,t}$, the capital stock of country i in period t. Our first (baseline) measure is constructed using the standard perpetual inventory method, and we denote it as $K_{i,t}^P$. We construct $K_{i,t}^P$ by multiplying GDP in each country by the capital-output ratio $\left(\frac{k}{y}\right)_{i,t}$ in that period. In 1989 we take capital-output ratios directly from Dhareshwar and Nehru (1993), who report both physical capital stock and GDP figures. After 1989 we construct $\left(\frac{k}{y}\right)_{i,t+1}$ using the following recursion:

$$
\left(\frac{k}{y}\right)_{i,t+1}\left(\frac{y_{i,t+1}}{y_{i,t}}\right)=(1-\delta)\left(\frac{k}{y}\right)_{i,t}+\left(\frac{x}{y}\right)_{i,t},
$$

where $\left(\frac{y_{i,t+1}}{y_{i,t}}\right)$ is the growth rate of GDP for country i (in constant PPP dollars from the OECD Quarterly National Accounts), $\left(\frac{x}{y}\right)_{i,t}$ is the ratio of investment to GDP (both in current prices, from the OECD Quarterly National Accounts), δ is the capital depreciation rate which we set equal to 6 percent for all countries and for all years (this value is also used by Kraay et al. 2005, and is consistent with the depreciation rate of capital in the model). Our second measure uses the fact that part of capital comprises assets of firms quoted on the stock market, and thus we can measure the growth of the value of this capital simply by measuring the growth of stock prices. For the remaining (non-publicly-traded) portion of $K_{i,t}$ we assume the growth rate corresponds to the growth of the capital stock computed using the perpetual inventory method $K_{i,t}^P$. We denote our alternative measure (in US dollars) of the capital stock K_{it}^S. We first assume that in 1989

$K^S_{i,t} = K^P_{i,t}$, while for subsequent years we use the following recursion:

$$K^S_{i,t+1} = (K^S_{i,t} - S_{i,t})g(K^P_{i,t}) + S_{i,t}g(P_{i,t})$$

where $S_{i,t}$ is the value of the stock market in country i at year t (from the World Bank Global Development Indicators), $g(K^P_{i,t})$ is the growth rate of the perpetual inventory method capital stock in country i, and $g(P_{i,t})$ is the growth rate for stock prices in country i (computed using the growth rate of the Morgan Stanley Capital International (MSCI) standard stock price index for country i). The complete dataset is available online on the authors' websites.

Appendix D: Controlling for nominal exchange rate risk

This is a simplified version of the model described in the text, extended to include nominal exchange rate risk and a forward foreign exchange market. There are two perfectly symmetric countries. Agents first make portfolio decisions, buying domestic and foreign equities and forward contracts. Then uncertainty is resolved, revealing the domestic and foreign dividends d and d^*, domestic and foreign labor incomes l and l^*, domestic and foreign price levels p and p^*, and the nominal exchange rate (price of foreign relative to domestic currency) s. We denominate d, d^*, l, and l^* in units of local currency (dollars) and assume that d, d^*, l, l^*, and s are jointly normally distributed.

The forward market works as follows. Prior to the resolution of uncertainty, agents can commit to purchase foreign currency at price f. Let μ (μ^*) denote the units of such forward contracts purchased by the domestic (foreign) representative agent. We set $f = E[s] = 1$, assume $E[d] = E[d^*]$, and normalize initial dollar wealth to one for both the domestic representative and foreign agents.

Given symmetry, equilibrium domestic and foreign stock prices are also both equal to one, and the fraction of wealth that domestic (foreign) agents invest in domestic (foreign) stocks is equal in equilibrium to the fraction of domestic (foreign) stocks owned by domestic (foreign) agents.

Thus, the problem for a representative domestic agent can be written as

$$\max_{\lambda,\mu} E\left[\log c\right]$$

subject to

$$c = \frac{\lambda d + (1-\lambda)d^* + \mu(s-f) + l}{p}.$$

The analogous problem for the foreign agent is

$$\max E\left[\log c^*\right]$$

subject to

$$c^* = \frac{\lambda^* d^* + (1-\lambda^*)d + \mu^*(s-f) + l^*}{p^* s}.$$

Note that since dividends and wages are denominated in dollars, to convert income into units of real consumption, we must divide by the nominal exchange rate to convert to units of foreign currency, and then divide by the foreign price level p^* (the price of a unit of foreign consumption in units of foreign currency).

The two first-order conditions for the representative domestic agent with respect to λ and μ are

$$\begin{aligned} E\left[u'(c)\frac{d}{p}\right] &= E\left[u'(c)\frac{d^*}{p}\right] \\ E\left[u'(c)\frac{s}{p}\right] &= E\left[u'(c)\frac{f}{p}\right]. \end{aligned}$$

We now proceed, via a series of steps, to provide a closed-form characterization of the expressions for λ and μ that solve these two equations.

Step 1. With log preferences, price-level risk plays no role in portfolio choice. In particular, the first-order conditions can be written as

$$\begin{aligned} E\left[u'(\tilde{c})d\right] &= E\left[u'(\tilde{c})d^*\right] \\ E\left[u'(\tilde{c})s\right] &= E\left[u'(\tilde{c})f\right], \end{aligned}$$

where

$$\tilde{c} = \lambda d + (1-\lambda)d^* + \mu(s-f) + l.$$

Given $E[d] = E[d^*]$, $E[s] = f$ and $cov(u'(\tilde{c}), f) = 0$, these simplify to

$$\begin{aligned} cov\left(u'(\tilde{c}), d\right) &= cov\left(u'(\tilde{c}), d^*\right) \\ cov\left(u'(\tilde{c}), s\right) &= 0. \end{aligned}$$

Step 2. Recall that d, d^*, s, and l are jointly Normally distributed. Then $\tilde{c}$ is the sum of Normals and is itself Normal, and $\tilde{c}$ on the one hand and d, d^*, and s on the other are jointly Normal. Then, using Stein's lemma, the first-order conditions can be rewritten as

$$\begin{aligned} cov(\tilde{c}, \Delta d) &= 0 \\ cov(\tilde{c}, s) &= 0, \end{aligned}$$

where $\Delta d = d^* - d$.

Step 3. At this point, we substitute in the definition for $\tilde{c}$ and use these two equations to solve for λ and μ :

$$cov(\tilde{c}, \Delta d) = -\lambda var(\Delta d) + cov(d^*, \Delta d) + \mu cov((s-f), \Delta d) + cov(l, \Delta d)$$

$$\Rightarrow \lambda = \frac{cov(d^*, \Delta d) + \mu cov((s-f), \Delta d) + cov(l, \Delta d)}{var(\Delta d)}.$$

Similarly,

$$cov(\tilde{c}, s) = -\lambda cov(\Delta d, s) + cov(d^*, s) + \mu var(s) + cov(l, s)$$

$$\Rightarrow \mu = \frac{\lambda cov(\Delta d, s) - cov(d^*, s) - cov(l, s)}{var(s)}.$$

Substituting the expression for μ into the expression for λ gives

$$\lambda\left(var(\Delta d)-\frac{cov(\Delta d,s)^2}{var(s)}\right)=cov(d^*,\Delta d)-\frac{cov(d^*,s)cov(\Delta d,s)}{var(s)}+cov(l,\Delta d)-\frac{cov(l,s)cov(\Delta d,s)}{var(s)}.$$

Step 4. Following the same steps for the representative foreign agent gives a similar expression for λ^* except that the signs on the $cov(d,\Delta d)$ and $cov(l^*,\Delta d)$ are flipped because we maintain the definition $\Delta d = d^* - d$:

$$\lambda^*\left(var(\Delta d)-\frac{cov(\Delta d,s)^2}{var(s)}\right)=-cov(d,\Delta d)+\frac{cov(d,s)cov(\Delta d,s)}{var(s)}-cov(l^*,\Delta d)+\frac{cov(l^*,s)cov(\Delta d,s)}{var(s)}.$$

Note that nominal exchange rate risk drops immediately out of the foreign agent's first-order conditions, just like price-level risk.

Step 5. Define our empirical measure for diversification as $1-\bar{\lambda}=\left((1-\lambda)+(1-\lambda^*)\right)/2$. Then,

$$\begin{aligned}
1-\bar{\lambda} &= 1-\frac{1}{2}\frac{var(\Delta d)-\frac{cov(\Delta d,s)^2}{var(s)}-cov(\Delta l,\Delta d)+\frac{cov(\Delta l,s)cov(\Delta d,s)}{var(s)}}{var(\Delta d)-\frac{cov(\Delta d,s)^2}{var(s)}} \\
&= \frac{1}{2}+\frac{1}{2}\frac{cov(\Delta l,\Delta d)-\frac{cov(\Delta l,s)cov(\Delta d,s)}{var(s)}}{var(\Delta d)-\frac{cov(\Delta d,s)^2}{var(s)}},
\end{aligned}$$

where $\Delta l = l^* - l$.

Step 6. In our empirical analysis we work with log differences rather than absolute differences. For a generic variable x, let $\bar{x}=E[x]$ and define $\hat{x}=\log x-\log\bar{x}$. Then,

$$x \approx \bar{x}\left(1+\hat{x}\right).$$

Thus,

$$1-\bar{\lambda}\approx\frac{1}{2}+\frac{1}{2}\frac{\bar{l}\bar{d}}{\bar{d}^2}\frac{cov(\Delta\hat{l},\Delta\hat{d})-\frac{cov(\Delta\hat{l},\hat{s})cov(\Delta\hat{d},\hat{s})}{var(\hat{s})}}{var(\Delta\hat{d})-\frac{cov(\Delta\hat{d},\hat{s})^2}{var(\hat{s})}}.$$

Step 7. At this point, note that if we regress any log variable $\hat{x}$ on the log nominal exchange rate $\hat{s}$, we get a coefficient $\beta_x = cov(\hat{x},\hat{s})/var(\hat{s})$ and an implied residual

$$\hat{x}_\varepsilon=\hat{x}-\frac{cov(\hat{x},\hat{s})}{var(\hat{s})}\hat{s}.$$

If we regress a second variable $\hat{y}$ on the nominal exchange rate and compute the covariance between the residuals $\hat{x}_\varepsilon$ and $\hat{y}_\varepsilon$, we get

$$cov(\hat{x}_\varepsilon,\hat{y}_\varepsilon)=cov(\hat{x},\hat{y})-\frac{cov(\hat{x},\hat{s})cov(\hat{y},\hat{s})}{var(\hat{s})}.$$

Given this result, it is immediate that the expression for $1-\bar{\lambda}$ can be written as

$$
\begin{aligned}
1-\bar{\lambda} &\approx \frac{1}{2}\left(1+\frac{\bar{l}\bar{d}}{\bar{d}^2}\frac{cov(\Delta\hat{l}_\varepsilon, \Delta\hat{d}_\varepsilon)}{var(\Delta\hat{d}_\varepsilon)}\right) \\
&= \frac{1}{2}\left(1+\frac{\bar{l}}{\bar{d}}M\right),
\end{aligned}
$$

which is exactly what we have in the paper (see eq. 14 and the proof of Proposition 2).

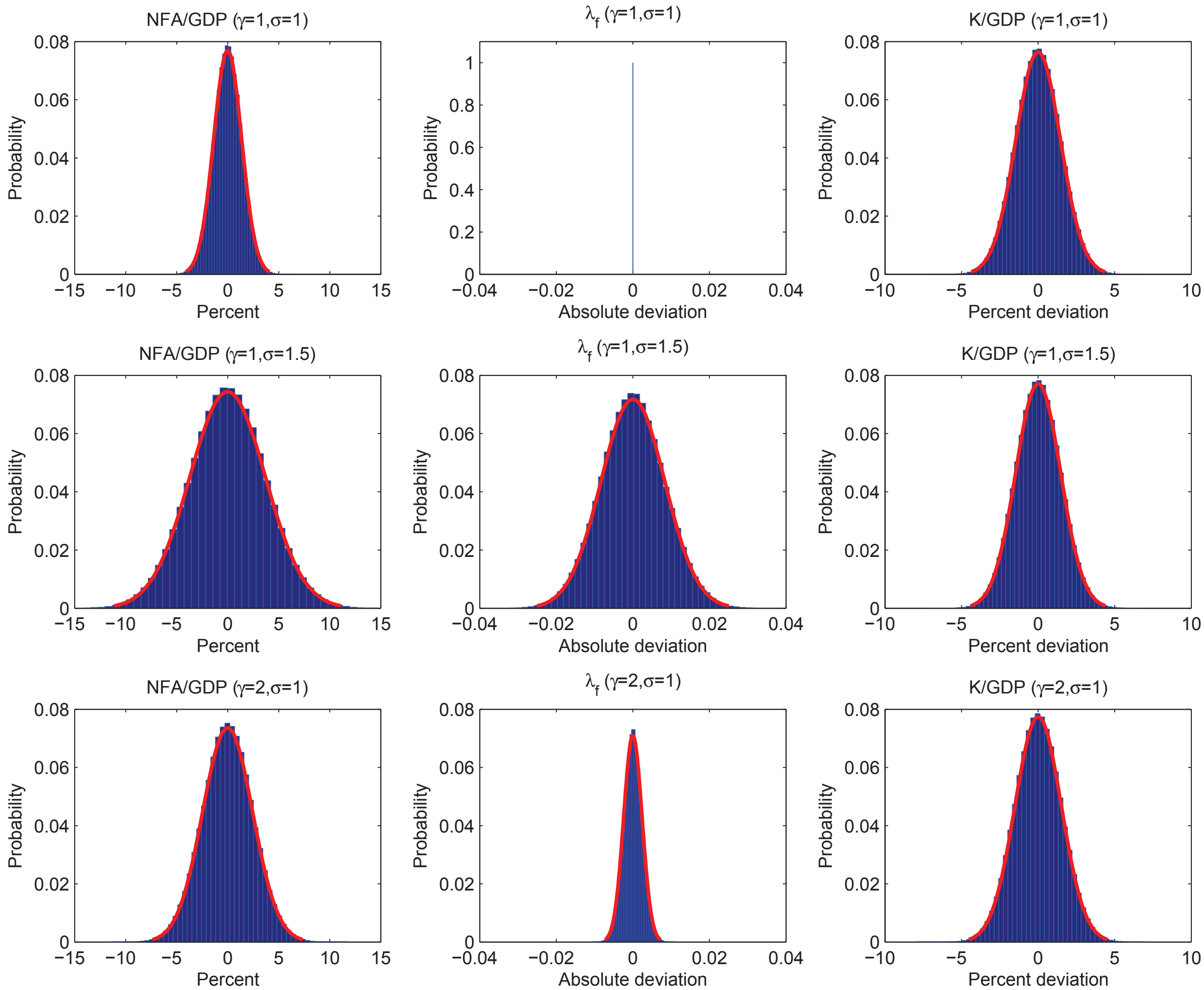
Appendix E: Numerical solution methods

For a local solution method to be appropriate, it is important to verify that all equilibrium variables, and in particular equilibrium diversification and the net foreign asset position, are stationary. To say anything about the dynamics of diversification, we need to approximate the optimality conditions for asset purchases to the third order, since with a second-order approximation to those conditions, it is impossible to characterize dynamics (see Devereux and Sutherland 2010). In addition, in the context of our model, it is important that the equilibrium features trade in stocks when assessing the stationarity of the net foreign asset position, since absent trade, changes in the net foreign asset position could be driven only by changes in relative asset prices.

We therefore solve the model using a third-order approximation to decision rules for asset purchases and a second-order approximation to the other equilibrium conditions. Given the resulting decision rules, we run many simulations of the model, where each simulation has an increasing number of periods. We find that as the number of periods gets large, the numerical densities of each variable in the model converge to an invariant distribution, indicating stationarity of our model. Figure $E1$ plots the long-run pdf of net foreign assets relative to GDP, of equilibrium portfolio diversification (λ_f in the figure) and of the capital-to-output ratio (K/GDP in the figure). The figure documents these distributions for our baseline calibration and for two alternatives: one with $\sigma = 1.5$ and another with $\gamma = 2$. In each case, the average net foreign asset position is zero, and the average values for diversification correspond to those shown in figure 4.

The reader might wonder why we get stationarity while standard two-country incomplete-markets models that are solved using local linear or quadratic approximations display nonstationary dynamics for the net foreign asset position (see, for example, Heathcote and Perri 2002). Stationarity in our model obtains because we solve the model to the third order of approximation. Indeed, if a standard two-country incomplete-markets model (as, for example, in Baxter and Crucini 1995) is solved using third order approximations to decision rules, the resulting net foreign asset position is stationary. Intuitively, a third-order approximation is necessary to capture the property that with CRRA preferences, the precautionary motive to accumulate additional net wealth is declining in current net worth, and thus the equilibrium net foreign asset position is mean reverting. For more details on the link between the order of numerical approximation and the dynamics of net foreign assets, see Heathcote, Perri, and Steinberg (2012).

Figure E1: Long-run distributions of selected variables



Appendix F: Calibration of baseline model and business cycle statistics

Table F1 reports the parameter values used to plot the model impulse responses in figure 1 and to derive business cycle statistics in table F2. Preference and technology parameters $\beta, V(.), \theta, \delta$ and ω are set in a standard fashion and they yield, respectively, a steady state real interest rate of 4 percent, a Frisch elasticity of labor supply of 1, a share of GDP going to capital of 36 percent, a steady state capital-output ratio of 2.34, and an import share of 15 percent. Regarding the parameters for technology shocks, we use the transition matrix for the shocks (z, z^*) estimated in Heathcote and Perri (2004). We then pick the variance and covariance matrix of the shock innovations $(\varepsilon, \varepsilon^*)$ so that the model matches the standard deviation of US GDP and the correlation of US GDP with an aggregate of foreign GDPs over the period 1980.1 to 2007.4.

Table F1: Parameter values

Preferences	
Discount factor	$\beta = 0.99$
Disutility from labor	$V(n) = \frac{n^2}{2}$
Technology	
Capital's share	$\theta = 0.36$
Depreciation rate	$\delta = 0.015$
Import share	$1 - \omega = 0.15$
Productivity Process	

$$\begin{bmatrix} z(s^t) \\ z^*(s^t) \end{bmatrix} = \begin{pmatrix} 0.91 & 0.00 \\ 0.00 & 0.91 \end{pmatrix} \begin{bmatrix} z(s^{t-1}) \\ z^*(s^{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{bmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.007^2 & 0.38 * 0.007^2 \\ 0.38 * 0.007^2 & 0.007^2 \end{pmatrix} \right)$$

Once the model has been parameterized we can use it to produce a standard set of business cycle statistics, which we report in table F2. Alongside the model statistics we report two sets of corresponding empirical statistics. One set is for the United States. The other is the median value for each statistic within our sample of 19 countries described in Appendix C.

Table F2: Business cycle statistics: baseline model and data

		Domestic volatilities					
		$\sigma(y)$	$\frac{\sigma(c)}{\sigma(y)}$	$\frac{\sigma(n)}{\sigma(y)}$	$\frac{\sigma(x)}{\sigma(y)}$	$\sigma(nx/y)$	$\sigma(e)$
Model:		1.24	0.21	0.41	4.41	0.41	0.30
Data:	US	1.24	0.63	0.64	2.84	0.40	6.33
	Median of countries in sample	1.20	0.72	0.71	3.20	0.95	5.09
		Domestic comovement					
		$corr(y, nx)$	$corr(y, e)$	$corr(e, \frac{c}{c^*})$			
Model:		-0.51	0.42	1			
Data:	US	-0.51	-0.01	-0.04			
	Median of countries in sample	-0.29	0.07	-0.01			
		International comovement					
		$corr(y, y^*)$	$corr(c, c^*)$	$corr(n, n^*)$	$corr(x, x^*)$		
Model:		0.43	0.39	0.51	0.22		
Data:	US	0.43	0.24	0.37	0.27		
	Median of countries in sample	0.47	0.27	0.35	0.47		

Note: All data are from the OECD Quarterly National Accounts and Main Economic Indicators. The sample for the data statistics is 1980.1–2007.4. For all statistics except employment the sample of countries is the one listed in Appendix C. For employment statistics the sample is G7 countries. A star next to a variable denotes a foreign variable i.e. an aggregate of all the countries in the sample except the domestic country. The variable y denotes real GDP, c denotes real consumption (including both private and public), n denotes civilian employment, x denotes real gross fixed capital formation, $\frac{nx}{y}$ denotes net exports over GDP (all nominal), e denotes the CPI based real exchange rate, which is a weighted average of bilateral real exchange rates of the domestic country with all the other countries in our sample, where the weights are proportional to real GDP. All variables except net exports are in logs. All variables are HP filtered with a smoothing parameter of 1600. Statistics from the model are produced by simulating the model for 112 periods and taking averages over 20 simulations.

Comparing model and data, we conclude that our baseline model shares the successes and failures of this class of international business cycle models. It does reasonably well in reproducing observed volatility and international co-movement in quantities (i.e., GDP, consumption, employment, investment, and net exports). It can even rationalize the so-called quantity anomaly, namely, the fact that the cross-country correlation of consumption is lower than the corresponding correlation for output. The model generates slightly more investment volatility than is observed in the data. This discrepancy could be addressed by introducing standard capital adjustment costs in the model, though such an extension would mean losing our closed-form expression for portfolios. We have solved numerically for equilibrium portfolios with capital adjustment costs calibrated to exactly replicate observed investment volatility. We found that portfolios in that model are almost identical to those in the baseline model without such costs. A more serious shortcoming of the model is that the real exchange rate is much less volatile and more pro-cyclical than in the data. In Section 4.1 of the paper we discuss this issue and explain why this discrepancy does not necessarily undermine our results on diversification.