The Great Resignation and Optimal Unemployment Insurance

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Abstract

How generous should social insurance be when quits account for a large share of transitions into non-employment? We address this question using a multi-sector directed search model extended to incorporate endogenous quits both to other jobs and to non-employment. Workers quit too often in the competitive equilibrium, and private markets co-ordinate on excessively high "efficiency" wages. Quantitatively, we find that unemployment insurance is optimally much less generous in an economy with quits than in one without. An extended Baily-Chetty formula is derived to illustrate the source of this difference.

JEL codes: E24, J31, J64, J65

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1 Introduction

In most labor search models, the rate at which workers transition to unemployment is exogenous. Optimal unemployment insurance (UI) design has focused on the trade-off between consumption smoothing versus preserving job search incentives. However, most jobs end because the worker quits, not because the worker is laid off. Some of these quits are immediate job-to-job transitions. But a large and growing share of quits are not immediately followed by a new job. How does the presence of a quitting margin change the optimal provision of UI? Should workers who chose to quit be treated differently from those who were fired? In practice, there is no consensus on these questions: quitters are eligible for UI benefits in many countries (after varying waiting periods) but are generally ineligible in the United States.[1]

We extend a directed search and matching model to incorporate quits. Workers in our model are subject to idiosyncratic shocks to the disutility of work and may choose to quit to non-employment when these shocks are large. In addition, we allow for on-the-job search, so a portion of quits involve immediate transitions to a new job.

Workers in the model are risk averse, while firms are risk neutral. Firms have commitment and can commit to dynamic wage contracts. Crucially, firms do not observe worker preference shocks and cannot directly verify the existence of outside offers. Our focus is on understanding how quits affect the optimal level of unemployment insurance. To start, we assume that the government cannot differentiate between workers who are non-employed because they were laid off versus those who are non-employed because they chose to quit.

We show that in this setting, workers tend to quit inefficiently often, breaking up matches that have positive joint surplus. In equilibrium, this translates into workers directing search toward high wage jobs: high "efficiency" wages partially mitigate the excessive quitting problem. In addition, firms offer wage contracts with two features designed to reduce quitting. First, wages rise with tenure: backloading wage payments reduces incentives to quit to non-employment. Second, firms stochastically match outside offers, which reduces the rate at which workers quit to take other jobs.

We show that optimal policy addresses the excessive quitting inefficiency by reducing the optimal UI replacement rate, relative to the rate that would be optimal without the quit margin. We illustrate this result most starkly in a simple static version of the model in which workers are risk

[1] See Table B2 in Venn (2012) for details of country-specific rules.

neutral and there is no role for any policy intervention in the absence of the quitting margin (Acemoglu and Shimer, 1999). Once that margin is introduced, the optimal transfer to non-workers becomes negative.

In a dynamic version of the model with risk averse workers, we show how introducing the quitting margin changes the Baily-Chetty formula defining the optimal UI replacement rate. A new term appears in this equation, which reflects the cost of UI in terms of a higher equilibrium quit rate and thus lower offered wages. We show that while the equilibrium elasticity of the wage to replacement rate is small, the channel is quantitatively important for the optimal level of UI, since wages are depressed for all workers.

A different interpretation of quitting behavior is that it reflects workers trying to improve match quality (e.g., Acemoglu and Shimer 2000 or Marimon and Zilibotti 1999). Perhaps UI policy should not seek to discourage quitting if workers need to sample multiple jobs in order to find a good match and if search is easier when not employed. The quantitative version of our model features stochastic match quality, and we replicate the observed rate of job-to-job transitions and the average wage growth associated with those transitions. Most quitters to non-employment in our model are in low quality matches. Because reducing UI lowers the quit rate, it also therefore slows down the rate at which workers reallocate from bad to good matches. The design of optimal UI policy in our quantitative model therefore takes into account both the inefficiency and reallocation aspects of quitting behavior.

We calibrate the vacancy posting cost, the match efficiency parameter, and the average utility cost of work to match observed unemployment, the job opening rate, and the quit rate. To calibrate the cross-sectional variance of the utility cost of work, we exploit cross-industry variation in wages and quit rates. To the extent that quits are heavily concentrated in low wage industries (such as food services), the model indicates a large average utility cost of work, but modest cross-worker dispersion in that cost.

Our main quantitative findings are as follows. In our baseline calibration, the optimal UI replacement rate is 38.4 percent. In a version of the model in which the variance of preference shocks is very small, so that the equilibrium quit rate is near zero, the corresponding optimal replacement rate is 48.9 percent. Thus, incorporating quitting has a big impact on the optimal policy.

Suppose the government can differentiate between non-workers who quit versus those who were fired. How different should transfers be for the quitters versus those who were laid off? In

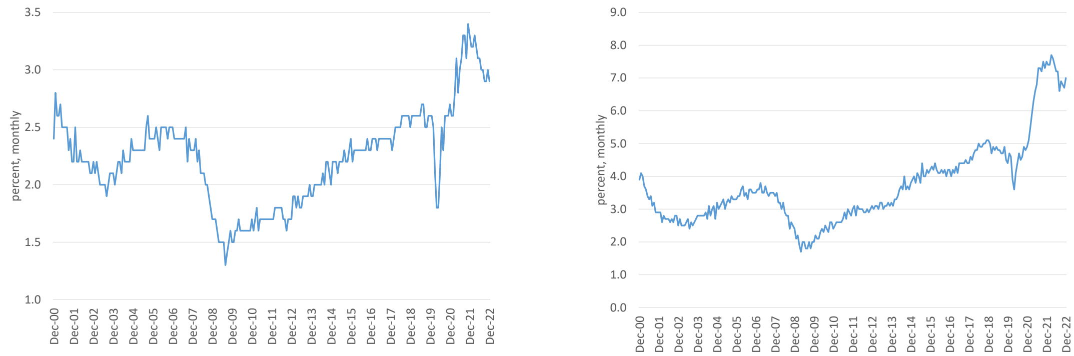

Figure 1: JOLTS Quit Rate (left) and Job Openings Rate (right)

our baseline calibration, we find a 48.5 percent optimal replacement rate for the former group, versus a much less generous 29.5 percent for the latter group. That suggests the government should penalize quitters, as a way to discourage excessive quitting. However, we find only a small welfare gain of 0.3 percent of consumption associated with moving from a universal non-worker benefit of 38.4 percent to this differentiated quit-versus-fired benefit model. To the extent that it is costly to elicit information about precisely how particular workers came to be non-employed, a universal benefit might be preferable.

We also use the model to interpret the rise in the quit rate between 2006 (the end of the pre-Global Financial Crisis expansion) and the summer of 2022. Data from JOLTS indicate much higher quit and job opening rates in 2021 relative to those in 2006, but a similar unemployment rate. The model can broadly replicate observed changes in unemployment, job openings and quits given a decline in the cost of posting vacancies. One possible interpretation is that new online platforms like Monster and Indeed have made it cheaper for firms to contact workers. We find that a decline in the vacancy posting cost reduces the optimal UI replacement in our model, because workers will quit more readily if they know that new jobs are easier to find. This translates into a two percentage point reduction in the optimal UI replacement rate between 2006 and 2021.

There is a large literature on optimal unemployment insurance. Baily (1978) and Chetty (2006) framed a trade-off between UI helping risk averse workers smooth consumption when unemployed, versus UI reducing incentives for job search and thus raising unemployment. Landais et al. (2018a,b) extend this analysis to consider a range of wage-setting mechanisms that might generate inefficient labor market tightness. We also develop a version of the Baily-Chetty formula, but in contrast to Landais et al. (2018a) our planner is not trying use UI as a lever to push market tightness in the efficient direction. In fact, in our directed search setup, tightness is efficient (conditional on the replacement rate). Rather, our planner wants to reduce the replacement rate – relative to what the standard Baily-Chetty equation would dictate – in order to reduce the quit rate and thereby boost equilibrium wages. In our richer quantitative model, the planner also internalizes how the UI replacement rate affects average match quality and thus productivity.

Acemoglu and Shimer (1999) was one of the first papers to study optimal unemployment insurance in a directed search setting with risk averse workers. Golosov et al. (2013) develop additional insights on the nature of optimal policy in a similar setting. Our key innovation relative to those papers is again to emphasize the impact of quitting on optimal policy design – both to non-employment and to other jobs – in a quantitative environment.[2]

There is strong evidence that economic considerations are important in quitting decisions. As we will show, quit rates decline quite steeply with earnings (see also Krueger and Summers 1988). In addition, fewer workers quit in recessions, suggesting that many quitters plan to return to work and are hesitant to quit when doing so is hard.

Our model predicts that more generous transfers should translate into a higher job separation rate and shorter employment duration (as long as quitters can receive those transfers). Consistent with this prediction, several papers document a dramatic spike in the fraction of jobs that end exactly at the moment that workers become eligible for UI.[3] These papers focus mostly on Canada, exploiting exogenous historical variation in the length of time required to build UI eligibility (see Christofides and McKenna 1996, Green and Riddell 1997, and Baker and Rea 1998). Note that at that time, quitters in Canada could collect UI benefits, as long as they were UI eligible, after a 6 week waiting period. Jager et al. (2023) find that a temporary expansion of UI benefits in Austria increased the job separation rate for treated workers by 11 percentage points (27 percent) over a five year period relative to a control group that did not receive extra benefits. Using a displaced workers survey, Jurajda (2003) estimates that being entitled to UI reduces employment durations in the United States. Schmieder et al. (2016) use age discontinuities in UI eligibility to estimate

[2]There is a related literature on how UI should optimally vary over the business cycle: see, for example, Mitman and Rabinovich (2015).

[3]There is a large literature on the impact of UI on *unemployment* duration. Johnston and Mas (2018) and Karahan et al. (2022) exploit an unexpected cut in maximum UI duration in Missouri in 2011; they find significant declines in non-employment duration and a large rise in market tightness. Ganong et al. (2022) find that the expansion of benefits during COVID had only a small negative impact on-the-job-finding rate, but argue that this outcome occurred because they were introduced at a time when the job finding rate was already depressed and thus could not fall much.

that UI extensions reduced job tenure for middle-aged workers in Germany.[4]

The empirical evidence suggests that the impact of UI on wages is small. Schmieder et al. (2016) estimate that UI extensions reduced wages in Germany. In contrast, Nekoei and Weber (2017) and Jager et al. (2020) find a small positive impact of UI on wages in Austria.[5] But Jager et al. (2020) conclude that they can reject an impact of UI on wages exceeding three cents per dollar increase in benefits. In our model, increasing UI leads workers to direct search toward higher wage jobs. In addition, more generous UI may boost wages by speeding up transitions to high quality matches. Working against these two conventional effects is our new channel, whereby more generous UI reduces wages by reducing expected job tenure. We will show that our model with quitting is consistent with the Jager et al. (2020) estimates, while an analogous model without quitting delivers a wage impact that exceeds their bound. But a key message of our paper is that the impact of UI on equilibrium wages via the quitting channel is a key determinant of the optimal UI replacement rate, even though the aggregate equilibrium impact on wages is very small.

There are many papers modeling quits, but these focus mostly on workers leaving their current jobs for different ones, so-called "job-to-job transitions". Early examples include Shimer (2006) in random search models and Delacroix and Shi (2006) in a directed search setting (see also Shi 2009 and Menzio and Shi 2011). More recently, Mercan and Schoefer (2020) and Elsby et al. (2022) explore the notion of "vacancy chains," illustrating the interactions between workers' quitting behavior and firm's replacement hiring, and how such interactions can lead to the amplification of labor market fluctuations.

Regarding how firms can attempt to reduce quitting, Salop (1979) was one of the first formal efficiency wage models, in which higher wages reduce turnover. Stevens (2004) and Burdett and Coles (2003) were the first to recognize that backloading compensation can further reduce quitting. Shi (2009) shows that this insight also applies in a directed search setting. Balke and Lamadon (2022) find that firms also want to backload wages when there is a moral hazard friction such that low worker effort can lead to job destruction.

Besides backloading wages, we also give firms a second tool to reduce quitting, which is to match outside offers. A common assumption in the literature (e.g., in Shi 2009) is that firms do not respond to outside offers. One motivation for this assumption has been that outside offers are typically not verifiable.[6] However, Moore (1985) shows that stochastic contracts can incentivize

[4] In Germany, quitters are eligible for unemployment benefits after a three month waiting period (Venn 2012).

[5] In Austria, quitters are eligible for unemployment benefits after a one month waiting period (Venn 2012).

[6] Burdett and Coles (2003) write, "An important assumption ... is that a firm does not respond to outside offers

truthful reporting of a privately observed worker reservation wage. We build on that insight in modeling stochastic offer matching, whereby firms either match a reported outside offer or fire the worker, and show that this is more profitable than simply ignoring such offers.[7] One attractive feature of this model is that it offers an endogenous explanation for a common exogenous assumption: that it is easier to find a job while not working than while employed. The explanation is simply that it is costly to target recruiting effort toward employed workers, since a large share of matches will generate matching counteroffers rather than new hires.

An emerging literature studies models where workers can quit into non-employment. The simple static version of our model in Section 2 is similar to the static version of the economy considered by Guerrieri (2008). The key differences in our dynamic economies are (1) our workers are risk averse – which is critical for the optimal insurance question – and (2) we explore repeated preference shocks rather than permanent preference heterogeneity. Hopenhayn and Nicolini (2009) frame optimal UI policy as a principal agent problem, where the agent chooses search effort while unemployed and can also quit. When the principal wants to prevent quits but cannot differentiate between quits and layoffs, consumption optimally rises during employment spells and drops discretely following separations. Blanco et al. (2023) consider an environment where workers quit because of productivity shocks coupled with wage rigidities, and explore the impact of monetary policy shocks. Qiu (2022) and Bagga et al. (2023) study the business cycle implications of the quitting channel. Mazur (2016) explores the welfare implications of extending UI benefits to quitters, though in a model that is very different from ours. In particular, wages are drawn from an exogenous distribution in his model, so the mechanism we emphasize whereby quitting reduces returns to vacancy creation and thus offered wages is absent.

2 Quitting and UI: A Simple Tractable Example

We consider first a tractable static model. Analytic tractability will make it easy to understand how the quitting margin impacts the directed search equilibrium and how it impacts optimal policy

received by any of its employees. Clearly this restriction is not satisfied in some labor markets such as the academic labor market in the U.S. Nevertheless, there are reasons to suspect our restriction holds in other labor markets, especially those markets where workers are homogeneous. First, outside offers may not be observable by firms. Indeed, why should a firm verify to another firm that it has made a particular offer to a worker? Of course given offers from other firms are not observed, they will be ignored."

[7] Outside offers can also be matched in the Postel–Vinay and Robin (2002) model. But note that theirs is a complete information setup, in which the details of outside offers are fully observable. In this tradition, Elsby and Gottfries (2022) consider a setting where firms can match offers with an exogenously-specified probability.

design. In the next section we will revisit and extend the analysis in a richer dynamic model.

A continuum of workers with linear utility are initially unmatched. The labor market is modeled in the directed search tradition. Firms post vacancies in submarkets indexed by the wage promised to a worker, w, and by market tightness θ, which is the ratio of vacancies v to searching workers (in fact, $\theta = v$, given a unit mass of searching workers). Vacancies can be created at cost ϕ. Workers find jobs with probability $p(\theta)$, and vacancies find workers with probability $q(\theta)$.

Once matched, workers draw an idiosyncratic cost of work shock χ from a cumulative distribution function $F(.)$. In our baseline model specification, the shock χ is not observable by the firm, and thus the wage w must be independent of χ.[8] After χ is realized, matched workers decide whether to remain with the firm and produce, or quit the firm and not work. There is a government, which imposes a lump-sum tax τ on all workers, and uses the revenue collected to pay a transfer b to all non-workers. If a matched worker with offered wage w and realized preference shock χ chooses not to quit, she enjoys utility given by $U^e = w - \tau - \chi$, while a non-employed worker (who either quit or failed to match) receives $U^n = b$. It is immediate that the worker will quit if and only if the draw for χ exceeds a threshold

$$\bar{\chi} = w - (\tau + b). \tag{1}$$

Note that the threshold $\bar{\chi}$ is increasing in w.

Firms are happy to post vacancies at any wage w that delivers positive expected profits, and free entry drives profits to zero in equilibrium. This zero profit condition is

$$q(\theta)F(\bar{\chi})(z - w) = \phi. \tag{2}$$

The right-hand side is the cost of positing a vacancy. The left-hand side is the expected profit from doing so: the job filling probability times the probability the worker does not quit times the firm surplus from a producing worker.

Since $pF(\bar{\chi})$ is the expected share of employed workers at the time of production, the government budget constraint is $pF(\bar{\chi})\tau = (1 - pF(\bar{\chi}))\, b$. Workers' expected utility is given by

$$pF(\bar{\chi})U^e + (1 - pF(\bar{\chi}))U^n. \tag{3}$$

We think of an unmatched worker as choosing across different labor submarkets characterized by different combinations (p, w) to maximize expected utility (3) subject to the zero profit condi-

[8]There is one way in which firms could potentially separate workers with different private χ realizations, even in our static model. In particular, they could do so by offering stochastic contracts (Moore, 1985). For example, a firm could offer matched workers two alternative contracts: a low wage with guaranteed employment, or a higher wage with a low employment probability. Workers with a low χ realization might prefer the first, while workers with a higher χ realization might prefer the second. We will not pursue these sorts of stochastic contracts in this simple example.

tion (5). This constraint ensures that firms are happy to post vacancies into the market where the worker wants to search.

We will also consider an alternative specification in which the firm observes the realization of χ and can offer contracts in which wages are contingent on χ, which we denote $w(\chi)$. In this "public χ" version of the model, the firm can reduce quitting without raising the average wage paid $E[w] = \int_{-\infty}^{\bar{\chi}} w(\chi) dF(\chi)$ by promising higher wages when the realization of χ is high, and lower wages when it is low. Spreading out wages in this fashion is not costly to the worker, given linear utility. The firm will pay a wage of up to z in order to retain a worker with a high realization for χ, and thus the quitting threshold in the public χ economy will be $\bar{\chi} = z - (\tau + b)$, which is independent of $E[w]$.[9] Note that this threshold is higher than the corresponding threshold in the private χ economy (equation 1).

We assume that the matching function is Cobb-Douglas with share parameter equal to 0.5.[10] We further assume that the preference shock is drawn from a uniform distribution with support $[0, a]$. With these assumptions, we can characterize the competitive equilibrium in closed form.

Figure 2 is a graphical illustration of the unmatched worker's choice regarding where to direct search.[11] The red solid line is the set of feasible (w, p) pairs derived from the firms' free entry condition in the baseline private χ economy, while the blue sold line is the set of feasible $(E[w], p)$ pairs in the public χ economy. One can think of these lines as tracing out the budget sets faced by workers. The dashed lines are indifference curves from the workers' perspective, where all points on a given line offer identical expected utility. The points where the two curves are tangent are the competitive equilibria.

Note that the budget line for the private χ economy in Figure 2 lies below the one for the public χ economy. The reason is that for any $w < z$, workers in the private χ economy quit more frequently than in the corresponding (i.e., $E[w] = w$) public χ economy. Anticipating a higher quit

[9]There are lots of contingent wage contracts that deliver efficient quitting. One is a piecewise-linear wage rule:

$$w(\chi) = \begin{cases} \chi + (\tau + b) & \bar{w} - (\tau + b) \leq \chi \leq z - (\tau + b), \\ \bar{w} & \text{otherwise} \end{cases}$$

where $\bar{w}$ satisfies

$$E[w] = \int_{-\infty}^{\bar{w} - (\tau + b)} \bar{w} dF(\chi) + \int_{\bar{w} - (\tau + b)}^{z - (\tau + b)} [\chi + (\tau + b)] \, dF(\chi).$$

This allocation delivers $\bar{w}$ for all workers with χ realizations below a threshold, and a wage that increases one-for-one with χ above that threshold before maxing out at $w = z$.

[10]This implies that the worker's job finding probability is $p(\theta) = A\sqrt{\theta}$, while the firm's job-filling probability can be expressed as a closed-form function of worker's job finding probability p: $q(\theta) = A/\sqrt{\theta} = A^2/p(\theta)$.

[11]The parameter values used to construct the plot are: $A = 1.5$, $a = 2$, $z = 1$, $\phi = 0.5$, and $b = 0$.

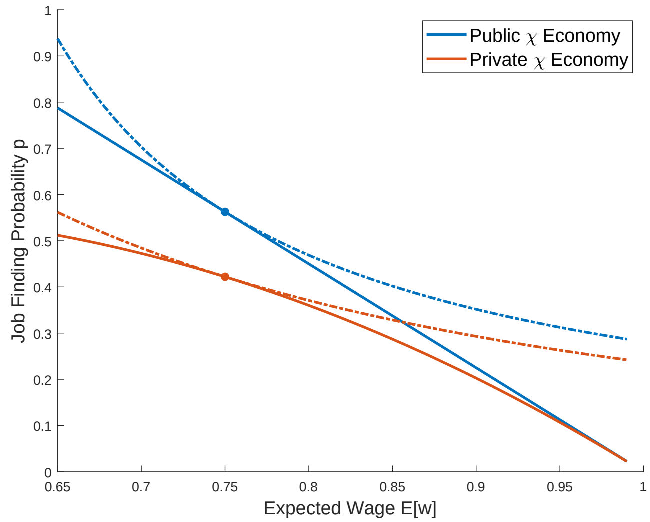

Figure 2: Equilibria in the Public and Private χ Economies. The solid lines show the set of submarkets available to searching workers. The dashed lines are indifference curves. The dots are the equilibrium choices.

rate, firms need to be compensated with a higher job-filling probability (i.e., a lower p) in order to be willing to post vacancies at the same expected wage level. Note also that the budget line is flatter in the private χ economy. Firms are willing to post higher wages in exchange for only a small increase in their job filling probability q (and thus a small decline in p) because part of the direct cost of paying a higher wage is offset by a smaller chance that the match dissolves.[12]

Proposition 2.1 *Given policy parameters b and τ, competitive equilibrium allocations in the economies in which χ is publicly observable and in which χ is not observable are given by*

	public χ		private χ
$\bar{\chi}$	$z-(\tau+b)$	$>$	$\frac{3}{4}z-\frac{3}{4}(\tau+b)$
$E[w]$	$\frac{3}{4}z+\frac{1}{4}(\tau+b)$	$=$	$\frac{3}{4}z+\frac{1}{4}(\tau+b)$
p	$\frac{A^2}{\phi}\frac{1}{a}\frac{1}{4}(z-(\tau+b))^2$	$>$	$\frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-(\tau+b))^2$

Proof: See Appendix A for all proofs.

Because workers face a smaller budget set in the private χ economy, they must choose some combination of a lower wage or a lower job finding probability. The proposition indicates that in this example they choose to pay the cost associated with the private information friction entirely

[12]That can be readily seen when $p = A^2/q$, $F(\bar{\chi}) = \bar{\chi}/a$ and $\bar{\chi} = w-(\tau+b)$ are substituted into eq. (5) to give $p = \frac{A^2}{\phi}\frac{w-(\tau+b)}{a}(z-w)$. A higher wage decreases flow profit $(z-w)$, necessitating a decline in p to maintain zero profit. But a higher wage also raises the matched worker retention rate $\frac{w-(\tau+b)}{a}$, which boosts profits.

in terms of a lower equilibrium job finding probability; there is no change in the average wage paid. They make this choice because, from the worker's standpoint, the fact that p is less sensitive to w makes searching in a relatively high wage market more attractive. Thus, in the private χ economy, workers and firms coordinate on a relatively high wage equilibrium. This outcome is consistent with the longstanding notion of "efficiency wages." One way to interpret this outcome is that even though workers cannot commit not to quit *ex ante*, workers can partially alleviate the quitting friction by choosing to direct search to a high-wage market, thereby making it more costly for them to quit *ex post*.

2.1 Efficiency and Optimal Policy

We now review the sense in which the competitive equilibrium in the private χ economy is inefficient, as a precursor to discussing optimal policy. Recall that in the competitive equilibrium, households make two choices: the job finding probability p (or, equivalently, market tightness θ) and the quitting threshold $\bar{\chi}$.

Proposition 2.2 *In the public χ economy, allocations are efficient when $b = \tau = 0$. In the private χ economy when $b = \tau = 0$, $\bar{\chi}$ is inefficiently low (the quit rate is inefficiently high), and p is inefficiently low (workers are too picky). However, the equilibrium value for p is identical to the one preferred by a planner who can dictate market tightness, but must respect the private quitting constraint (eq. 1). In this sense, equilibrium market tightness is conditionally efficient.*

Given linear utility, the efficient values for p and $\bar{\chi}$ are the ones that maximize per capita consumption net of the utility cost of work; that is, they solve

$$\max_{p,\bar{\chi}} p \int_0^{\bar{\chi}} (z - \chi)\, dF(\chi) - \theta\phi. \tag{4}$$

It is immediate that, independent of the value for p, the efficient value for the quitting threshold is $\bar{\chi} = z$, where the value of output created by the match is exactly equal to the cost of producing it. From Proposition 2.1, the quitting threshold is efficient in the public χ economy when $b = \tau = 0$. Thus, state-contingent wage contracts deliver efficient quitting. In the private χ version of the model, there is too much quitting at $b = \tau = 0$: $\bar{\chi} < z$. Quitting is inefficiently high because workers with $\chi \in (\bar{\chi}, z)$ quit when the value to the firm from continuing to produce, $z - w$, exceeds the value to the worker from quitting, $\chi - w$. In the decentralized equilibrium, this inefficiency shows up in the form of offered wages that are depressed relative to the public χ economy for any given p.

In the public χ economy, the equilibrium value for p at $b = \tau = 0$ described in Proposition 2.1 maximizes equation (4) when $\bar{\chi} = z$. Thus, directed search delivers efficient market tightness and vacancy posting. The intuition is straightforward: workers simply choose to search in the submarket that offers the highest expected welfare, subject to firms making zero profits. Acemoglu and Shimer (1999) consider a directed search environment similar to ours and also find that a zero unemployment benefit delivers the first best allocation.

In the baseline private χ economy, there is an equilibrium relationship between the quitting threshold and job finding probability $\bar{\chi}(p)$ via equations (1) and (2). We have already noted that at $b = \tau = 0$, workers are too picky (p is too low) relative to the efficient value attained in the public χ economy. But it is straightforward to verify that if the planner maximizing (4) must take as given the quitting rule (1) (rather than the efficient value $\bar{\chi} = z$), then that planner would choose the same value for p that emerges in equilibrium at $b = \tau = 0$. In this sense, wage setting in the private χ economy is conditionally efficient. Thus, the motivation for policy intervention in this paper is different from that in Landais et al. (2018b), who consider models with inefficient wage setting.

We now address optimal policy. In the private χ economy, policy can be used to reduce the quit rate. In particular, a negative value for b raises $\bar{\chi}$, pushing it closer to the efficient level. But when $b < 0$, workers perceive the cost of non-employment to be larger than the true social cost, and they become too desperate to find a job. Thus, the equilibrium value for p exceeds the value that maximizes per capita consumption, and the equilibrium wage is too low.

Since the government has only one independent policy instrument b it cannot, in general, deliver the efficient values for both $\bar{\chi}$ and p.[13] The optimal policy, which is $\tau^* + b^* = -\frac{z}{5}$, strikes a balance.[14] It delivers $\bar{\chi} = \frac{9}{10}z < z$, so there is still too much quitting, but also a job finding rate p that is too high (conditional on quitting behavior), so there is also too much vacancy creation.[15]

Proposition 2.3 *In the public χ economy, the optimal unemployment benefit b^* and tax rate τ^* are 0, while in the private χ economy, b^* and τ^* are negative:*

[13] The government can also decide on τ, but it needs to maintain a balanced budget, so it really only has only one independent policy lever.

[14] We summarize policy by the optimal value for $b + \tau$, since policy parameters enter all equilibrium variables in the form of this sum (see the previous proposition). It is straightforward to compute the optimal value for b, given $b + \tau$, using the equilibrium expressions for $\bar{\chi}$ and p and the government budget constraint.

[15] An additional potential instrument is a wage policy. One can show that a combination of a minimum wage policy and less generous UI can decentralize the first best. Intuitively, an appropriate minimum wage ensures that p is at the efficient (low) level, while the UI benefit is set sufficiently negative to deliver the efficient (low) quit rate.

	public χ	*private* χ
$\tau^* + b^*$	0	$-\frac{z}{5}$

In the public χ economy, the optimal policy delivers the first-best allocation. In the private χ economy, welfare under the optimal policy is strictly less than under the first-best allocation.

In the quantitative model, which we describe next, the key insight from this simple model will survive: workers will quit too frequently, translating into depressed equilibrium wages, which motivate reducing UI benefits to reduce quits and boost wages. However, in our quantitative model, workers have concave utility, and thus it will always be optimal to set benefits positive. As a result, rather than being too desperate to find jobs (as when $b < 0$ in the simple model), we will see that workers will typically be too picky at the search stage, choosing a value for p that is too low relative to the efficient one. This will reflect a classic fiscal externality: workers will not internalize that by taking longer to find a job, they are raising the equilibrium tax rate. Thus, relative to the simple static model, our quantitative model will introduce a new motive to bump up the UI replacement rate (insurance with concave utility) and a new motive to not make benefits too generous (the fiscal externality). Note that both quits to other jobs and quits to non-employment can be inefficient, in that workers leave firms when the joint value of remaining matched is positive. But quits to non-employment tend to be more costly, because the fiscal externality applies to them.

3 A Dynamic Quantitative Model

We now describe the model we will use for our quantitative exploration of optimal policy. Our focus will be on designing social insurance, which we label "unemployment insurance", for individuals with a strong labor force attachment who experience temporary spells of non-employment when they are either quit or are fired. Our analysis abstracts from the subset of the U.S. population that is persistently out of the labor force, owing to advanced age, disability, or other factors.[16]

Time is discrete and the horizon is infinite. There are no aggregate shocks. There is a unit mass of infinitely-lived workers. The economy is composed of different sectors indexed by n. Workers are *ex ante* heterogeneous with respect to the sector to which they belong and cannot move across sectors. Workers start each period either matched to a firm or unmatched. Matched workers are

[16] Of course, the size of the labor force is somewhat endogenous to policy, and exploring the joint design of social insurance targeted at different groups is an important research direction (see, e.g., Pavoni and Violante 2007).

further differentiated by the quality of their match z, where match quality is drawn at the time a new match is formed, and remains fixed for the duration of the match. We assume two possible values for match quality, $z \in \{z_H, z_L\}$, which are drawn with probabilities μ_H and $\mu_L = 1 - \mu_H$. A worker in sector n with match quality z produces zY_n units of output each period for as long as the match survives.

Workers have concave time-separable utility over consumption, c, and a disutility cost of work, χ. Workers and firms discount at a common rate β. Expected lifetime utility at date 0 for an individual i is $E_0 \sum_{t=0}^{\infty} \beta^t \left(U(c_{it}) - \chi_{it} \right)$, where $U(.)$ is a concave function. Individual i's wage at t is denoted by w_{it}, and wage income is taxed at rate τ. We will describe wage determination shortly. If an individual does not work, they receive a benefit $b(n_i)$ that depends on i's sector. Workers have no access to private insurance or credit.[17] Thus,

$$c_{it} = \begin{cases} w_{it}(1-\tau) & \text{if } i \text{ is working at } t \\ b(n_i) & \text{if } i \text{ is not working at } t. \end{cases}$$

The utility cost of work is idiosyncratic and stochastic, and drawn independently each period from a distribution F, where F is common across sectors. Note that the utility cost is paid only in periods in which the individual is working.

Firms are risk neutral and seek to maximize profits. They post vacancies at a sector-specific cost ϕ_n. Firms observe match quality once a match is formed, but they do not observe preference shocks χ, the worker's on-the-job search behavior, or their receipt of outside offers. There are two ways a match can end. First, each period every match is destroyed with fixed exogenous probability $1 - \gamma$.[18] The second way a match can end is if the worker chooses to quit, either to move to another job, or to transit to non-employment.

Firms commit to flexible dynamic contracts that specify how wages will vary with the realization of match quality and with tenure. In contrast, workers cannot commit to future job search or quitting choices. We will formalize the firm problem recursively, and summarize the state of a particular firm-worker match by the pair (V, z), where V is the expected present value of utility promised to the worker and z is the match quality.

Before describing how promised values and wages evolve, we first summarize the sequence of events during a period.

1. All workers (matched and unmatched) choose the submarket to which they direct job search.

[17] In Section 6.2 we consider a specification with private insurance.

[18] Exogenous match destruction shocks could be interpreted as reflecting large negative shocks to match productivity.

(a) Unmatched workers search in submarkets indexed by sector n, by promised value V^s, and by market tightness θ.

(b) Matched workers search in submarkets indexed by the characteristics of the incumbent match (V, z), in addition to the triple (n, V^s, θ).

2. Workers who started the period matched report to their current employer whether or not they have received an outside offer. If the worker reports an offer, the incumbent firm either matches the expected value of the offer V^s, in which case the worker stays with the incumbent firm, or else the incumbent firm tells the worker to leave the firm.

3. New matches draw match quality z, which is immediately observed by both firm and worker.

4. With probability $(1 - \gamma)$ each match is destroyed, generating an involuntary separation.

5. Workers who remain matched draw an idiosyncratic cost-of-work shock χ. Given their realization of this shock, a worker chooses whether or not to quit.

6. Workers who remain matched produce, and all workers consume.

We now describe the search stage and the firm's dynamic contracting problem. To simplify notation, we temporarily suppress the sector notation.

Consider the vacancy posting problem for firms posting in markets for unmatched workers. For any possible submarket indexed by (θ, V^s), let $J^u(\theta, V^s)$ denote the expected profit from posting an additional vacancy. That is equal to the probability that the vacancy translates into a match times expected profits conditional on a match, minus the vacancy posting cost; that is,

$$J^u(\theta, V^s) = q(\theta) E[\Pi(V^s)] - \phi,$$

where $E[\Pi(V^s)]$ denotes the maximum expected present value of profits for a firm exiting the search and matching stage newly matched to a worker promised V^s, before match quality is revealed. Unmatched workers solve

$$\max_{V^s, \theta} \{p(\theta) V^s + (1 - p(\theta)) V^u\},$$

subject to $J^u(\theta, V^s) = 0$.

For any possible submarket for matched workers indexed by (θ, V^s, V, z), let $J^m(\theta, V^s, V, z)$ denote the profit from posting an extra vacancy. For firms posting vacancies in markets for matched workers, only meetings in which the incumbent firm does not match the outside offer translate into new hires. Let $\bar{\zeta}(V^s, V, z)$ denote the probability that an incumbent firm that has promised

V with match quality z matches an outside offer of V^s. The payoff from posting an additional vacancy is now

$$J^m(\theta, V^s, V, z) = q(\theta)(1 - \bar{\zeta}(V^s, V, z))E[\Pi(V^s)] - \phi. \tag{5}$$

Matched workers in state (V, z) solve

$$\max_{V^s, \theta} \{p(\theta) V^s + (1 - p(\theta)) V\} \tag{6}$$

subject to $J^m(\theta, V^s, V, z) = 0$.

Once a worker transitions to a firm offering an expected value V^s, match quality is revealed. Let $\Pi(V, z)$ denote the maximum expected present value of profits for a firm that has promised V to a worker when match quality is z. Firms choose contingent values V_H and V_L, conditional on the realization of match quality, subject to a promise-keeping constraint. Thus,

$$E[\Pi(V^s)] = \max_{V_H, V_L} \{\mu_H \Pi(V_H, z_H) + (1 - \mu_H) \Pi(V_L, z_L)\}, \tag{7}$$

subject to

$$\mu_H V_H + (1 - \mu_H) V_L \geq V^s.$$

We next describe the firm dynamic profit maximization problem, for a firm exiting the search and matching stage (i.e., at step 4 in the timeline outlined above) matched to a worker with a promise V and known match quality z . We will think of the firm as directing the choices of the worker about when to quit and about where to direct on-the-job search, recognizing that those choices must be consistent with utility maximization by the worker. The firm chooses (i) the current period wage, w; (ii) the current period quitting threshold, $\bar{\chi}$; (iii) a promised continuation value V' if the match survives into the next period and the worker does not report an outside offer after the search phase at the start of the next period; (iv) the promised value $V^{s'}$ and (v) tightness θ' of the market to which the worker will direct job search in the next period; and (vi) the probability ζ' that the firm will match outside offers in the next period.

The firm makes these six choices to maximize the expected present value of profits. Flow current period profits are given by $zY - w$, but these are realized only if the match is not destroyed and the worker does not quit, which occurs with probability $\gamma F(\bar{\chi})$. Expected continuation profits depend on whether the worker receives an outside offer at the start of the next period (the probability of which is $p(\theta')$), and, if they do, on whether the firm matches the offer. Thus, the firm solves

$$\Pi(V, z) \quad = \max_{w, V', \bar{\chi}, \theta', V^{s\prime}, \zeta'} \gamma F(\bar{\chi}) \left[zY - w + \beta\left(1 - p(\theta')\right)\Pi(V', z) + \beta p(\theta')\zeta'\Pi(V^{s\prime}, z)\right], \tag{8}$$

subject to the following constraints.

First, the quitting threshold must be consistent with optimal quitting behavior on the part of the worker:

$$U(w(1-\tau)) - \bar{\chi} + \beta p(\theta^{'}) V^{s\prime} + \beta \left(1 - p(\theta^{'})\right) V^{\prime} = V^{u}. \tag{9}$$

The left-hand side of this expression defines the worker's expected present value when they do not quit: they receive the wage w, pay the utility cost of work, and enter the next period matched. In the next period, the worker receives an outside offer with probability $p(\theta^{'})$, in which case their continuation value is $V^{s\prime}$, while with reciprocal probability their continuation value is $V^{\prime}$. If they quit, the worker gets the present value of not working in the current period, which we denote V^{u}.

Second, the contract offered by the firm must deliver V to the worker. This promise-keeping constraint can be written as

$$\gamma F(\bar{\chi}) \left[U(w(1-\tau)) - E\left[\chi | \chi \leq \bar{\chi}\right] + \beta p(\theta^{'}) V^{s\prime} + \beta \left(1 - p(\theta^{'})\right) V^{\prime}\right] + (1 - \gamma F(\bar{\chi})) V^{u} \geq V. \tag{10}$$

The term in square brackets is expected lifetime utility if the worker does not separate in the current period. If the worker does separate, they receive V^{u}.

Third, the choices for $V^{s\prime}$ and $\theta^{\prime}$ must be consistent with optimal job search behavior on the part of the worker; this is, they must solve equation (6), subject to $J^{m}(\theta^{\prime}, V^{s\prime}, V^{\prime}, z) = 0$, where the offer matching functions in equation (5) are taken as given.

Finally, the firm's choice for the probability of matching outside offers $\zeta^{\prime}$ must be such that a worker without an outside offer weakly prefers not to falsely report an offer:

$$\zeta^{\prime} V^{s\prime} + (1 - \zeta^{\prime}) V^{u} \leq V^{\prime}, \tag{11}$$

where the left hand side is the payoff to a worker without an outside offer who falsely reports having an offer, while the right hand side is the payoff if the worker truthfully reports no outside offer. Note that if the worker reports having an outside offer, the incumbent firm will infer that the value of that offer is the value $V^{s\prime}$ that solves the worker's on-the-job search problem.

We define a stationary equilibrium for this economy in Appendix B.

3.1 Backloading of Wages

We now characterize some properties of the solution to the dynamic contracting problem described above. To start, we abstract from on-the-job search and focus on the impact of preference shocks driving quits to non-employment.[19] In this problem, the firm seeks to deliver the promised

[19] Specifically, this simplified contracting problem is:

value V in the most profitable way. Given that worker utility is concave, it is conceivable that the firm would like to smooth wages over time. On the other hand, the quitting friction creates an incentive to backload wages, because by doing so, the firm can reduce the worker's future incentives to quit. The following inverse Euler equation characterizes the solution to the firm's problem.

Proposition 3.1 *Wage growth under the optimal contract satisfies:*

$$\frac{1}{U'(w_{t+1})} - \frac{1}{U'(w_t)} = \frac{f(\bar{\chi}_{t+1})}{F(\bar{\chi}_{t+1})}\left[zY - w_{t+1} + \beta\Pi_{t+2}\right]. \tag{13}$$

The wage rises for as long as the worker and firm remain matched, and converges to zY.

Condition (13) can be understood in the following way. Imagine that the firm needs to deliver some promised value V_t to a worker, and is contemplating some wage sequence that delivers that value. Now consider perturbing the worker's value from period t to period $t+1$. The left-hand side of the equation measures the net pecuniary cost to the firm, where the firm's profit is increased by $\frac{1}{U'(w_t)}$ in period t and reduced by $\frac{1}{U'(w_{t+1})}$ in period $t+1$. The right-hand side measures the benefit of increasing future promised value: a higher future value V_{t+1} raises the $t+1$ quitting threshold $\bar{\chi}_{t+1}$, leading to reduced quitting in proportion to the normalized density function $\frac{f(\bar{\chi}_{t+1})}{F(\bar{\chi}_{t+1})}$. The value of the match is then increased by the change in the probability of quitting times the future expected profit if the worker remains with the firm, $zY - w_{t+1} + \beta\Pi_{t+2}$. Under the optimal wage sequence, the cost of an additional delay to worker compensation must equal the benefit, resulting in (13).

As long as the match remains profitable, the right hand side of condition (13) will be positive, and thus the left hand side must be too. Given concave utility, this implies that w_{t+1} must be greater than w_t; this is, wages are backloaded. A very simple way to understand this backloading result is that a higher wage at t reduces quitting at t but not at $t+1$, while a higher promised wage at $t+1$ reduces quitting at both t *and* $t+1$. As wage growth continues, wages converge to productivity, zY.

$$\Pi(V,z) = \max_{w,V',\bar{\chi}}\left\{\gamma F(\bar{\chi})(zY - w + \beta\Pi(V',z))\right\} \tag{12}$$

$$U(w) - \bar{\chi} + \beta V' = V^u$$

$$\gamma F(\bar{\chi})\left(U(w) - E\left[\chi|\chi \le \bar{\chi}\right] + \beta V'\right) + (1-\gamma F(\bar{\chi}))V^u \ge V,$$

where the first constraint states that the quitting threshold $\bar{\chi}$ satisfies the worker's indifference condition (9), while the second constraint is the promise-keeping condition (10).

3.2 Outside Offer Matching

We now turn to the on-the-job search feature of the model. In equilibrium, workers search for offers that yield higher value than their current match: $V^{s\prime} \geq V'$. Whether the incumbent firm will consider matching an offer $V^{s\prime}$ depends on whether the match would remain profitable at the higher promised value. But even if the match would remain profitable, the incumbent firm cannot promise to match with probability one, because that would incentivize workers to falsely report such offers. If $\Pi(V^{s\prime}, z) \geq 0$, then profit maximization implies that firms choose the highest matching probability ζ' consistent with the incentive constraint (11), which ensures that workers do not want to fake offers:

$$\zeta' = \frac{V' - V^u}{V^{s\prime} - V^u}.$$

The matching probability is decreasing in $V^{s\prime}$ because a higher $V^{s\prime}$ makes mimicking more attractive to workers who do not have an outside offer. Thus, the firm needs to impose a harsher punishment (a higher probability of firing instead of matching) to deter such mimicking. When $V^{s\prime}$ is sufficiently high, the value of the match to the incumbent firm drops below zero: $\Pi(V^{s\prime}, z) < 0$. In this case the offer matching rate is zero.

Proposition 3.2 *The equilibrium offer matching rule $\zeta'\left(V^{s\prime}, V', z\right)$ is given by:*

$$\zeta'\left(V^{s\prime}, V', z\right) = \begin{cases} \frac{V' - V^u}{V^{s\prime} - V^u} & \textit{if } \Pi(V^{s\prime}, z) \geq 0 \\ 0 & \textit{otherwise.} \end{cases}$$

4 Calibration

The model period length is set to one month. We assume workers have logarithmic utility from consumption, and set $\beta = 0.96$ on an annual basis. We assume a Cobb-Douglas matching technology, with productivity A, such that if v vacancies are posted and u workers search, the number of meetings is $m = A\sqrt{uv}$. The preference shock χ is assumed to be drawn from a lognormal distribution, with mean μ_χ and variance $\sigma_\chi^2 : \chi \sim LN(\mu_\chi, \sigma_\chi^2)$. The cost of posting vacancies is assumed proportional to average sectoral earnings: $\phi_n = \hat{\phi} Y_n$. The average quality of new matches is normalized to 1: $\mu_H z_H + (1 - \mu_L) z_L = 1$.

We identify different model sectors with the sectors for which data on job openings, layoffs and quits are available in the JOLTS database, and for which data on employment and average weekly earnings are also available in the CES survey.[20] Most of our empirical targets are averages for the U.S. private sector over the 12 months from July 2021 to June 2022. We think of the U.S. economy as being in steady state during this period.[21] For each of our sectors, we define the share of the population in sector n, λ_n, as employment in that sector relative to total employment across all sectors. And we define the productivity of sector n, Y_n, as average weekly earnings in that sector relative to average earnings across all sectors. Thus, $\sum_{n=1}^{N} \lambda_n Y_n = 1$. [22]

The unemployment benefit system is assumed to take the following form:

$$b(n) = \delta + \kappa \min \{Y_n, 1\}. \tag{14}$$

The parameter δ captures a floor on benefits, while the parameter κ captures the component of benefits that is proportional to earnings, up to a cap.[23] We will set $\delta = 0.05$ and $\kappa = 0.5$. The logic for these choices is as follows. The two most important sources of benefits for unemployed Americans are unemployment insurance and SNAP (food stamps) (see, e.g., Bitler et al. 2020, or Elsby et al. 2022). While the generosity of unemployment benefits varies significantly across different U.S. states, many states offer replacement rates of around 50 percent, up to a cap of around \$500 per week, which is about half of average weekly earnings. That is our rationale for setting $\kappa = 0.5$. SNAP benefits are worth around \$250 per month (not per week) per recipient household, suggesting a modest floor δ on benefits. This transfer model is similar to standard calibrations in the literature (e.g., Braxton et al. 2023, Birinci and See 2023, Guimaraes and Lourenco 2023). We discuss it further in Section 4.1.

Seven parameters remain: the exogenous match destruction rate $1 - \gamma$, the vacancy cost and match efficiency parameters, $\hat{\phi}$ and A, the mean and variance of the disutility parameter, μ_χ and σ_χ^2, and the ratio of high to low match quality and the share of high quality matches, z_H/z_L and μ_H. Three of our targets for these parameters come from JOLTS. We set γ to match the sum of the JOLTS layoff rate plus the JOLTS "other separations" rate (which includes retirements, deaths and disability), which is $1.66 + 0.28 = 1.94$ percent per month.[24] We also target the JOLTS job opening

[20]These sectors are: mining and logging, construction, durable manufacturing, non-durable manufacturing, wholesale trade, retail trade, transportation, warehousing and utilities, information, financial activities, professional and business services, education and health, arts, entertainment and recreation, accommodation and food services, and other services.

[21]The COVID-era expansions of UI benefits ended on September 6, 2021. Bagga et al. (2023) show that total UI expenditure per unemployed worker had returned to the pre-COVID level by June 2021.

[22]In the equilibrium of our model, individual earnings are endogenous, and vary within sectors with idiosyncratic match quality, tenure, and history of outside offers. But average equilibrium earnings within a sector n turn out to be very close to the exogenous sectoral productivity parameter Y_n.

[23]Our assumption that benefits are linked to average sectoral earnings rather than to individual earnings simplifies the analysis, because it implies that all non-workers in a sector have the same expected value V_n^u.

[24]We incorporate the adjustments proposed by Davis et al. (2010) to the published JOLTS numbers, and scale up

rate (8.03 percent per month) and the JOLTS quit rate (3.69 percent). One more target is the CPS unemployment rate, which was 4.15 percent for the private sector.[25] Loosely speaking, the job opening rate and the unemployment rate identify the vacancy posting cost $\hat{\phi}$ and match efficiency parameter, A, while the quit rate identifies the average disutility cost of work, μ_χ.

An important target for us is the share of quits that involve transitions to a period of non-employment, versus quits that are associated with an immediate transfer to another job. For this, we turn to the Census Job to Job Explorer, which is based on LEHD data. Averaging over quarterly data from 2000 Q2 to 2021 Q2, 32.1 percent of all separations are job-to-job separations with continuous employment. Applied to our total JOLTS separation rate of $1.94 + 3.69 = 5.63$ percent, that suggests a monthly job-to-job quit rate of $0.321 \times 5.63 = 1.81$ percent. Many parameters affect the model job-to-job transition rate, but an important one is the share of matches that are initially good, μ_H : if drawing a good match is less likely, there is more equilibrium churn as workers repeatedly sample new firms and quit low quality matches. Given a total quit rate of 3.69 percent, the targeted quit to non-employment rate is $3.69 - 1.81 = 1.88$ percent.[26]

To identify the productivity differential between good and bad matches, z_H/z_L, we target average wage growth associated with job-to-job transitions. Birinci et al. (2022) estimate 9 percent growth in earnings for continuously employed workers upon a job change using the LEHD.

One parameter remains, which is the variance of idiosyncratic preference shocks, σ_χ^2. This parameter is important because it determines how sensitive the model quit rate is to policy parameters. We set σ_χ^2 to replicate the observed elasticity of the sectoral quit rate to sectoral average earnings. Intuitively, if the quit rate is much higher in high wage sectors (like finance) than low wage sectors (like accommodation and food services), that would point to a relatively low value for σ_χ^2, while if quits are largely uncorrelated with wages, that would point to a high value. Figure 6 in Appendix D plots sectoral quit rates against sectoral average wages and indicates that quits are indeed heavily concentrated in low wage sectors.[27]

the published layoff, other separation, job openings, and quit rates by factors of 1.64, 1.29, 1.08 and 1.15, respectively, following their Table 5.4.

[25] In our model, we count all non-workers as unemployed except for those who quit in the current period.

[26] Fujita et al. (2023) estimate a job-to-job transition rate from the CPS, and show that a change in the CPS survey methodology can partly explain a puzzling decline in the measured CPS J2J rate. Their proposed correction gives a J2J rate of around 2 percent, similar to our target. However, they highlight two important open data puzzles. One is that even with their correction, the CPS-measured total quit rate looks pretty flat over the post-Global Financial Crisis period, while there is a dramatic upward trend in the JOLTS quit rate. A second puzzle is that the level of quits in the CPS is much higher than in JOLTS. Both these puzzles are clearly illustrated in Figure 11 of their Online Appendix.

[27] Krueger and Summers (1988) find a positive (negative) relationship between industry wage premia and tenure (quits).

Table 1: Calibration

Parameters		Value	Source/Targets
Externally Calibrated			
Discount factor	β	$0.99^{1/3}$	Monthly model
Non-employed consumption	δ, κ	0.05,0.5	SNAP+UI
Exogenous separation rate	$1-\gamma$	1.94%	JOLTS layoffs + other separations
Sector weights/earnings	$\{\lambda_n, Y_n\}$		CES
Internally Estimated			
Labor disutility shocks, mean	μ_χ	-0.95	Quit to non-employment rate 1.88%
Labor disutility shocks, variance	σ_χ^2	0.25	Elasticity of quits to sectoral earnings
Vacancy posting cost	$\hat{\phi}$	0.165	Job opening rate 8.03%
Match quality dispersion	z_H/z_L	1.14	Wage growth for job switchers 9%
Share of high quality matches	μ	0.5	Job-to-job transition rate 1.81%
Matching efficiency	A	0.75	Unemployment rate 4.15%

All parameter values are reported in Table 1.

4.1 Discussion of Model for Non-wage Income

In our simple model, non-workers have no other sources of income besides government transfers. This assumption follows most of the existing literature, and allows for a transparent analysis of the optimal UI replacement rate. Empirical evidence is consistent with a strong impact of the level of UI benefits on income and consumption for job losers. For example, Ganong et al. (2022) use bank data to track spending of UI-receiving households from employment to unemployment and from the initial phase of unemployment, when UI benefits are paid, through the point at which benefits are exhausted. They show that the impacts of UI benefit exhaustion on household income and household consumption are around twice as large as the impacts of job loss itself, even though benefit exhaustion is entirely predictable.

At the same time, however, workers exiting employment may have additional sources of income. The most important of these is likely spousal income (see Figure 2 in Elsby et al. 2019). Another source of replacement income is savings and credit: Braxton et al. (2023) estimate that financially unconstrained job losers replace around 5 percent of lost earnings using credit markets. In Section 6.2, we show that introducing private insurance reduces the optimal level of public insurance almost dollar for dollar.[28]

[28] Braxton et al. (2023) find that eliminating access to credit reduces the optimal UI replacement rate by 6.6 percentage

A second, related feature of the data that we do not model is cross-sectional variation in individuals' ability to use spousal income or savings to replace lost labor earnings, which translates into cross-sectional variation in workers' willingness to quit jobs (see Birinci and See 2023.). Our model will attribute all variation in the willingness to quit, holding fixed wages, to variation in the utility cost of work.

A specific potential concern with our baseline calibration is that we assume that workers who are fired and those who quit collect identical transfers. However, in the United States, workers who quit are typically ineligible – in theory – for unemployment insurance. One defense for our calibration choice is that the differential in UI receipt between workers who were fired versus those who quit is smaller than one might think. Birinci and See (2023) report that only 57 percent of the unemployed are eligible for UI, and only 61 percent of the eligible group actually collects benefits (see also Chodorow-Reich and Karabarbounis, 2016). In addition, quitters are sometimes successful in collecting UI benefits, and may also qualify for other transfers (see Chodorow-Reich and Karabarbounis, 2016 and Jurajda, 2003).[29] In addition, the fact that quitters have chosen to quit suggests they may have greater than average access to other transfers besides UI and, more broadly, to other sources of non-wage income. In Section 6.2, we will consider an alternative model calibration, in which quitters receive lower benefits than those who are fired. That calibration delivers an optimal UI replacement rate that is even lower than in the baseline model.

5 Quantitative Results

Figure 3 plots various industry-level labor market statistics generated by the model (red dots), against those in the data (blue dots). Given our calibration strategy, the model matches quite well the variation in quit rates across different industries (Panel A). In terms of untargeted moments, the model also performs reasonably well with respect to cross-industry variation in job opening rates and unemployment rates (Panels C and D, respectively), both of which tend to be higher in low wage industries.

Quits in the model can be decomposed into quits to non-employment and quits to another job. Panel B of Figure 3 plots such a decomposition across different industries. The purple dots show the share of workers who quit into non-employment, while the green dots show job-to-job tran-

points.

[29] However, Rothstein and Valletta (2017) estimate that other transfers offset only a small portion of lost UI for the unemployed who exhaust eligibility.

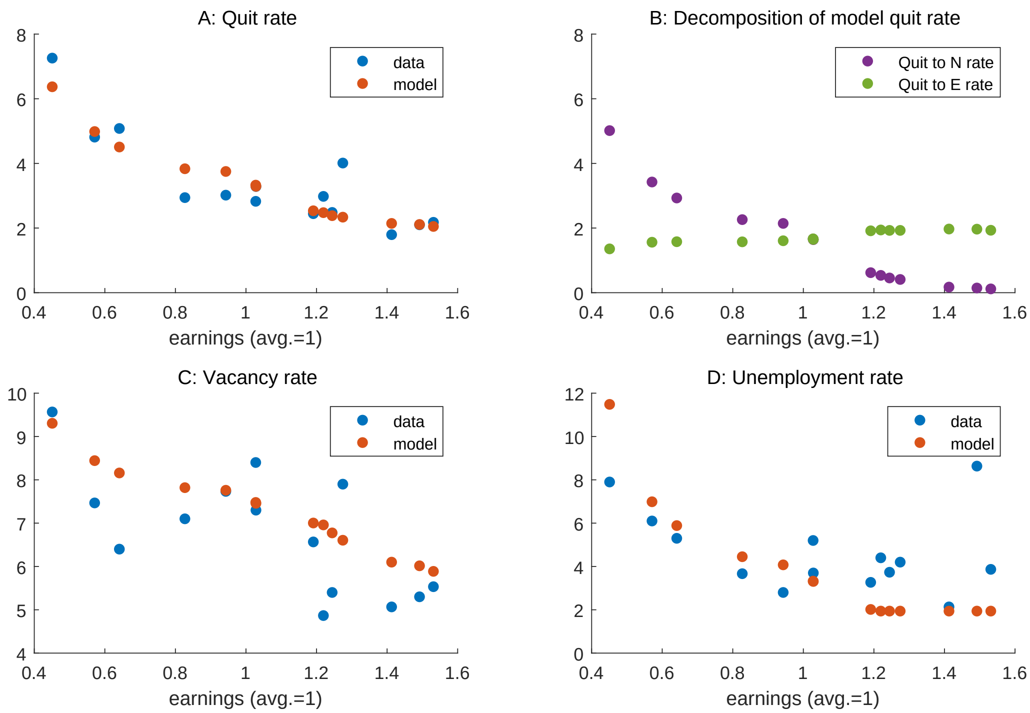

Figure 3: Quit Rate, Vacancy Rate, and Unemployment Rate by Sector

sitions. The negative relationship in the model between industry earnings and the industry quit rate is driven entirely by variation in the quit-to-non-employment rate; the job-to-job transition rate varies little with sectoral earnings.[30]

The fact that the quit-to-non-employment rate is quite sensitive to sector-level earnings suggests a preference shock distribution with significant density in the region where low wage workers will optimally quit but high wage workers will not.[31] A compressed labor disutility shock distribution has important consequences for our policy analysis. It means that reducing unemployment insurance can have a large impact on workers' quit rates and hence can lead to large welfare improvements. We will illustrate this point further in Section 6.

Figure 4 plots various value and policy functions conditional on whether the worker is in a high-quality (left column) or low-quality (right column) match. The first row plots continuation

[30]Bosler and Petrosky-Nadeau (2016) document that the job-to-job transition rate is similar across different occupations among recent cohorts of workers.

[31]There is an analogy here to the unemployment volatility puzzle (Shimer, 2005). The resolution proposed by Hagedorn and Manovskii (2008) is that the value of unemployment is high, so that workers are close to indifferent between working or not. This indifference generates high unemployment sensitivity with respect to aggregate labor productivity shocks. In our context, we observe a high sensitivity of the quit rate to sectoral earnings in cross-section, suggesting that the mean of the labor disutility shock must be high and its variance must be low.

values $V'(V)$ as a function of current promised value (red line). For high quality matches, workers start with low promised value but experience growth in promised value and wages (second row, left panel) with tenure. Workers with short tenure and low current promised values engage in on-the-job search (third row, left panel).[32] If they succeed in finding an outside offer, their incumbent firm matches the offer with a high probability (fourth row), so only a few such workers switch employers. Lastly, the worker's quit-to-non-employment rate decreases as promised value increases.

Turning to low quality matches, we see that wages never exceed those for workers in high quality matches. Badly matched workers choose to search in submarkets where their job finding probability is around 50 percent (third row, right panel). The incumbent firm does not match those offers, because doing so would imply negative profits. Finally, the quit-to-non-employment rate is much higher for workers in low quality matches than for those in high quality matches (last row).

Figure 5 shows how wages rise with tenure and how the quit rate declines with tenure. High quality matches feature more scope for wage growth, while in low quality matches, workers are paid their labor productivity starting from the second month of employment. This is due to an insurance mechanism. In particular, recall that firms need to allocate promised values conditional on the realization of match quality (eq. 7). Given concave utility, the firm optimally cross subsidizes workers who realize low match quality, so their initial wage is close to their marginal product. But the firm does not perfectly insure match quality risk, because paying higher wages to workers in high quality matches reduces the quit rate for those profitable matches.[33]

5.1 Optimal Policy

We turn now to optimal policy. We assume that benefits to non-employed workers are always specified by the function described in equation (14) and optimize with respect to the parameter κ. We maximize with respect to the average (across sectors) expected value for a non-working individual in steady state, $\sum_{n=1}^{N} \lambda_n V_n^u$.[34]

[32] These policy functions pertain to workers in a high productivity sector ($Y_n = 1.5$). Interestingly, for workers in high quality matches, the model only features on-the-job search in high productivity sectors. The reason is that for such workers, the benefit replacement rate is low. Thus, unmatched workers in high productivity sectors direct search to submarkets with high job finding probabilities but low promised values. Because their initial equilibrium promised values are low (relative to expected productivity), it is profitable for other firms to try to poach recent hires.

[33] Figure 7 in Appendix D plots a sample path of income for a worker in a sector with average productivity.

[34] We have also considered an objective of average lifetime utility in cross section; results were very similar. In this section we do not consider transitional dynamics: for any κ we evaluate welfare given the corresponding steady state distribution over employment and match quality. In Section 6, we compare policies assuming all workers are initially unmatched, incorporating transition. In our directed search setting, the only difference between the two approaches is

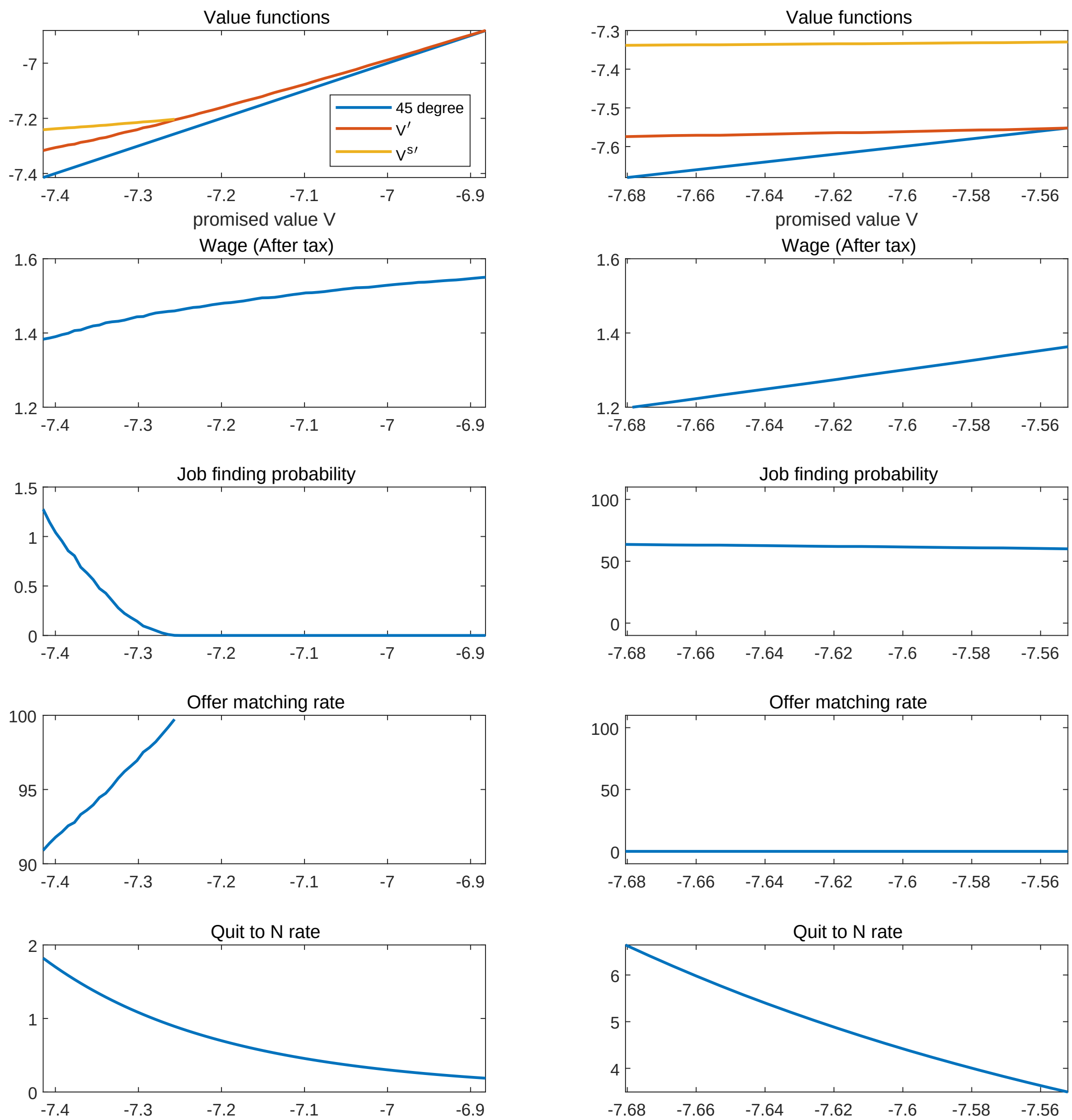

Figure 4: Value and Policy Functions ($Y_n = 1.5$). The left-side panels are for workers in a high quality match. The right-side panels are for workers in a low quality match. The x axis for each figure shows current promised value V. The x axes ranges show the set of values for V observed in equilibrium, conditional on match quality.

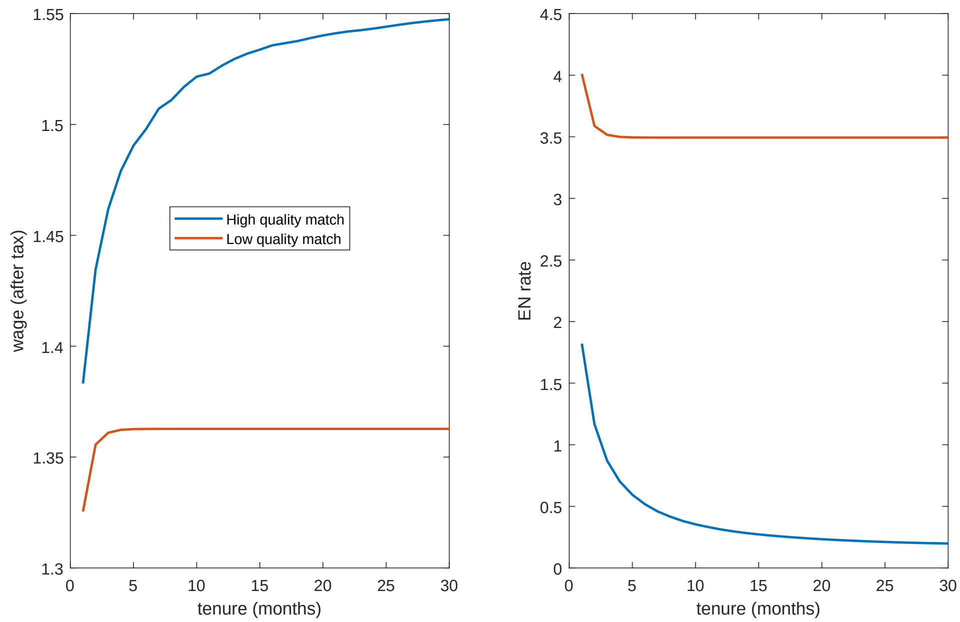

Figure 5: Wages and Quit Rate by Tenure (worker in sector with $Y_n = 1.5$)

The welfare maximizing policy involves reducing κ from 50.0 percent to 38.4 percent. Table 2 shows how this changes key labor market statistics. Optimally reducing the replacement rate cuts the quit to non-employment rate from 1.8 to 0.5 percent. Non-employed workers also become less picky, directing search to markets where they almost surely find jobs immediately. The combined impact of these changes is that the unemployment rate is cut in half.

How important is the presence of the quitting margin in driving our baseline policy prescription? To explore this question, we now consider several counterfactuals. In column 4 of Table 2, we shrink the variance σ_χ^2 of the idiosyncratic preference shock to near zero (while adjusting the mean parameter μ_χ so as to leave the expected value for χ unchanged). In this scenario the optimal replacement rate is 48.9 percent, which is 10.5 percentage points higher than the optimum in the baseline model and very similar to our calibrated value.

At first sight, a 49 percent replacement rate might still seem low. The only behavioral margin in this specification is job pickiness, and at the optimum replacement rate, 87.5 percent of unemployed workers find a job within a month. The reason the planner does not prefer a higher replacement rate is that the job finding rate is quite sensitive to κ.[35]

the value for the tax rate τ required to balance the government budget.

[35] In our calibration, the expected utility cost of working is $E[\chi] = \exp(\mu_\chi + \sigma_\chi^2/2) = 0.439$. Given that cost, a household is indifferent between working and not working when $\log((1-\tau)*1) - 0.439 = \log(\kappa)$. Given $\tau = 0.011$

Table 2: Optimal Policies under Counterfactuals

	Actual Policy	Optimal Policies				
		Baseline	$\sigma_\chi^2 = 0.01$	No OJS	$\frac{z_H}{z_L} = 1$	$\kappa^*_{EU} \neq \kappa^*_{EN}$
κ^* (%)	50.0	38.4	48.9	44.0	33.5	κ^*_{EU} 48.5 κ^*_{EN} 29.8
EN rate (%)	1.80	0.46	0.07	1.42	1.08	0.01
EE rate (%)	1.85	2.09	1.78	0.00	0.04	1.97
u rate (%)	4.13	1.98	2.32	2.38	1.95	2.26
v rate (%)	7.69	6.82	5.19	7.42	5.43	5.26
p rate (%)	78.1	98.7	87.5	92.7	99.7	p_U 87.5 p_N 100

In the next experiment (column 5 of Table 2), we switch off quits to other jobs, by assuming that working individuals cannot search for new jobs. In this case, the optimal replacement rate is 44.0 percent, 5.6 percentage points higher than in the baseline version of the model. When on-the-job search is ruled out, the only way workers can escape from low quality matches is to quit to non-employment, and then search. The lower are benefits, the less inclined workers will be pursue this route, so the planner increases benefits relative to the baseline to preserve incentives to transition out of bad matches.

Next, we eliminate variation in match quality (column 6 of Table 2). Now, there is no efficiency rationale for job-to-job transitions: all jobs are equally productive. We find an even lower optimal replacement rate in this case: 33.5 percent. Our interpretation is that in the baseline model (with match quality risk), the planner is less concerned about quitting to non-employment, because most quitters are workers in bad matches and the surplus from these matches is small. In addition, workers who quit transition to better matches at a faster rate than those who remain employed: under our calibration to the United States, only 50 percent of employed workers in bad matches transition to a new match each month via on-the-job search (Figure 4), compared with 78 of non-employed workers. Thus, in the baseline specification, quitting to non-employment speeds up reallocation to better matches. When we switch off variation in match quality, this beneficial effect of quitting on reallocation is no longer operative. Thus, the optimal replacement rate is lower.

In our last policy experiment, we assume that the government can perfectly identify two different types of non-workers: those who voluntarily quit from their previous job and those who

(the budget balancing rate at the optimum) this suggests a maximum possible replacement rate of around 64 percent. But even that is an upper bound, because if the planner were to increase κ, p would decline, implying more benefit recipients, necessitating a larger τ.

experienced an exogenous separation shock. We allow the government to pay different benefits to those who quit and those who were fired. In the United States, the government does try to separately identify these two groups, and the former is usually ineligible for unemployment benefits.

The last column of Table 2 reports the results from this experiment. The planner chooses different replacement rates for the two groups, with the firees getting 48.5 of earnings replacement (up to the cap), while the quitters get only 29.8 percent. The planner gives the quitters less to reduce excessive quitting. Note, however, that it is not optimal to give the quitters nothing: workers who choose to quit do so because they face a high cost of working, and the planner does not want to enforce zero quitting by threatening quitters with starvation. Note also that the optimal replacement rate for firees remains below 50 percent (as it is in the experiment in which we effectively eliminate quitting), so workers who separate exogenously still face large declines in consumption.

Suppose the planner can verify whether a non-employed individual quit or was fired, but that the planner must pay to acquire this information. How large a cost is worth paying? To answer this question, we compare the welfare gains of two different policy reforms. The first is our baseline reform (column 3 of Table 2), in which we move from a uniform 50 percent replacement rate to a uniform 38.4 percent replacement rate. This reform raises welfare for a non-working firee by an amount equivalent to a permanent 1.0 percent in increase in consumption. The second reform is replacing a uniform 38.4 percent replacement rate with the type-specific replacement rates for quitters and firees reported in the last column of Table 2. The corresponding welfare gain from this reform is much smaller, at 0.3 percent of consumption. Whether it is worth paying the cost of ascertaining which workers quit and which were fired depends on whether the cost of doing so is less than 0.3 percent of aggregate consumption. Guimaraes and Lourenco (2023) estimate that administration costs amount to 10 percent of the value of UI benefits paid in the United States, which amounts to much less than 0.3 percent of US consumption.[36] But workers applying for benefits likely bear a larger portion of the cost of administering the current system. In their model, Birinci and See (2023) attribute the fact that many unemployed workers choose not to apply for UI to such costs. If a uniform benefit system would allow for a much simpler application process, it might be preferable.

[36]Total UI spending is typically only around 0.15 percent of US GDP, though it surged to 2.3 percent in fiscal year 2020.

6 Exploring the Mechanism: An Extended Baily-Chetty Formula

In this section, we explore the sources of the large welfare gain we find from reducing UI in our economy with quitting. We develop a version of the Baily-Chetty optimal replacement rate formula in a simplified version of our model. This provides a clear understanding of how the quitting margin changes the optimal provision of social insurance.

The economy we consider here is identical to our baseline, except that (i) there is a single sector and no variation in match quality, so all employed workers produce identical output z, (ii) there is no on-the-job search, and (iii) wages must be constant for the duration of a match. We also consider a simplified UI scheme, according to which the government chooses a constant replacement rate κ, so transfers to non-workers are κz, financed by a constant proportional tax τ on earnings. All workers have utility over consumption given by the concave function $U(.)$.

The initial state is one in which all workers are unmatched and social welfare is expected lifetime utility for one such worker, which we denote by W. Given a time invariant policy (τ, κ), social welfare can be written as a weighted average of the per-period utility when employed and when non-employed:

$$(1-\beta)W\left(p,\bar{\chi},\tau,\kappa\right) = (1-\tilde{u})\left(U\left((1-\tau)\,w(p,\bar{\chi})\right) - E[\chi_{|\chi\leq\bar{\chi}}]\right) + \tilde{u}U(\kappa z), \tag{15}$$

where $1-\tilde{u} = \frac{\gamma p F(\bar{\chi})}{1-\beta\gamma(1-p)F(\bar{\chi})}$ is the expected fraction of time (appropriately discounted) that an initially unmatched worker will spend employed. The first term on the right-hand side of equation (15) is $(1-\tilde{u})$ times expected per period flow if the worker is employed, and the second term is $\tilde{u}$ times utility if non-employed. The equilibrium wage $w(p,\bar{\chi})$ is pinned down by the firms' free entry condition.[37] A worker will quit when $\chi \geq \bar{\chi}$, where $\bar{\chi}$ is the solution to $U\left(w\left(p,\bar{\chi}\right)(1-\tau)\right) - \bar{\chi} + \beta V' = V^u$, which is equation (9), but without on-the-job search. This equation, after substituting in expressions for V' and V^u, pins down the quitting threshold $\bar{\chi}$ as a function of (p,τ,κ).[38]

A worker who is unmatched at the start of the period chooses p to maximize W. When doing so, the worker internalizes the impact of the probability p on the wage w via equation (16); search

[37]This condition is given by

$$\phi = q(p)\frac{\gamma F(\bar{\chi})}{1-\beta\gamma F(\bar{\chi})}(z-w), \tag{16}$$

where $q(p) = A^2/p$.

[38] $\bar{\chi}\left(p;\tau,\kappa\right)$ is defined implicitly by

$$-\beta\gamma F(\bar{\chi})(1-p)\left(\bar{\chi} - E[\chi_{|\chi\leq\bar{\chi}}]\right) = U\left(w\left(p,\bar{\chi}\right)(1-\tau)\right) - \bar{\chi} - U(\kappa z). \tag{17}$$

in a tighter market with a lower p translates to a higher wage w. The searching worker also internalizes that his choice for p (and thus w) will affect his future quitting threshold $\bar{\chi}$ via equation (17). Of course, workers do not internalize the equilibrium impact of either of these choices on the policy variables. Thus, an unmatched worker solves

$$\max_{p} W\left(p, \bar{\chi}\left(p; \tau, \kappa\right); \tau, \kappa\right)$$

with optimality condition

$$\frac{\partial W}{\partial p} + \frac{\partial W}{\partial \bar{\chi}} \frac{\partial \bar{\chi}}{\partial p} = 0, \tag{18}$$

where $\frac{\partial \bar{\chi}}{\partial p}$ is taken with respect to the no-quitting condition (17).

We now move to the planner's problem. Our planner's policy problem is to choose κ to maximize W. The planner internalizes three constraints. The first is that given a choice for κ, the planner must adjust τ to achieve present value budget balance:

$$\tau(1 - \tilde{u}\left(p, \bar{\chi}\right))w\left(p, \bar{\chi}\right) = \kappa \tilde{u}\left(p, \bar{\chi}\right) z. \tag{19}$$

The planner also internalizes how policy choices (τ, κ) affect the private equilibrium $\left(p, \bar{\chi}\right)$ via the no-quitting condition (17) and the optimal job-search condition (18). These three equations pin down a mapping from κ to $\left(\tau, p, \bar{\chi}\right)$. We can compactly write the general form of this problem as

$$\max_{\kappa} W\left(p(\kappa), \bar{\chi}\left(p\left(\kappa\right); \tau\left(\kappa\right), \kappa\right), \tau(\kappa), \kappa\right). \tag{20}$$

This notation captures the idea that changes in κ affect social welfare directly, and indirectly via the impacts of κ on p, $\bar{\chi}$ and τ.

The optimal choice for κ is defined by a first order condition. After we substitute in the optimality condition for job search (18), this equation can alternatively be expressed in terms of marginal utilities and elasticities capturing how changes in the replacement rate affect labor market equilibrium, similarly to Baily (1978) and Chetty (2006).

Proposition 6.1 *At the optimum of the social planner's problem defined in (20), the following extended Baily-Chetty formula holds:*

$$\underbrace{\frac{U'\left(c^u\right) - U'\left(c^w\right)}{U'\left(c^w\right)}}_{\text{consumption insurance}} + \underbrace{-\left[\frac{1}{1-\tilde{u}}\varepsilon_{\tilde{u},\kappa} - \varepsilon_{w,\kappa}\right]}_{\text{fiscal externality}} + \underbrace{\frac{1-\tau}{\tau}\varepsilon_{w,\kappa|p}}_{\text{quitting externality}} = 0, \tag{21}$$

where

c^u (c^w) is the consumption of the unemployed (employed),

$\varepsilon_{\tilde{u},\kappa}$ is the general equilibrium elasticity of present value unemployment $\tilde{u}$ with respect to κ,

$\varepsilon_{w,\kappa}$ is the general equilibrium elasticity of the wage w with respect to κ, and
$\varepsilon_{w,\kappa|p} = \frac{\kappa}{w}\frac{\partial w}{\partial \bar{\chi}}\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right)$ is the partial elasticity of w with respect to κ via $\bar{\chi}$, holding fixed p.

The first term in equation (21), labeled "consumption insurance," captures the mechanical impact of an increase in κ on consumption inequality, in the absence of any behavioral responses via p or $\bar{\chi}$. A higher κ means non-workers enjoy higher consumption, while workers enjoy lower consumption. This term appears identically in the standard Baily-Chetty formula – it is exactly the left-hand side of equation (8) in Chetty (2006).

The second term, labeled "fiscal externality," captures the behavioral impact of raising κ on tax revenue. In the model, a higher replacement rate will lower the quitting threshold $\bar{\chi}$ and lower the job finding probability p. These effects will increase the equilibrium share of non-workers, necessitating an additional increase in the tax rate τ to balance the government budget. In the standard Baily-Chetty formula, the analogous term is the elasticity of unemployment to benefit duration. Here, with a quitting margin, the replacement rate affects the frequency of non-employment, as well as the duration of non-employment spells: the elasticity of $\tilde{u}$ with respect to κ captures both margins. A second difference relative to the standard Baily-Chetty condition is the $\varepsilon_{w,\kappa}$ term. This term is present because the budget-balancing tax rate must adjust to changes in the equilibrium wage in addition to changes in equilibrium unemployment.[39]

The third term, labeled "quitting externality," is novel. It captures the fact that in our economy, a change in κ changes the equilibrium wage via the impact on the quitting threshold $\bar{\chi}$. A higher κ makes matched workers more willing to quit, lowering $\bar{\chi}$. Anticipating shorter match durations, firms will offer lower wages for any given p, which is welfare reducing. This quitting effect term scales with the partial elasticity of the wage to a change in κ, holding p constant, which we denote $\varepsilon_{w,\kappa|p}$. Recall from equation (16) that changes in κ change w both via an impact on p and via an impact on $\bar{\chi}$. Why does the planner worry only about the impact via the $\bar{\chi}$ channel? The reason is that unmatched workers are choosing p optimally, internalizing the impact of p on wages via $\bar{\chi}$ as part of that optimization. So the planner perceives no marginal welfare gain from using κ to manipulate w via changes in p. That is why there is no analogue of the quitting term in the (textbook) version of the model in which workers cannot quit.

The partial elasticity $\varepsilon_{w,\kappa|p}$ is given by $\frac{\kappa}{w}\frac{\partial w}{\partial \bar{\chi}}\left[\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right]$. Wages are increasing in $\bar{\chi}$ because less quitting raises the expected match surplus: $\frac{\partial w}{\partial \bar{\chi}} > 0$. Increases in κ or τ make workers more likely

[39] The reason the $\varepsilon_{\tilde{u},\kappa}$ elasticity is pre-multiplied by $1/(1-\tilde{u})$ and the $\varepsilon_{w,\kappa}$ term is not is that a rise in $\tilde{u}$ means both more benefit recipients and fewer taxpayers, while a decline in w reduces tax revenue without increasing costs.

to quit, by raising the value of the worker's outside option, or reducing the value of employment; hence, $\frac{\partial \bar{\chi}}{\partial \kappa} < 0$ and $\frac{\partial \bar{\chi}}{\partial \tau} < 0$. Taken together, these properties imply that the partial elasticity and thus the quitting effect are negative. Thus, the quitting term pushes the optimal replacement rate downward.

We now turn to a quantification of these different terms. The calibration is mostly identical to our baseline. Utility is logarithmic, and the baseline replacement rate is $\kappa = 0.5$. We recalibrate match efficiency A, the vacancy posting cost ϕ, and the mean of the cost-of-work distribution μ_χ to replicate 2021-2022 values for (i) the unemployment rate (4.15 percent), (ii) the job openings rate (8.03 percent), and (iii) the quit rate (1.88 percent). The implied values are in Panel A of Table 5 in Appendix 6.1.

Panel B of Table 5 reports the values of the different terms in our extended Baily-Chetty formula, along with some other equilibrium variables. The first row evaluates these terms at $\kappa = 0.5$. The second row evaluates the same terms at the optimal replacement rate, which is $\kappa = 0.328$. The key findings are as follows.

First, the quitting term is quantitatively important. At $\kappa = 0.5$, the welfare cost of a marginal increase in κ via a lower equilibrium wage because of quits is larger than the gain from better consumption insurance. Second, the quitting effect is not large because the equilibrium wage is very sensitive to κ. In fact, the general equilibrium elasticity is tiny, and the partial equilibrium elasticity (holding constant p) is also quite small. Rather, the reason the quitting term is quantitatively important is that this partial equilibrium elasticity is pre-multiplied by $(1 - \tau)/\tau$, which is large because τ is small. Intuitively, small declines in equilibrium wages are costly because they apply to employed workers, who constitute the vast majority of the population, while the other terms in the Baily-Chetty formula are effectively proportional to the much smaller share of non-employed workers.

Second, at the much lower optimal replacement rate of $\kappa = 0.328$, the marginal gain from increasing κ via better consumption insurance is much larger, while the marginal costs in terms of higher equilibrium tax rates and lower equilibrium wages are of very similar magnitude. The existing literature has focused entirely on lower equilibrium employment as the cost of more generous unemployment insurance. Our finding here suggests that the cost of lower equilibrium wages is quantitatively just as important when the quitting friction is taken into account.

The Baily-Chetty formula has proven very popular because it offers an intuitive formulation of the trade-offs in setting UI and because the elasticities that appear in the formula can in principle be estimated and used to calibrate actual policy. Unfortunately, however, one elasticity in equation (21) is the elasticity of wages to κ, holding constant p. How one would go about estimating that elasticity is unclear. And because this elasticity is scaled by a large constant, a very precise estimate would be required. Jager et al. (2020) estimate the aggregate elasticity of wages to the UI replacement rate and conclude that it is less than 3 cents per dollar of additional benefits: at $\kappa = 0.5$, the corresponding value in our model is 0.4 cents.[40]

6.1 Sensitivity 1: Quitting Less Sensitive to UI

We now show how the Baily-Chetty decomposition changes in an experiment in which we retain the quitting margin but make the quit rate less sensitive to UI policy. In particular, we consider an alternative model calibration in which the standard deviation of χ is much higher. This makes the quit rate much less sensitive to the UI replacement rate κ, since only a small mass of workers are close to indifferent about quitting. We recalibrate A, ϕ and μ_χ so that at $\kappa = 0.5$, this high preference variance specification delivers the same unemployment rate, job openings rate, and quit rate as the baseline calibration. Panel A of Table 5 shows that implies no change in the calibrated value for A, a much lower value for μ_χ, and a fourfold increase in the the vacancy cost ϕ.

Rows 3 and 4 of Panel B report results for this high preference variance calibration. At $\kappa = 0.5$, the quitting term is much smaller than in the corresponding baseline model, reflecting a much smaller partial elasticity of wages with respect to κ, $\varepsilon_{w,\kappa|p}$. This partial elasticity is smaller because the quit rate is much less sensitive to κ. Because this negative partial elasticity is smaller than in the baseline model, the full general equilibrium elasticity of wages to κ is larger. Now, a one dollar increase in benefits leads to a 5.2 cent increase in the wage, which exceeds the upper bound estimated by Jager et al. (2020).

The optimal replacement rate in the high variance calibration is much higher than in the baseline calibration, at $\kappa = 0.455$. Again, that is because the costs of more generous benefits, in terms of shorter expected match duration and thus lower wages, are much smaller.[41]

Note that an important message from this experiment is that it is not the presence of quitting

[40] There are many estimates of the elasticity of non-employment $\tilde{u}$ with respect to κ; see Schmieder and von Wachter (2016) for a recent survey.

[41] We have also conducted an experiment in which we assume that the quitting threshold $\bar{\chi}$ is exogenously fixed at the baseline value, implying a constant quit rate of 1.88 percent, irrespective of κ. Hence, the quitting externality term is exactly zero. The optimal κ in this case is 43.7 percent, similar to what it is in the high variance case. Note that in this case, the standard Baily-Chetty formula holds, and the consumption inequality term is exactly equal to the fiscal externality term at the optimum.

Table 3: Welfare Decomposition with Baily-Chetty Formula

Panel A: Parameter values

	A	ϕ	μ_χ	σ^2_χ	γ
Baseline	0.563	0.103	-1.22	0.25	0.9806
$\sigma^2_\chi = 100$	0.563	0.405	-20.83	100	0.9806

Panel B: Terms in Baily-Chetty formula and elasticities

	κ	c ineq.	fiscal extn.	quit extn.	$\varepsilon_{\tilde{u},\kappa}$	$\varepsilon_{w,\kappa}$	$\varepsilon_{w,\kappa\|p}$
Baseline	0.500	0.918	-4.545	-1.364	4.271	0.002	-0.046
Optimum	0.328	1.996	-1.012	-0.984	0.980	-0.008	-0.008
$\sigma^2_x = 100$	0.500	0.867	-0.983	-0.202	0.948	0.027	-0.007
Optimum	0.455	1.057	-0.838	-0.220	0.814	0.025	-0.006

Panel C: Model moments

	κ	$\tilde{u}$ (%)	w	τ (%)	p	Quit rate (%)	$\Delta w / \Delta \kappa$
Baseline	0.500	6.06	0.9913	3.26	0.628	1.88	0.004
Optimum	0.328	2.36	0.9911	0.80	1.000	0.43	-0.024
$\sigma^2_x = 100$	0.500	6.06	0.9658	3.34	0.628	1.88	0.052
Optimum	0.455	5.58	0.9634	2.79	0.677	1.85	0.053

per se that dictates a less generous optimal UI replacement rate. Rather, quitting is a problem only if a significant share of quitters are close to marginal workers and would be induced not to quit – thereby preserving matches with positive joint surplus – if UI was slightly less generous. If the quit rate is insensitive to the benefit level, then quitters and firees should be treated equally.

Why is the calibrated value for vacancy posting costs four times smaller in the baseline calibration than in the high preference variance specification? With a much lower vacancy posting cost, firms are willing to offer either higher wages or a higher job finding probability. But recall that these two economies replicate identical values for unemployment, job openings and quits. Given identical values for p and $F(\bar{\chi})$, it is immediate from the firm's free entry condition that lower vacancy costs in the baseline model must translate into a higher wage level w, which is evident in Panel C, column 3. Thus, workers in the baseline model choose to benefit from low vacancy costs in the form of high wages rather than a high job finding probability. The reason they do so is again the efficiency wage logic: higher wages signal lower future quitting, which firms are happy to reward. Thus, one lesson from this experiment is that models with endogenous quitting can generate significant unemployment with very low job creation costs. In this sense, the quitting margin amplifies frictional unemployment.

We have also explored a calibration in which there are no exogenous separations ($\gamma = 1$). In this parameterization, discussed in detail in Appendix C, the optimal UI replacement rate parameter is $\kappa = 0.078$. The huge difference between the optimal replacement rates in the high variance case (in which all separations are essentially exogenous) and the $\gamma = 1$ case (in which all separations are endogenous) starkly illustrates the importance of understanding what drives separations.

6.2 Sensitivity 2: Alternative Models of Insurance

We now discuss how the optimal level of UI changes under two alternative calibrations reflecting different assumptions about income when not working. In the first alternative calibration, we assume that the planner pays lower benefits to quitters relative to those who were fired. In particular, we assume that those who were fired get $\kappa = 0.5$, as in the baseline, while quitters receive only half as much. Given that assumption, we recalibrate A, ϕ and μ_χ so that the model replicates the baseline values for unemployment and for the job opening and quit rates (see Table 6 in Appendix C).

This recalibration delivers a higher average utility cost of work, which is required to maintain the observed quit rate when quitters anticipate reduced benefits. Given this recalibration, we ask what benefit levels are optimal. If the planner has to pay quitters and firees the same benefit, the optimal value for κ is now 0.217, which is much lower than the value in the baseline calibration. Intuitively, with a higher utility cost of work, more workers are close to indifferent about quitting, and the planner therefore has a stronger incentive to lower κ. If the planner can discriminate between quitters and firees, they would choose a very low transfer value for quitters: $\kappa_{EN} = 0.051$.

Finally, we consider a calibration in which all non-workers receive an exogenous amount of income κ_P in addition to government benefits. We do not micro-found the source of this private insurance. We assume that $\kappa_P = 0.25$ and that the government also provides a UI replacement rate of $\kappa = 0.25$, so that total income for non-workers is the same as in our baseline calibration. The only difference relative to that model is that the budget-balancing tax rate τ is lower. In this case, the optimal value for κ is 0.105, implying a total replacement rate of $\kappa_P + \kappa = 0.355$, which is similar to the value in the baseline model (0.328). Thus, introducing private insurance crowds out optimal public insurance almost one-for-one.

6.3 Explaining the Great Resignation

Both the quit rate and the vacancy rate are much higher at the end of our sample period than they were at the end of the previous expansion in 2006 (see Figure 1). It is difficult to separate cycle from trend over the relatively short sample period for which JOLTS data are available, but the figure suggests a secular upward trend in both series. We now use the model to offer a tentative explanation for observed changes in labor market dynamics. In particular, we show that a decline in vacancy posting costs can rationalize most of the key dynamics observed over this period. One interpretation is that new technologies and platforms such as Monster and Indeed have made it cheaper for firms to advertise open positions.

Table 4: The Great Resignation via Lower Vacancy Costs

	2006	2021-22	Δ (pp)	Δ Model
EN rate (%)	0.8	1.8	1.0	0.9
EE rate (%)	1.8	1.8	0.0	0.3
u rate (%)	4.6	4.1	-0.5	-1.0
v rate (%)	4.0	7.7	3.7	3.5

Table 4 reports the results from the following experiment. We start from our multi-sector model calibrated to 2021-2022. We change only one parameter, which is the vacancy posting cost parameter $\hat{\phi}$. We search for the value $\hat{\phi}_{2006}$ so that in steady state, the model comes as close as possible to matching the 2006 values for the unemployment rate, the job openings rate, the EN transition rate, and the EE transition rate.[42] The resulting estimate for $\hat{\phi}_{2006}$ is 0.320, almost twice the value for 2021/22, which is 0.165. Comparing across steady states shows that moving from $\hat{\phi}_{2006}$ to $\hat{\phi}_{2021-22}$ increases the model quit-to-non-employment rate by 0.9 percentage points while increasing the job-openings rate by 3.5 percentage points. Both changes are very similar to those observed in the data. The model predicts a small increase in the job-to-job transition rate, while there was no such increase in the data.

For the 2006 calibration, the optimal value for the replacement rate parameter κ is 0.403, which is larger than the optimal value of 0.384 we found for 2021. The planner wants to make UI less generous in response to a smaller vacancy posting cost because more plentiful vacancies increase the equilibrium quit rate and the quitting externality.

[42] Our data targets are 12 month averages of these statistics for 2006. Our distance metric is the sum of squared deviations between model and data moments.

7 Conclusion

In a model in which quits to non-employment are driven by private, idiosyncratic preference shocks, workers quit too readily, destroying matches with positive joint surplus. This translates into depressed wages and wasteful vacancy creation, and motivates designing public insurance so as to discourage quits. If the government cannot differentiate between separations in which the worker was fired versus those in which the worker quit, this consideration reduces the optimal common UI replacement rate. If the government can distinguish the two groups, the replacement rate for quitters should be lower than the one for firees.

In our calibrated directed search model, firms and workers understand that quits can be costly, and adapt to reduce quits in three ways. First, firms offer contracts in which wages are backloaded, as a way to increase worker retention. Second, firms stochastically match outside offers, which reduces the rate at which workers quit to other jobs. Third, workers choose to direct search to high "efficiency" wage jobs, understanding that this is a way to partially commit to not quitting. However, while all these arrangements reduce the quit rate, it remains inefficiently high. And while high efficiency wages are privately welfare maximizing, holding out for high wage jobs translates to a higher equilibrium unemployment rate and higher taxes. Thus, the quitting margin exacerbates the standard fiscal externality associated with unemployment insurance.

The quantitative relevance of the quitting margin for the optimal UI replacement rate depends not on the quit rate *per se* but rather on the elasticity of quits to the replacement rate, which in turn is linked to the variance of idiosyncratic preference shocks. We identified this variance using cross-industry variation in the quit rate. We also derived an extended Baily-Chetty formula in which the quitting margin introduces a new term involving the partial elasticity of wages to the replacement rate. In our baseline quantitative calibration, we find an optimal UI replacement rate for workers in lower wage industries of 38.4%, which is over 10 percentage points lower than the rate that would apply without idiosyncratic preference shocks.

Our model could be extended to introduce persistent preference shocks in order to generate more persistent non-employment. Another possible extension would introduce repeated shocks to match productivity throughout employment spells, to generate additional wage dispersion. One could also explicitly model savings and other sources of self-insurance against unemployment risk. In all such extensions, the impact of UI policy parameters on quits and thus on other labor market variables would remain an important consideration in optimal social insurance design.

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Appendix: Not for Publication

A Proofs

A.1 Proof of Proposition 2.1

We start by solving for the equilibrium in the public χ economy. Given policy parameters, the problem of private agents is given by the following:

$$\max_{\theta,w,\bar{\chi}} p(\theta) \int^{\bar{\chi}} (w-\tau-\chi)\, dF(\chi) + [1-p(\theta)F(\bar{\chi})]\, b$$

$$s.t.$$

$$q(\theta) \int^{\bar{\chi}} (z-w)\, dF(\chi) = \phi$$

$$\bar{\chi} = z-\tau-b.$$

Plug the second constraint $\bar{\chi} = z-\tau-b$ into the objective function and into the first constraint

$$\max_{\theta,w,\bar{\chi}} p(\theta) \int^{z-\tau-b} (w-\tau-\chi)\, dF(\chi) + [1-p(\theta)F(z-\tau-b)]\, b$$

$$s.t.$$

$$q(\theta) \int^{z-\tau-b} (z-w)\, dF(\chi) = \phi.$$

With the assumption that χ is uniformly distributed, we have

$$\max_{\theta,w} p(\theta) \frac{\left[(w-\tau)(z-\tau-b) - \frac{1}{2}(z-\tau-b)^2\right]}{a} + b - p(\theta)\frac{z-\tau-b}{a} b$$

$$s.t.$$

$$q(\theta)\frac{z-\tau-b}{a}(z-w) = \phi.$$

Collecting terms, we get that the objective function becomes

$$p(\theta)\left[\frac{(z-\tau-b)(w-\tau-b)}{a} - \frac{(z-\tau-b)^2}{2a}\right] + b.$$

With the assumptions on the matching function

$$p(\theta) = A\theta^{0.5}$$

$$q(\theta) = A\theta^{-0.5}$$

we can substitute out θ and obtain q as a function of p:

$$q(p) = \frac{A^2}{p}.$$

With this expression we can transform this problem into

$$\max_{w,p} p\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+b$$

$$s.t.$$

$$\frac{A^2}{p}\frac{z-\tau-b}{a}(z-w)=\phi.$$

The constraint can be rearranged to express p as a function of all other variables

$$p=\frac{A^2}{\phi}\frac{z-\tau-b}{a}(z-w).$$

Plug this into the objective

$$\max_{w}\frac{A^2}{\phi}\frac{z-\tau-b}{a}(z-w)\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+b.$$

Take a first order condition with respect to w

$$-\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+(z-w)\frac{(z-\tau-b)}{a}=0$$

$$-(w-\tau-b)+\frac{(z-\tau-b)}{2}+z-w=0$$

$$w=\frac{3}{4}z+\frac{1}{4}(\tau+b).$$

With the wage given by the above expression, the job finding probability p is given by

$$\begin{aligned}p &= \frac{A^2}{\phi}\frac{z-\tau-b}{a}(z-w)\\ &= \frac{A^2}{\phi}\frac{z-\tau-b}{a}\left(z-\left(\frac{3}{4}z+\frac{1}{4}(\tau+b)\right)\right)\\ &= \frac{A^2}{\phi}\frac{(z-\tau-b)^2}{4a}.\end{aligned}$$

The case of public χ is now fully characterized.

Next, we move to the case with private χ. The problem is given by

$$\max_{\theta,w,\bar{\chi}} p(\theta)\int^{\bar{\chi}}(w-\tau-\chi)\,dF(\chi)+[1-p(\theta)F(\bar{\chi})]\,b$$

s.t.

$$q(\theta)\int^{\bar{\chi}}(z-w)\,dF(\chi)=\phi$$

$$w-\tau-\bar{\chi}=b.$$

Substitute in $\bar{\chi}=w-\tau-b$

$$\max_{\theta,w,\bar{\chi}} p(\theta)\int^{w-\tau-b}(w-\tau-\chi)\,dF(\chi)+[1-p(\theta)F(w-\tau-b)]\,b$$

s.t.

$$q(\theta)\int^{w-\tau-b}(z-w)\,dF(\chi)=\phi.$$

Plug in the uniform distribution of χ

$$\max p(\theta)\left[(w-\tau-b)\frac{(w-\tau-b)}{a}-\frac{1}{2a}(w-\tau-b)^2\right]+b$$

s.t.

$$q(\theta)\frac{w-\tau-b}{a}(z-w)=\phi.$$

Plug in

$$q(p)=\frac{A^2}{p},$$

we have

$$\max p\left[(w-\tau-b)\frac{(w-\tau-b)}{a}-\frac{1}{2a}(w-\tau-b)^2\right]+b$$

subject to

$$\frac{A^2}{p}\frac{w-\tau-b}{a}(z-w)=\phi$$

or

$$p=\frac{A^2}{\phi}\frac{w-\tau-b}{a}(z-w).$$

Plug this into the objective:

$$\max\frac{A^2}{\phi}\frac{w-\tau-b}{a}(z-w)\left[(w-\tau-b)\frac{(w-\tau-b)}{a}-\frac{1}{2a}(w-\tau-b)^2\right]+b$$

Simplifying, we obtain

$$\max\frac{A^2}{\phi}\frac{(w-\tau-b)^3}{a}(z-w)\frac{1}{2a}+b.$$

Take the FOC with respect to w

$$3(w-\tau-b)^2(z-w)+-(w-\tau-b)^3=0$$

$$3z-3w-w+\tau+b=0$$

$$w=\frac{3}{4}z+\frac{1}{4}(\tau+b).$$

Hence, the quitting threshold is given by

$$\begin{aligned}\bar{\chi} &= w-\tau-b\\ &= \frac{3}{4}z+\frac{1}{4}(\tau+b)-\tau-b\\ &= \frac{3}{4}(z-\tau-b),\end{aligned}$$

and the job finding probability is given by

$$\begin{aligned} p &= \frac{A^2}{\phi}\frac{w-\tau-b}{a}(z-w) \\ &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2. \end{aligned}$$

A.2 Proof of Proposition 2.2

We will first show that the public χ economy can achieve a first-best allocation. For the public χ economy, denote this value function $V(\tau,b)$:

$$\begin{aligned} V(\tau,b) &= p\int^{z-\tau-b}[w-\tau-\chi]\,dF(\chi)+(1-pF(z-\tau-b))\,b \\ &= pF(z-\tau-b)(w-\tau)-p\int^{z-\tau-b}\chi dF(\chi)+(1-pF(z-\tau-b))\,b \\ &= pF(z-\tau-b)(w-\tau-b)+b-\frac{p}{2a}(z-\tau-b)^2. \end{aligned}$$

Using the government budget constraint:

$$pF(z-\tau-b)\,\tau=(1-pF(z-\tau-b))\,b,$$

we have

$$pF(z-\tau-b)(b+\tau)=b.$$

Using this relation to substitute out the middle term b, we have:

$$\begin{aligned} V(\tau,b) &= pF(z-\tau-b)(w-\tau-b)+pF(z-\tau-b)(b+\tau)-\frac{p}{2a}(z-\tau-b)^2 \\ &= pF(z-\tau-b)\,w-\frac{p}{2a}(z-\tau-b)^2 \\ &= p\frac{z-\tau-b}{a}w-\frac{p}{2a}(z-\tau-b)^2. \end{aligned}$$

Plug in the expressions for w and p :

$$w=\frac{3}{4}z+\frac{1}{4}(\tau+b)$$

$$p=\frac{A^2}{\phi}\frac{(z-\tau-b)^2}{4a}$$

We have

$$\begin{aligned}
V(\tau, b) &= p\frac{z-\tau-b}{a}w - \frac{p}{2a}(z-\tau-b)^2 \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^3}{4a^2}\left[\frac{3}{4}z + \frac{1}{4}(\tau+b) - \frac{2}{4}(z-\tau-b)\right] \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^3}{16a^2}[z-\tau-b] \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^4}{16a^2}.
\end{aligned}$$

It is easy to see that V is minimized when $\tau + b = 0$. By the budget constraint, if $\tau + b = 0$, it follows that b must also be zero. Hence, the Acemoglu-Shimer result holds in this environment. We need to show that this coincides with the first-best allocation.

Under the first best allocation, the planner solves:

$$\max_{\theta,\bar{\chi}} p(\theta)\int^{\bar{\chi}}(z-\chi)\,dF(\chi) - \theta\phi.$$

Plug in the uniform distribution and $p(\theta) = A\theta^{\frac{1}{2}}$, and substitute in $\bar{\chi} = z$:

$$\max_{\theta} A\theta^{\frac{1}{2}}\frac{\frac{1}{2}z^2}{a} - \theta\phi.$$

Take the FOC:

$$\frac{1}{2}A\theta^{-\frac{1}{2}}\frac{\frac{1}{2}z^2}{a} = \phi$$

$$\frac{1}{4}A\frac{z^2}{a\phi} = \theta^{\frac{1}{2}}$$

$$\theta = \left(\frac{1}{4}A\frac{z^2}{a\phi}\right)^2.$$

Plug this back into the objective. The value becomes:

$$\begin{aligned}
& A\frac{1}{4}A\frac{z^2}{a\phi}\frac{\frac{1}{2}z^2}{a} - \left(\frac{1}{4}A\frac{z^2}{a\phi}\right)^2\phi \\
& \frac{1}{8}A^2\frac{z^4}{a^2\phi} - \frac{1}{16}A^2\frac{z^4}{a^2\phi} \\
=\ & \frac{1}{16}A^2\frac{z^4}{a^2\phi}.
\end{aligned}$$

which is exactly equal to the value in the public χ economy. Hence, we confirm that the first best allocation can be achieved in the public χ economy. It then follows that in the private χ economy, agents quit too often, and they are also too picky relative to the first-best case.

We will now show that a government that respects the same no-quitting condition would pick the same submarket as private agents.

The government's problem is given by:

$$\max_{p,w} p(\theta) \int_0^{\bar{\chi}} (z-\chi)\, dF(\chi) - \theta\phi$$

$$s.t.$$

$$\bar{\chi} = w$$

$$q(\theta) F(\bar{\chi})(z-w) = \phi$$

$$p = A\theta^{\frac{1}{2}}.$$

The first constraint is the no-quitting condition (with no taxes and subsidies). The second is the zero profit condition that pins down θ. The third condition determines p as a function of market tightness θ.

It is easy to see that quitting is inefficient: $\bar{\chi} < z$. Suppose instead that $\bar{\chi} = z$. That would imply $w = \bar{\chi} = z$, which means that firm makes zero profit *ex post*; hence, $\theta = p = 0$, which is inconsistent with welfare maximization.

We need to compare the solution to this problem with the private agent's problem, which is given by:

$$\max_{p,w} p \int^{\bar{\chi}} [w-\chi]\, dF(\chi)$$

$$s.t.$$

$$\bar{\chi} = w$$

$$q(\theta) F(\bar{\chi})(z-w) = \phi$$

$$p = A\theta^{\frac{1}{2}}.$$

Those constraints are exactly the same; hence, it is sufficient to show that the objective functions are equivalent for given values of p and w. From the zero profit condition,

$$A\theta^{-\frac{1}{2}} F(\bar{\chi})(z-w) = \phi,$$

where we have plugged in that

$$q(\theta) = A\theta^{-\frac{1}{2}}.$$

Multiply both sides by θ :

$$A\theta^{\frac{1}{2}} F(\bar{\chi})(z-w) = \theta\phi$$

or

$$p(\theta) F(\bar{\chi})(z-w) = \theta\phi.$$

Noting that

$$p(\theta) = A\theta^{\frac{1}{2}},$$

use this equation to substitute out the $\theta\phi$ term in the social planner's problem:

$$\begin{aligned}
& p(\theta) \int_0^{\bar{\chi}} (z-\chi) \, dF(\chi) - \theta\phi \\
= \;& p(\theta) \int_0^{\bar{\chi}} (z-\chi) \, dF(\chi) - p(\theta) F(\bar{\chi})(z-w) \\
= \;& p(\theta) \int_0^{\bar{\chi}} (w-\chi) \, dF(\chi).
\end{aligned}$$

Hence, the objective functions are exactly the same, implying that the planner and the private agents would choose exactly the same submarket, or allocation pair (p, w).

A.3 Proof of Proposition 2.3

We first derive the indirect value function for the workers, then show that there exists a closed form in both the public χ and the private χ economy.

For the public χ economy, denote this value function $V(\tau, b)$:

$$\begin{aligned}
V(\tau, b) &= p \int^{z-\tau-b} [w-\tau-\chi] \, dF(\chi) + (1 - pF(z-\tau-b)) \, b \\
&= pF(z-\tau-b)(w-\tau) - p \int^{z-\tau-b} \chi dF(\chi) + (1 - pF(z-\tau-b)) \, b \\
&= pF(z-\tau-b)(w-\tau-b) + b - \frac{p}{2a}(z-\tau-b)^2.
\end{aligned}$$

Using the government budet constraint

$$pF(z-\tau-b) \, \tau = (1 - pF(z-\tau-b)) \, b,$$

we have

$$pF(z-\tau-b)(b+\tau) = b.$$

Using this relation to substitute out the middle term b, we have

$$\begin{aligned}
V(\tau,b) &= pF(z-\tau-b)(w-\tau-b)+pF(z-\tau-b)(b+\tau)-\frac{p}{2a}(z-\tau-b)^2 \\
&= pF(z-\tau-b)\,w-\frac{p}{2a}(z-\tau-b)^2 \\
&= p\frac{z-\tau-b}{a}w-\frac{p}{2a}(z-\tau-b)^2.
\end{aligned}$$

Plug in the expressions for w and p :

$$w=\frac{3}{4}z+\frac{1}{4}(\tau+b)$$

$$p=\frac{A^2}{\phi}\frac{(z-\tau-b)^2}{4a}.$$

We have

$$\begin{aligned}
V(\tau,b) &= p\frac{z-\tau-b}{a}w-\frac{p}{2a}(z-\tau-b)^2 \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^3}{4a^2}\left[\frac{3}{4}z+\frac{1}{4}(\tau+b)-\frac{2}{4}(z-\tau-b)\right] \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^3}{16a^2}[z-\tau-b] \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^4}{16a^2}.
\end{aligned}$$

It is easy to see that V is minimized when $\tau+b=0$. By the budget constraint, if $\tau+b=0$, it follows that b must also be zero. Hence, the Acemoglu-Shimer result holds in this environment. We need to show that this coincides with the first-best allocation.

Under the first best allocation, the planner solves:

$$\max_{\theta,\bar{\chi}} p(\theta)\int^{\bar{\chi}}(z-\chi)\,dF(\chi)-\theta\phi$$

Plug in the uniform distribution and $p(\theta)=A\theta^{\frac{1}{2}}$, and substitute in $\bar{\chi}=z$:

$$\max_{\theta} A\theta^{\frac{1}{2}}\frac{\frac{1}{2}z^2}{a}-\theta\phi.$$

Take FOC:

$$\frac{1}{2}A\theta^{-\frac{1}{2}}\frac{\frac{1}{2}z^2}{a}=\phi$$

$$\frac{1}{4}A\frac{z^2}{a\phi}=\theta^{\frac{1}{2}}$$

$$\theta=\left(\frac{1}{4}A\frac{z^2}{a\phi}\right)^2.$$

Plug it back into the objective. The value becomes:

$$A\frac{1}{4}A\frac{z^2}{a\phi}\frac{\frac{1}{2}z^2}{a}-\left(\frac{1}{4}A\frac{z^2}{a\phi}\right)^2\phi$$

$$\begin{aligned} & \frac{1}{8}A^2\frac{z^4}{a^2\phi} - \frac{1}{16}A^2\frac{z^4}{a^2\phi} \\ = & \frac{1}{16}A^2\frac{z^4}{a^2\phi}, \end{aligned}$$

which is exactly equal to the value in the public χ economy. Hence we confirm that the first best allocation can be achieved in the public χ economy.

Let's move to the private χ economy. Denote this value function $V(\tau, b)$:

$$\begin{aligned} V(\tau, b) &= p\int^{w-\tau-b}[w-\tau-\chi]\,dF(\chi) + (1 - pF(w-\tau-b))\,b \\ &= pF(w-\tau-b)(w-\tau) - p\int^{w-\tau-b}\chi dF(\chi) + (1 - pF(w-\tau-b))\,b \\ &= pF(w-\tau-b)(w-\tau-b) + b - \frac{p}{2a}(w-\tau-b)^2. \end{aligned}$$

Using the government budet constraint

$$pF(w-\tau-b)\,\tau = (1 - pF(w-\tau-b))\,b,$$

we have

$$\begin{aligned} V(\tau, b) &= pF(w-\tau-b)(w-\tau-b) + pF(w-\tau-b)(b+\tau) - \frac{p}{2a}(w-\tau-b)^2 \\ &= pF(w-\tau-b)\,w - \frac{p}{2a}(w-\tau-b)^2 \\ &= p\frac{w-\tau-b}{a}w - \frac{p}{2a}(w-\tau-b)^2. \end{aligned}$$

Plug in

$$w = \frac{3}{4}z + \frac{1}{4}(\tau+b)$$

$$p = \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2.$$

We have

$$\begin{aligned} V(\tau, b) &= p\frac{(w-\tau-b)}{a}\left[w - \frac{w-\tau-b}{2}\right] \\ &= p\frac{(w-\tau-b)}{a}\left[\frac{w+\tau+b}{2}\right] \\ &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2\frac{\left(\frac{3}{4}z + \frac{1}{4}(\tau+b) - \tau - b\right)}{a}\left[\frac{\frac{3}{4}z + \frac{1}{4}(\tau+b) + \tau + b}{2}\right] \\ &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2\frac{\left(\frac{3}{4}z - \frac{3}{4}(\tau+b)\right)}{a}\left[\frac{\frac{3}{4}z + \frac{5}{4}(\tau+b)}{2}\right] \\ &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\frac{3}{4}\frac{(z-(\tau+b))^3}{a}\left[\frac{\frac{3}{4}z + \frac{5}{4}(\tau+b)}{2}\right]. \end{aligned}$$

Taking the derivative with respect to

$$x = \tau + b,$$

we have:

$$-3(z-x)^2(3z+5x)+5(z-x)^3=0$$

$$-3(3z+5x)+5(z-x)=0$$

$$5z-5x=9z+15x$$

$$20x=-4z$$

$$x=-\frac{z}{5}.$$

Hence, the optimal policy in this case calls for taxing the unemployed:

$$\tau+b=-\frac{z}{5}.$$

With the optimal policy, the value function is given by

$$\begin{aligned}
V(\tau,b) &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\frac{3}{4}\frac{\left(z+\frac{z}{5}\right)^3}{a}\left[\frac{\frac{3}{4}z-\frac{5}{4}\frac{z}{5}}{2}\right] \\
&= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\frac{3}{4}\left(\frac{6}{5}\right)^3\frac{1}{4}\frac{z^3}{a}z \\
&= \frac{A^2}{\phi}\frac{3}{4}\frac{1}{4}\frac{3}{4}\left(\frac{6}{5}\right)^3\frac{1}{4}\frac{z^4}{a^2} \\
&= \frac{3}{4}\frac{1}{4}\frac{3}{4}\left(\frac{6}{5}\right)^3\frac{1}{4}\frac{A^2}{\phi}\frac{z^4}{a^2} \\
&= 0.0607\frac{A^2}{\phi}\frac{z^4}{a^2}<\frac{1}{16}\frac{A^2}{\phi}\frac{z^4}{a^2}=\text{ value at the first best allocation.}
\end{aligned}$$

Hence, we have shown that the private χ economy cannot achieve the welfare of the first best allocation.

A.4 Proof of Proposition 3.1

$$\Pi(V)=\max_{w,V'}\left\{\gamma F(\bar{\chi}(w,V'))(z-w+\beta\Pi(V'))\right\}$$

s.t.

$$\log(w)-\bar{\chi}(w,V')+\beta V'=V^u : \textit{quitting\_rule}$$

$$\gamma F(\bar{\chi}(w,V'))\left(U(w)-E\left[\chi|\chi\leq\bar{\chi}(w,V')\right]+\beta V'\right)+(1-\gamma F(\bar{\chi}(w,V')))V^u\geq V : \textit{promise\_keeping}$$

The quitting rule gives a closed form for $\bar{\chi}(w,V')$:

$$\bar{\chi}(w,V')=U(w)+\beta V'-V^u.$$

We can plug that into the second constraint and into the objective

So we have an objective with choice variables w and V':

$$\Pi(V) = \max_{w,V'} \left\{ \gamma F(U(w) + \beta V' - V^u)(z - w + \beta \Pi(V')) \right\}$$

s.t.

$$\gamma F(U(w) + \beta V' - V^u) \left(U(w) - E\left[\chi | \chi \leq U(w) + \beta V' - V^u \right] + \beta V' \right) + (1 - \gamma F(U(w) + \beta V' - V^u))V^u = V : \mu$$

where we denote the multiplier μ.

The promise keeping constraint can be rewritten as:

$$\gamma F(U(w) + \beta V' - V^u) \left(U(w) + \beta V' \right) - \gamma \int_{-\infty}^{U(w)+\beta V' - V^u} \chi f(\chi) \, d\chi + (1 - \gamma F(U(w) + \beta V' - V^u))V^u = V : \mu.$$

The Lagrangian is given by

$$\begin{aligned} L &= \gamma F(U(w) + \beta V' - V^u)(z - w + \beta \Pi(V')) \\ &\quad + \mu \left(V - \gamma F(U(w) + \beta V' - V^u) \left(U(w) + \beta V' \right) + \gamma \int_{-\infty}^{U(w)+\beta V' - V^u} \chi f(\chi) \, d\chi - (1 - \gamma F(U(w) + \beta V' - V^u))V^u \right). \end{aligned}$$

Take FOCs

$$\begin{aligned} w \quad : \quad & -\gamma F(U(w) + \beta V' - V^u) + \gamma f(U(w) + \beta V' - V^u)(z - w + \beta \Pi(V'))U'(w) \\ & -\mu \begin{pmatrix} \gamma F(U(w) + \beta V' - V^u)U'(w) + \gamma f(U(w) + \beta V' - V^u) \left(U(w) + \beta V' \right) U'(w) \\ -\gamma \left(U(w) + \beta V' - V^u \right) f\left(U(w) + \beta V' - V^u \right) U'(w) \\ -\gamma f\left(U(w) + \beta V' - V^u \right) V^u U'(w) \end{pmatrix} \\ = \quad & 0 \end{aligned}$$

and divide through by $f(.), U'(w)$, and γ :

$$\begin{aligned} w \quad : \quad & -\frac{F(U(w) + \beta V' - V^u)}{f(U(w) + \beta V' - V^u)} \frac{1}{U'(w)} + (z - w + \beta \Pi(V')) \\ & -\mu \begin{pmatrix} \frac{F(U(w)) + \beta V' - V^u)}{f(U(w) + \beta V' - V^u)} + \gamma \left(U(w) + \beta V' \right) \\ - \left(U(w) + \beta V' - V^u \right) \\ -V^u \end{pmatrix} \\ = \quad & 0 \end{aligned}$$

$$w : -\frac{F(U(w) + \beta V' - V^u)}{f(U(w) + \beta V' - V^u)} \frac{1}{U'(w)} + z - w + \beta \Pi(V') - \mu \frac{F(U(w) + \beta V' - V^u)}{f(U(w) + \beta V' - V^u)} = 0,$$

Rearranging w, we have the following relation:

$$w : \frac{f(\bar{\chi})}{F(\bar{\chi})}\left[z - w + \beta\Pi(V')\right] = \mu + \frac{1}{U'(w)}.$$

The other first order condition with respect to V' is:

$$\begin{aligned}
V' \quad : \quad & \beta\gamma f(U(w) + \beta V' - V^u)(z - w + \beta\Pi(V')) + \beta\gamma F(U(w) + \beta V' - V^u)\Pi'(V') \\
& -\mu \left(\begin{array}{c} \beta\gamma f(U(w) + \beta V' - V^u)\,(U(w) + \beta V') + \beta\gamma F(U(w) + \beta V' - V^u) \\ -\beta\gamma\,(U(w) + \beta V' - V^u)\, f\,(U(w) + \beta V' - V^u) \\ -\beta\gamma f(U(w) + \beta V' - V^u))V^u \end{array} \right) \\
= \quad & 0.
\end{aligned}$$

Divide through with $\beta\gamma f\,(.)$:

$$\begin{aligned}
V' \quad : \quad & (z - w + \beta\Pi(V')) + \frac{F}{f}\Pi'(V') \\
& -\mu\frac{F}{f} \\
= \quad & 0.
\end{aligned}$$

Thus we have

$$\frac{f(\bar{\chi})}{F(\bar{\chi})}\left[z - w + \beta\Pi(V')\right] = \mu - \Pi'(V').$$

Combining the two we have

$$\frac{1}{U'(w_t)} = -\Pi'(V_{t+1}).$$

Now, we need to obtain an expression for $\Pi'(V_{t+1})$. We take the envelope theorem:

$$\Pi'(V_{t+1}) = \mu_{t+1} = \frac{f(\bar{\chi}_{t+1})}{F(\bar{\chi}_{t+1})}\left[zY - w_{t+1} + \beta\Pi(V_{t+1})\right] - \frac{1}{U'(w_{t+1})}.$$

Hence we have

$$\frac{1}{U'(w_{t+1})} - \frac{1}{U'(w_t)} = \frac{f(\bar{\chi}_{t+1})}{F(\bar{\chi}_{t+1})}\left[zY - w_{t+1} + \beta\Pi(V_{t+1})\right].$$

A.5 Proof of Proposition 6.1

The proof is organized into the following steps. We first derive the social welfare function (equation 15), the optimal quitting condition (equation 17), and the government budget constraint (equation 19).

To derive the Baily-Chetty formula in the elasticity form (equation 21), we first derive its non-elasticity form by taking a first order condition with respect to the replacement rate in the government's problem. At this step, we don't need to assume any specific functional forms for the various functions, such as the social welfare function or the tax function. We then plug in the specific functional forms for these functions and derive the Baily-Chetty formula in its elasticity form.

The Social Welfare Function

We first derive the social welfare function (equation 15). Let W^u and W^e denote the values of being unmatched and matched at the start of the period, before search and matching. Let V^u and V^e be the values of being unmatched and matched after the search and matching stage, but before separations happen. These values are given by

$$\begin{aligned}
W^u &= pV^e + (1-p)V^u \\
W^e &= V^e \\
V^u &= U(\kappa z) + \beta W^u \\
V^e &= \gamma F(\bar{\chi})(U(w(1-\tau)) - E[\chi_{|\chi \leq \bar{\chi}}] + \beta W^e) + (1-\gamma F(\bar{\chi}))V^u.
\end{aligned}$$

Let us denote $F = F(\bar{\chi})$; $E = E[\chi|\chi \leq \bar{\chi}]$; and $u = U(w(1-\tau))$:

$$W^u = pV^e + (1-p)V^u$$

$$V^u = U(\kappa z) + \beta W^u$$

$$W^e = V^e$$

$$V^e = \gamma F(u - E + \beta W^e) + (1-\gamma F)V^u.$$

First, let's substitute out W^u and W^e:

$$\begin{aligned}
V^u &= U(\kappa z) + \beta\left(pV^e + (1-p)V^u\right) \\
V^u(1-\beta(1-p)) &= U(\kappa z) + \beta p V^e.
\end{aligned}$$

We obtain:

$$V^e = \gamma F(u - E + \beta V^e) + (1-\gamma F)V^u.$$

Hence, the expression for V^e is given by:

$$V^e = \frac{\gamma F(u-E)}{(1-\beta\gamma F)} + \frac{(1-\gamma F)V^u}{(1-\beta\gamma F)}. \tag{22}$$

Now substitute the V^e expression into V^u :

$$\begin{aligned}
V^u(1-\beta(1-p)) &= U(\kappa z)+\beta p\left(\frac{\gamma F(u-E)}{(1-\beta\gamma F)}+\frac{(1-\gamma F)V^u}{(1-\beta\gamma F)}\right)\\
V^u\left(1-\beta(1-p)-\beta p\frac{(1-\gamma F)}{(1-\beta\gamma F)}\right) &= U(\kappa z)+\beta p\frac{\gamma F(u-E)}{(1-\beta\gamma F)}\\
V^u\left(\frac{1-\beta}{1-\beta\gamma F}\left(1-\beta\gamma F(1-p)\right)\right) &= U(\kappa z)+\beta p\frac{\gamma F(u-E)}{(1-\beta\gamma F)}\\
V^u\left(1-\beta\right) &= \frac{(1-\beta\gamma F)}{(1-\beta\gamma F(1-p))}\log(\kappa z)+\frac{\beta p\gamma F}{(1-\beta\gamma F(1-p))}(u-E).
\end{aligned}$$

Now,

$$V^e(1-\beta\gamma F)=\gamma F(u-E)+(1-\gamma F)V^u.$$

We can obtain the following expression for W^u:

$$\begin{aligned}
W^u &= pV^e+(1-p)V^u\\
&= p\frac{\gamma F(u-E)+(1-\gamma F)V^u}{(1-\beta\gamma F)}+(1-p)V^u\\
&= \frac{p\gamma F}{(1-\beta\gamma F)}(u-E)+\frac{p(1-\gamma F)+(1-p)(1-\beta\gamma F)}{(1-\beta\gamma F)}V^u\\
&= \frac{p\gamma F}{(1-\beta\gamma F)}(u-E)+\frac{1-p\gamma F-(1-p)\beta\gamma F}{(1-\beta\gamma F)}V^u\\
&= \left[\frac{p\gamma F}{(1-\beta)\left(1-\beta\gamma F(1-p)\right)}\right](u-E)+\frac{1}{(1-\beta)}\left(\frac{1-p\gamma F-(1-p)\beta\gamma F}{(1-\beta\gamma F(1-p))}U(\kappa z)\right).
\end{aligned}$$

Normalizing this expression by multiplying $1-\beta$, we obtained the social welfare function W in the main paper (equation 15).

The optimal quitting condition

We now derive the optimal quitting condition (equation 17).

We start with the original expression for the optimal quitting condition:

$$u-\bar{\chi}+\beta V^e=V^u,$$

where V^e is the value of employment (same with V'). Plug in the expression for V^e (equation 22):

$$\begin{aligned}
u-\bar{\chi}+\beta\left(\frac{\gamma F(u-E)}{(1-\beta\gamma F)}+\frac{(1-\gamma F)}{(1-\beta\gamma F)}V^u\right) &= V^u\\
u-\bar{\chi}+\beta\frac{\gamma F}{(1-\beta\gamma F)}(u-E) &= \left(1-\frac{\beta(1-\gamma F)}{(1-\beta\gamma F)}\right)V^u\\
u-\bar{\chi}+\beta\frac{\gamma F}{(1-\beta\gamma F)}(u-E) &= \left(\frac{1-\beta}{1-\beta\gamma F}\right)V^u.
\end{aligned}$$

This gives an expression for V^u:

$$V^u(1-\beta)=\frac{(1-\beta\gamma F)}{(1-\beta\gamma F(1-p))}U(\kappa z)+\frac{\beta p\gamma F}{(1-\beta\gamma F(1-p))}(u-E).$$

Plug this expression back into the last expression:

$$\begin{aligned}
u-\bar{\chi}+\beta\frac{\gamma F}{(1-\beta\gamma F)}(u-E) &= \frac{1-\beta}{1-\beta\gamma F}\left(\frac{(1-\beta\gamma F)}{(1-\beta)\left(1-\beta\gamma F(1-p)\right)}U(\kappa z)+\frac{1}{1-\beta}\left(1-\frac{(1-\beta\gamma F)}{(1-\beta\gamma F(1-p))}\right)(u-E)\right) \\
u-\bar{\chi} &= \frac{1}{(1-\beta\gamma F(1-p))}U(\kappa z)+\left(1-\frac{1}{(1-\beta\gamma F(1-p))}\right)(u-E) \\
-\bar{\chi} &= \frac{1}{(1-\beta\gamma F(1-p))}U(\kappa z)-\frac{1}{(1-\beta\gamma F(1-p))}u-\left(1-\frac{1}{(1-\beta\gamma F(1-p))}\right)E \\
\bar{\chi} &= \frac{-1}{(1-\beta\gamma F(1-p))}U(\kappa z)+\frac{1}{(1-\beta\gamma F(1-p))}u+\left(1-\frac{1}{(1-\beta\gamma F(1-p))}\right)E.
\end{aligned}$$

Rearrange those terms:

$$\begin{aligned}
(1-\beta\gamma F(1-p))\left(\bar{\chi}-E\right) &= u-E-U(\kappa z) \\
(1-\beta\gamma F(1-p))\left(\bar{\chi}-E\right)+(E-\bar{\chi}) &= u-E-U(\kappa z)+(E-\bar{\chi}) \\
(1-\beta\gamma F(1-p))\left(\bar{\chi}-E\right)-(\bar{\chi}-E) &= u-\bar{\chi}-U(\kappa z) \\
-\beta\gamma F(1-p)\left(\bar{\chi}-E\right) &= u-\bar{\chi}-U(\kappa z).
\end{aligned}$$

Hence, we obtained the no-quitting condition as in equation 17.

Government budget constraint

We now derive the (present-value) government budget constraint (equation 19). We denote the number of unmatched worker in period t to be u_t.

The number of workers in the first period is

$$1-u_0=p\gamma F(\bar{\chi}).$$

In the next period it is

$$\begin{aligned}
1-u_1 &= (1-u_0)\gamma F(\bar{\chi})+u_0 p\gamma F(\bar{\chi}) \\
&= p\gamma F(\bar{\chi})+(1-u_0)\left(\gamma F(\bar{\chi})-p\gamma F(\bar{\chi})\right) \\
&= p\gamma F(\bar{\chi})+\left(\gamma F(\bar{\chi})\right)^2 p\left(1-p\right).
\end{aligned}$$

Generally, the number of workers evolves according to

$$1-u_{t+2}=a+b(1-u_{t+1}),$$

where

$$\begin{aligned}
a &= p\gamma F(\bar{\chi}) \\
b &= \gamma F(\bar{\chi})\left(1-p\right).
\end{aligned}$$

So the present value of workers is

$$
\begin{aligned}
& a \\
& +\beta\left(a+ab\right) \\
& +\beta^2\left(a+ab+ab^2\right) \\
& +\beta^3\left(a+ab+ab^2+ab^3\right) \\
& +...
\end{aligned}
$$

Rearrange terms:

$$
\begin{aligned}
& a\left(1+\beta(1+b)+\beta^2(1+b+b^2)+...\right) \\
= \; & a\left(1+\beta+\beta^2+...\right) \\
& +ab\beta(1+\beta+\beta^2+...)+... \\
= \; & \frac{a}{1-\beta}\left(1+\beta b+\beta^2 b^2\right) \\
= \; & \frac{a}{1-\beta}\frac{1}{1-\beta b} \\
= \; & \frac{p\gamma F(\bar{\chi})}{(1-\beta)(1-\beta\gamma F(\bar{\chi})\,(1-p))}.
\end{aligned}
$$

The present value of tax revenue is thus

$$
w\tau\frac{p\gamma F(\bar{\chi})}{(1-\beta)(1-\beta\gamma F(\bar{\chi})\,(1-p))}.
$$

The present value of the number of unemployed is

$$
\begin{aligned}
& 1-a \\
& +\beta\left(1-(a+ab)\right) \\
& +\beta^2\left(1-\left(a+ab+ab^2\right)\right) \\
& +\beta^3\left(1-\left(a+ab+ab^2+ab^3\right)\right) \\
& +...,
\end{aligned}
$$

which is

$$
\begin{aligned}
& \frac{1}{1-\beta}-\frac{p\gamma F(\bar{\chi})}{(1-\beta)(1-\beta\gamma F(\bar{\chi})\,(1-p))} \\
= \; & \frac{1}{1-\beta}\left(1-\frac{p\gamma F(\bar{\chi})}{(1-\beta\gamma F(\bar{\chi})\,(1-p))}\right) \\
= \; & \frac{1}{1-\beta}\left(\frac{(1-\beta\gamma F(\bar{\chi})\,(1-p))-p\gamma F(\bar{\chi})}{(1-\beta\gamma F(\bar{\chi})\,(1-p))}\right).
\end{aligned}
$$

We can define $\tilde{u}$ as the total fraction of time an initially unemployed worker spend in non-employment, which is exactly the $\tilde{u}$ shown up in the social welfare function:

$$\begin{aligned}
\tilde{u} &= 1-\frac{p\gamma F(\bar{\chi})}{(1-\beta\gamma F(\bar{\chi})(1-p))}=\frac{1-\beta\gamma F(\bar{\chi})(1-p)-p\gamma F(\bar{\chi})}{(1-\beta\gamma F(\bar{\chi})(1-p))}=\frac{1-\beta\gamma F(\bar{\chi})+(\beta-1)\,p\gamma F(\bar{\chi})}{(1-\beta\gamma F(\bar{\chi})(1-p))} \\
1-\tilde{u} &= \frac{p\gamma F(\bar{\chi})}{(1-\beta\gamma F(\bar{\chi})(1-p))}.
\end{aligned}$$

Hence, the government budget constraint is given by equation 19.

Baily-Chetty formula

We now derive equation 21. We start with the following social welfare function (equation 15) which is abbreviated as:

$$W\left(p,\bar{\chi},\tau,\kappa\right).$$

There is a no-quitting condition (equation 17):

$$U\left[w\left(p,\bar{\chi}\right)(1-\tau)\right]-E[\chi_{|\chi\leq\bar{\chi}}]-U(\kappa z)=\left(1-(1-p)\beta\gamma F(\bar{\chi})\right)\left(\bar{\chi}-E[\chi_{|\chi\leq\bar{\chi}}]\right).$$

One could denote this condition as

$$H\left(p,\bar{\chi},\tau,\kappa\right)=0.$$

Note that we have not substituted any specific functional form into this condition.

The equation H can be solved implicitly as

$$\bar{\chi}=\bar{\chi}\left(p,\tau,\kappa\right).$$

The private agent solves the following problem, taking as given τ and κ, and the function $H(.)$, or

$$\max_{p} W\left(p,\bar{\chi}\left(p,\tau,\kappa\right),\tau,\kappa\right).$$

This gives the private optimality condition

$$G\left(p,\bar{\chi},\tau,\kappa\right)=\frac{\partial W}{\partial p}+\frac{\partial W}{\partial\bar{\chi}}\frac{\partial\bar{\chi}}{\partial p}=0. \tag{23}$$

Lastly, we have a government budget condition (equation 19), which boils down to expressing τ as a function of all other parameters:

$$\tau=\tau\left(p,\bar{\chi},\kappa\right),$$

because both $\tilde{u}$ and wages are functions of p and $\bar{\chi}$. Now the social planner's problem is to maximize social welfare $W(.)$ subject to the following three constraints:

$$H\left(p,\bar{\chi},\tau,\kappa\right)=0$$

$$G\left(p,\bar{\chi},\tau,\kappa\right)=0$$

and

$$\tau=\tau\left(p,\bar{\chi},\kappa\right).$$

These three constraints give a mapping from κ to $(p, \bar{\chi}, \tau)$:

$$p(\kappa), \bar{\chi}(\kappa), \tau(\kappa).$$

Note that

$$\bar{\chi}(\kappa) = \bar{\chi}(p(\kappa), \tau(\kappa), \kappa)$$

$$\tau(\kappa) = \tau(p(\kappa), \bar{\chi}(\kappa), \kappa).$$

Plug these back into the social welfare function:

$$\max_{\kappa} W(p(\kappa), \bar{\chi}(p(\kappa), \tau(\kappa), \kappa), \tau(\kappa), \kappa).$$

Take the FOC with respect to κ :

$$\frac{\partial W}{\partial p}\frac{\partial p}{\partial \kappa} + \frac{\partial W}{\partial \bar{\chi}}\frac{\partial \bar{\chi}}{\partial p}\frac{\partial p}{\partial \kappa} + \frac{\partial W}{\partial \bar{\chi}}\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right) + \frac{\partial W}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial W}{\partial \kappa} = 0.$$

Because of the envelope theorem (see equation 23), the first two terms cancel out. Rearrange the remaining terms and plug in the total derivative derived from:

$$\tau(\kappa) = \tau(p(\kappa), \bar{\chi}(\kappa), \kappa),$$

which is

$$\frac{d\tau}{d\kappa} = \frac{\partial \tau}{\partial p}\frac{dp}{d\kappa} + \frac{\partial \tau}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial \kappa}.$$

We have:

$$\underbrace{\frac{\partial W}{\partial \kappa} + \frac{\partial W}{\partial \tau}\frac{\partial \tau}{\partial \kappa}}_{\text{Consumption insurance}} + \underbrace{\frac{\partial W}{\partial \tau}\left(\frac{\partial \tau}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial p}\frac{dp}{d\kappa}\right)}_{\text{Fiscal externality}} + \underbrace{\frac{\partial W}{\partial \bar{\chi}}\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right)}_{\text{Quitting externality}} = 0.$$

Now we want to plug in specific forms for social welfare and the government budget constraint.

Start with the social welfare function:

$$W(p, \bar{\chi}, \tau, \kappa) = \underbrace{\left(\frac{p\gamma F(\bar{\chi})}{(1-\beta\gamma(1-p)F(\bar{\chi}))}\right)}_{1-\tilde{u}}\left(U((1-\tau)w(p,\bar{\chi})) - E[\chi_{|\chi\leq\bar{\chi}}]\right) + \underbrace{\frac{1-\beta\gamma F(\bar{\chi}) - (1-\beta)\gamma p F(\bar{\chi})}{(1-\beta\gamma(1-p)F(\bar{\chi}))}}_{\tilde{u}}U(\kappa z).$$

And the government budget constraint is given by

$$\underbrace{\left(\frac{p\gamma F(\bar{\chi})}{(1-\beta\gamma(1-p)F(\bar{\chi}))}\right)}_{1-\tilde{u}}\tau w(p, \bar{\chi}) = \underbrace{\frac{1-\beta\gamma F(\bar{\chi}) - (1-\beta)\gamma p F(\bar{\chi})}{(1-\beta\gamma(1-p)F(\bar{\chi}))}}_{\tilde{u}}\kappa z$$

$$\tau(p, \bar{\chi}, \kappa) = \frac{\tilde{u}}{1-\tilde{u}}\frac{\kappa z}{w(p, \bar{\chi})}.$$

We need to fill in those terms, and we do it one at a time:

$$\underbrace{\frac{\partial W}{\partial \kappa} + \frac{\partial W}{\partial \tau}\frac{\partial \tau}{\partial \kappa}}_{\text{Consumption insurance}} + \underbrace{\frac{\partial W}{\partial \tau}\left(\frac{\partial \tau}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial p}\frac{dp}{d\kappa}\right)}_{\text{Fiscal externality}} + \underbrace{\frac{\partial W}{\partial \bar{\chi}}\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right)}_{\text{Quitting externality}} = 0.$$

The consumption insurance term

Taking the derivative with respect to the social welfare function and the tax function:

$$\frac{\partial W}{\partial \kappa} = \tilde{u} U'\left(\kappa z\right) z$$

$$\frac{\partial W}{\partial \tau} = -\left(1-\tilde{u}\right) U'\left(\left(1-\tau\right) w\right) w$$

$$\frac{\partial \tau}{\partial \kappa} = \frac{\tilde{u}}{1-\tilde{u}} \frac{z}{w}.$$

Combine:

$$\begin{aligned}
\frac{\partial W}{\partial \kappa} + \frac{\partial W}{\partial \tau}\frac{\partial \tau}{\partial \kappa} &= \tilde{u} U'\left(\kappa z\right) z - \left(1-\tilde{u}\right) U'\left(\left(1-\tau\right) w\right) w \frac{\tilde{u}}{1-\tilde{u}} \frac{z}{w} \\
&= \tilde{u} U'\left(\kappa z\right) z - U'\left(\left(1-\tau\right) w\right) \tilde{u} z \\
&= \tilde{u} z \left(U'\left(c^{u}\right) - U'\left(c^{w}\right)\right).
\end{aligned}$$

The fiscal externality Term

The fiscal term is given by

$$\frac{\partial \tau}{\partial \bar{\chi}} \frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial p} \frac{dp}{d\kappa}.$$

Now, given that

$$\tau\left(p, \bar{\chi}, \kappa\right) = \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)},$$

We take the partial derivatives with respect to p and $\bar{\chi}$:

$$\begin{aligned}
\frac{\partial \tau}{\partial \bar{\chi}} &= \frac{\frac{\partial \tilde{u}}{\partial \bar{\chi}}\left(1-\tilde{u}\right) - \tilde{u}\left(-\frac{\partial \tilde{u}}{\partial \bar{\chi}}\right)}{\left(1-\tilde{u}\right)^{2}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)} - \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w^{2}} \frac{\partial w}{\partial \bar{\chi}} \\
&= \frac{1}{\left(1-\tilde{u}\right)^{2}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)} \frac{\partial \tilde{u}}{\partial \bar{\chi}} - \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w^{2}} \frac{\partial w}{\partial \bar{\chi}} \\
\frac{\partial \tau}{\partial p} &= \frac{\frac{\partial \tilde{u}}{\partial p}\left(1-\tilde{u}\right) - \tilde{u}\left(-\frac{\partial \tilde{u}}{\partial p}\right)}{\left(1-\tilde{u}\right)^{2}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)} - \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w^{2}} \frac{\partial w}{\partial p} \\
&= \frac{1}{\left(1-\tilde{u}\right)^{2}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)} \frac{\partial \tilde{u}}{\partial p} - \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w^{2}} \frac{\partial w}{\partial p}.
\end{aligned}$$

Hence

$$\begin{aligned}
& \frac{\partial \tau}{\partial \bar{\chi}} \frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial p} \frac{dp}{d\kappa} \\
= & \left[\frac{1}{\left(1-\tilde{u}\right)^{2}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)} \frac{\partial \tilde{u}}{\partial \bar{\chi}} - \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w^{2}} \frac{\partial w}{\partial \bar{\chi}}\right] \frac{d\bar{\chi}}{d\kappa} \\
& + \left[\frac{1}{\left(1-\tilde{u}\right)^{2}} \frac{\kappa z}{w\left(p, \bar{\chi}\right)} \frac{\partial \tilde{u}}{\partial p} - \frac{\tilde{u}}{1-\tilde{u}} \frac{\kappa z}{w^{2}} \frac{\partial w}{\partial p}\right] \frac{dp}{d\kappa}.
\end{aligned}$$

Regroup items, recognizing that

$$\frac{d\tilde{u}}{d\kappa} = \frac{\partial \tilde{u}}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tilde{u}}{\partial p}\frac{dp}{d\kappa}$$

$$\frac{dw}{d\kappa} = \frac{\partial w}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial w}{\partial p}\frac{dp}{d\kappa},$$

we have

$$\begin{aligned}
&\frac{\partial \tau}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial p}\frac{dp}{d\kappa} \\
= \quad & \frac{1}{(1-\tilde{u})^2}\frac{\kappa z}{w}\frac{d\tilde{u}}{d\kappa} - \frac{\tilde{u}}{1-\tilde{u}}\frac{\kappa z}{w^2}\frac{dw}{d\kappa}.
\end{aligned}$$

Hence, the whole tax benefit term is

$$\begin{aligned}
&\frac{\partial W}{\partial \tau}\left(\frac{\partial \tau}{\partial \bar{\chi}}\frac{d\bar{\chi}}{d\kappa} + \frac{\partial \tau}{\partial p}\frac{dp}{d\kappa}\right) \\
= \quad & -(1-\tilde{u})\,U'\left((1-\tau)\,w\right)w\left[\frac{1}{(1-\tilde{u})^2}\frac{\kappa z}{w}\frac{d\tilde{u}}{d\kappa} - \frac{\tilde{u}}{1-\tilde{u}}\frac{\kappa z}{w^2}\frac{dw}{d\kappa}\right] \\
= \quad & -U'\left((1-\tau)\,w\right)z\tilde{u}\left[\frac{1}{1-\tilde{u}}\frac{1}{\tilde{u}}\kappa\frac{d\tilde{u}}{d\kappa} - \frac{\kappa}{w}\frac{dw}{d\kappa}\right] \\
= \quad & -U'\left(c^w\right)z\tilde{u}\left[\frac{1}{1-\tilde{u}}\varepsilon_{\tilde{u},\kappa} - \varepsilon_{w,\kappa}\right].
\end{aligned}$$

The quitting externality term:

The quitting effect is given by:

$$\frac{\partial W}{\partial \bar{\chi}}\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right) = \left(-\frac{\partial \tilde{u}}{\partial \bar{\chi}}\left(U\left(c^w\right) - E[\chi_{|\chi\le\bar{\chi}}] - U(\kappa z)\right) + (1-\tilde{u})\left(U'\left(c^w\right)(1-\tau)\frac{\partial w}{\partial \bar{\chi}} - \frac{\partial E[\chi_{|\chi\le\bar{\chi}}]}{\partial \bar{\chi}}\right)\right)\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right).$$

We will argue that a component of this equation is zero:

$$-\frac{\partial \tilde{u}}{\partial \bar{\chi}}\left(U\left(c^w\right) - E[\chi_{|\chi\le\bar{\chi}}] - U(\kappa z)\right) - (1-\tilde{u})\frac{\partial E[\chi_{|\chi\le\bar{\chi}}]}{\partial \bar{\chi}} = 0, \tag{24}$$

which is implied from the optimal quitting condition:

$$U\left(c^w\right) - E[\chi_{|\chi\le\bar{\chi}}] - U(\kappa z) = (1-(1-p)\beta\gamma F(\bar{\chi}))\left(\bar{\chi} - E[\chi_{|\chi\le\bar{\chi}}]\right).$$

Plugging this into the last equation 24, we have:

$$-\frac{\partial \tilde{u}}{\partial \bar{\chi}}\left[(1-(1-p)\beta\gamma F(\bar{\chi}))\left(\bar{\chi} - E[\chi_{|\chi\le\bar{\chi}}]\right)\right] - (1-\tilde{u})\frac{\partial E[\chi_{|\chi\le\bar{\chi}}]}{\partial \bar{\chi}}.$$

Now, plug in:

$$\begin{aligned}
\frac{\partial E[\chi_{|\chi\le\bar{\chi}}]}{\partial \bar{\chi}} \quad &= \quad \frac{F(\bar{\chi})\bar{\chi}f(\bar{\chi}) - \int_{-\infty}^{\bar{\chi}}\chi dF(\chi)f(\bar{\chi})}{F(\bar{\chi})^2} \\
&= \quad \frac{f\left(\bar{\chi}\right)}{F\left(\bar{\chi}\right)}\left[\frac{F(\bar{\chi})\bar{\chi}}{F(\bar{\chi})} - \frac{\int_{-\infty}^{\bar{\chi}}\chi dF(\chi)}{F\left(\bar{\chi}\right)}\right] \\
&= \quad \frac{f\left(\bar{\chi}\right)}{F\left(\bar{\chi}\right)}\left(\bar{\chi} - E[\chi_{|\chi\le\bar{\chi}}]\right).
\end{aligned}$$

Now,

$$
\begin{aligned}
-\frac{\partial \tilde{u}}{\partial \bar{\chi}} &= \frac{\partial\left(1-\tilde{u}\right)}{\partial \bar{\chi}} = \frac{\partial \frac{p\gamma F(\bar{\chi})}{(1-\beta\gamma(1-p)F(\bar{\chi}))}}{\partial \bar{\chi}} \\
&= \frac{p\gamma f(\bar{\chi})\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right) - p\gamma F(\bar{\chi})\left(-\beta\gamma\left(1-p\right)f\left(\bar{\chi}\right)\right)}{\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right)^2} \\
&= \frac{p\gamma f(\bar{\chi})}{\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right)^2}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& -\frac{\partial \tilde{u}}{\partial \bar{\chi}}\left[\left(1-(1-p)\beta\gamma F(\bar{\chi})\right)\left(\bar{\chi}-E[\chi_{|\chi\leq\bar{\chi}}]\right)\right] - (1-\tilde{u})\frac{\partial E[\chi_{|\chi\leq\bar{\chi}}]}{\partial \bar{\chi}} \\
= \;& \frac{p\gamma f(\bar{\chi})}{\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right)^2}\left[\left(1-(1-p)\beta\gamma F(\bar{\chi})\right)\left(\bar{\chi}-E[\chi_{|\chi\leq\bar{\chi}}]\right)\right] - \frac{p\gamma F(\bar{\chi})}{\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right)}\frac{f\left(\bar{\chi}\right)}{F\left(\bar{\chi}\right)}\left(\bar{\chi}-E[\chi_{|\chi\leq\bar{\chi}}]\right) \\
= \;& \frac{p\gamma f(\bar{\chi})}{\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right)}\left(\bar{\chi}-E[\chi_{|\chi\leq\bar{\chi}}]\right) - \frac{p\gamma F\left(\bar{\chi}\right)}{\left(1-\beta\gamma\left(1-p\right)F\left(\bar{\chi}\right)\right)}\frac{f\left(\bar{\chi}\right)}{F\left(\bar{\chi}\right)}\left(\bar{\chi}-E[\chi_{|\chi\leq\bar{\chi}}]\right) = 0.
\end{aligned}
$$

Hence we can use this equation to simply the quitting effect term. We have the following equation:

$$
\frac{\partial W}{\partial \bar{\chi}}\left(\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right) = (1-\tilde{u})U'\left(c^w\right)\left(1-\tau\right)\frac{\partial w}{\partial \bar{\chi}}\left[\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right].
$$

Hence we have:

$$
\underbrace{\tilde{u}z\left(U'\left(c^u\right) - U'\left(c^w\right)\right)}_{\text{C insurance (conventional)}} + \underbrace{-U'\left(c^w\right)z\tilde{u}\left[\frac{1}{1-\tilde{u}}\varepsilon_{\tilde{u},\kappa} - \varepsilon_{w,\kappa}\right]}_{\text{Fiscal externality (conventional)}} + \underbrace{(1-\tilde{u})U'\left(c^w\right)\left(1-\tau\right)\frac{\partial w}{\partial \bar{\chi}}\left[\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right]}_{\text{Quitting externality (new)}} = 0.
$$

Dividing everything with $z\tilde{u}U'\left(c^w\right)$, we have

$$
\underbrace{\frac{U'\left(c^u\right) - U'\left(c^w\right)}{U'\left(c^w\right)}}_{\text{C insurance}} + \underbrace{-\left[\frac{1}{1-\tilde{u}}\varepsilon_{\tilde{u},\kappa} - \varepsilon_{w,\kappa}\right]}_{\text{Fiscal externality}} + \underbrace{\frac{(1-\tilde{u})U'\left(c^w\right)\left(1-\tau\right)}{z\tilde{u}U'\left(c^w\right)}\frac{\partial w}{\partial \bar{\chi}}\left[\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right]}_{\text{Quitting externality}} = 0.
$$

Note that, given the GBC:

$$
\begin{aligned}
\tau &= \frac{\tilde{u}}{1-\tilde{u}}\frac{\kappa z}{w} \\
\frac{(1-\tilde{u})U'\left(c^w\right)\left(1-\tau\right)}{z\tilde{u}U'\left(c^w\right)} &= \frac{\left(1-\tilde{u}\right)\left(1-\tau\right)}{z\tilde{u}} = \frac{1-\tau}{\tau}\frac{\kappa}{w}.
\end{aligned}
$$

Hence we have:

$$
\underbrace{\frac{U'\left(c^u\right) - U'\left(c^w\right)}{U'\left(c^w\right)}}_{\text{Cons insurance}} + \underbrace{-\left[\frac{1}{1-\tilde{u}}\varepsilon_{\tilde{u},\kappa} - \varepsilon_{w,\kappa}\right]}_{\text{Fiscal externality}} + \underbrace{\frac{1-\tau}{\tau}\frac{\kappa}{w}\frac{\partial w}{\partial \bar{\chi}}\left[\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right]}_{\text{Quitting externality}} = 0
$$

$$
\underbrace{\frac{U'\left(c^u\right) - U'\left(c^w\right)}{U'\left(c^w\right)}}_{\text{Cons insurance}} + \underbrace{-\left[\frac{1}{1-\tilde{u}}\varepsilon_{\tilde{u},\kappa} - \varepsilon_{w,\kappa}\right]}_{\text{Fiscal externality}} + \underbrace{\frac{1-\tau}{\tau}\varepsilon_{w,\kappa|p}}_{\text{Quitting externality}} = 0,
$$

where $\varepsilon_{w,\kappa|p} = \frac{\kappa}{w}\frac{\partial w}{\partial \bar{\chi}}\left[\frac{\partial \bar{\chi}}{\partial \tau}\frac{d\tau}{d\kappa} + \frac{\partial \bar{\chi}}{\partial \kappa}\right]$.

B Definition of Equilibrium

A stationary equilibrium is a set of values for unemployed workers, $\{V_n^u\}$ (one for each sector n), search choices for unemployed workers $\left\{\theta_n^u, V_{0,n}^u\right\}$, decision rules
$\{V_H(V), V_L(V), w_n(V,z), \bar{\chi}_n(V,z), V_n'(V,z), V_n^{s\prime}(V,z), \theta_n'(V,z), \zeta_n'(V,z)\}$ for all (V,z) and for all n, profit value functions $\Pi_n(V,z)$ for all (V,z,n) and $E\left[\Pi_n\left(V^s\right)\right]$ for all (V^s,n) and offer matching probability functions $\bar{\zeta}_n\left(V^s, V, z\right)$ for all (V^s, V, Z, n) s.t.

1. given V_n^u and $(V_n'(V,z), V_n^{s\prime}(V,z), \theta_n'(V,z))$, the decision rule for quitting thresholds $\bar{\chi}_n(V,z)$ satisfies the worker optimal quitting condition (9) for all (V,z,n);

2. the decision rule for offer matching $\zeta_n'(V,z)$ satisfies eq. (11) with equality when $\Pi_n(V_n^{s\prime}(V,z),z) \geq 0$ and is zero otherwise (firms retain profitable matches as often as possible while preserving truth telling);

3. the job matching probabilities that posting firms take as given are consistent with optimal offer matching: $\bar{\zeta}_n\left(V^{s\prime}, V', z\right) = \zeta_n'(V,z)$ when $V^{s\prime} = V_n^{s\prime}(V,z)$ and $V' = V_n'(V,z)$;

4. job search choices $(V_n^{s\prime}(V,z), \theta_n'(V,z))$ are optimal for workers at the searching stage in the following period, i.e. they maximize $p(\theta_n')V_n^{s\prime} + (1 - p(\theta_n')V'$ subject to $J^m(\theta_n', V_n^{s\prime}, V', z) = q\left(\theta_n'\right)\left(1 - \bar{\zeta}\left(V_n^{s\prime}, V', z\right)\right)E\left[\Pi\left(V_n^{s\prime}\right)\right] - \phi = 0$, taking as given offer matching probabilities $\bar{\zeta}_n\left(V_n^{s\prime}, V', z\right)$ and expected profits $E\left[\Pi_n\left(V_n^{s\prime}\right)\right]$, when $V' = V_n'(V,z)$.

5. Given V_n^u and the functions $\bar{\chi}_n(V,z)$, $\bar{\zeta}_n\left(V^{s\prime}, V', z\right)$, and $\left(V_n^{s\prime}(V,z), \theta_n'(V,z)\right)$, the decision rules $(w_n(V,z), \bar{\chi}_n(V,z), V_n'(V,z))$ satisfy the firm's profit maximization problem for all (V,z) and corresponding profits are given by $\Pi_n(V,z)$;

6. given $\Pi_n(V,z_H)$ and $\Pi_n(V,z_L)$ the decision rules $V_H(V)$ and $V_L(V)$ solve problem (7) and $E\left[\Pi_n\left(V^s\right)\right]$ is the associated expected profit value for all V^s;

7. $\left(\theta_n^*, V_{0,n}^*\right)$ maximizes welfare for unmatched workers within set of values (θ_n, V_n^s) that satisfy $q\left(\theta_n\right)E\left[\Pi_n\left(V_n^s\right)\right] - \phi_n = 0$;

8. unmatched values satisfy $V_n^u = U(b(n)) + \beta\left(p(\theta_n^*)V_{0,n}^* + (1 - p(\theta_n^*))V_n^u\right)$;

9. revenue from taxes at rate τ finances benefits to unmatched workers;

10. in each sector, the measure of unmatched workers and the joint distribution of workers over states (V,z) is constant over time.

C Sensitivity

C.1 No Exogenous Separations

We now report results for an experiment in which there are no exogenous separations ($\gamma = 1$). Again, we recalibrate so that the model replicates the baseline unemployment, job openings, and quit rates. All else equal, eliminating exogenous separations would lower unemployment. To replicate the same target unemployment rate requires a lower job finding probability, which in turn dictates a lower value for match efficiency, A.

Table 5: Welfare Decomposition, Economy with No Exogenous Separations

Panel A: Parameter values

	A	ϕ	μ_χ	σ^2_χ	γ		
Baseline	0.563	0.103	-1.22	0.25	0.9806		
$\gamma = 1$	0.296	0.049	-0.944	0.25	1		

Panel B: Terms in Baily-Chetty formula and elasticities

	κ	c ineq.	fiscal extn.	quit extn.	$\varepsilon_{\tilde{u},\kappa}$	$\varepsilon_{w,\kappa}$	$\varepsilon_{w,\kappa\|p}$
Baseline	0.500	0.918	-4.545	-1.364	4.271	0.002	-0.046
Optimum	0.328	1.996	-1.012	-0.984	0.980	-0.008	-0.008
$\gamma = 1$	0.500	0.926	-13.274	-2.138	12.431	-0.027	-0.073
Optimum	0.078	11.805	-1.440	-10.364	1.440	-0.00015	-0.00015

Panel C: Model moments

	κ	$\tilde{u}$ (%)	w	τ (%)	p	Quit rate (%)	$\Delta w / \Delta \kappa$
Baseline	0.500	6.06	0.9913	3.26	0.628	1.88	0.004
Optimum	0.328	2.36	0.9911	0.80	1.000	0.43	-0.024
$\gamma = 1$	0.500	6.16	0.9958	3.30	0.328	1.88	-0.054
Optimum	0.078	0.02	0.9980	0.000014	1.000	0.01	-0.002

In this parameterization, the optimal UI replacement rate parameter is $\kappa = 0.078$. Reducing UI to this low value almost eliminates quits, but workers who do quit suffer huge consumption declines. Still, from the standpoint of an unmatched worker, the benefits of a low replacement rate in terms of higher wages and lower tax rates exactly balance the costs of painful but infrequent non-employment spells. The huge difference between the optimal replacement rates in the high variance case (in which all separations are essentially exogenous) and the $\gamma = 1$ case (in which all separations are endogenous) starkly illustrates the importance of understanding what drives separations.

C.2 Alternative Insurance Models

Table 6 reports parameter values and optimal replacement rates for the two economies discussed in Section 6.2.

Table 6: Sensitivity Analysis

	Parameters						Optimal UI rates	
	A	ϕ	μ_χ	σ^2_χ	κ	τ (%)	κ^*	
Baseline model								
	0.563	0.103	-1.22	0.25	0.50	3.26	0.328	
Less UI for quitters: $\kappa_{EN} < \kappa_{EU}$								
	0.590	0.095	-0.67	0.25	κ_{EU} 0.50; κ_{EN} 0.25	2.65	0.217	κ^*_{EU} 0.379; κ^*_{EN} 0.051
Private UI insurance: $\kappa_P > 0$								
	0.565	0.103	-1.21	0.25	κ_P 0.25; κ 0.25	1.56	0.105	

D Additional Figures

Figure 6 plots JOLTS quit rates by industry against average weekly earnings by industry.

Figure 7 plots the sample path for a worker who starts out unemployed (red dots). They find a job at month two, and the match turns out to be low quality (represented by blue dots). At month four they quit the job because of a high χ preference shock. After several months of unemployment, they find another job, which this time turns out to be a high quality match (green dots). This lasts for quite a while, and wages rise with tenure. Then, the match ends exogenously. The next job they find is low quality, but through on-the-job search, they eventually transition to a high-quality match.

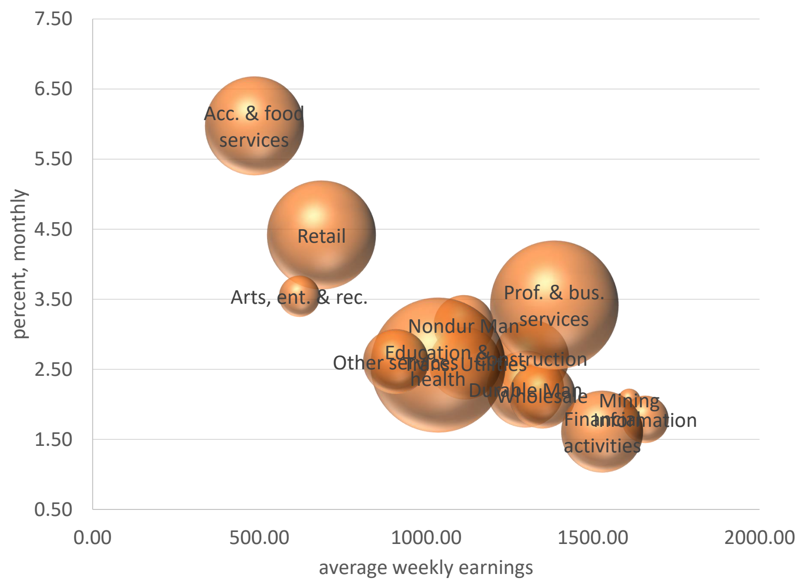

Figure 6: JOLTS Quit Rates by Industry, 2021-22. The bubble sizes reflect industry employment shares.

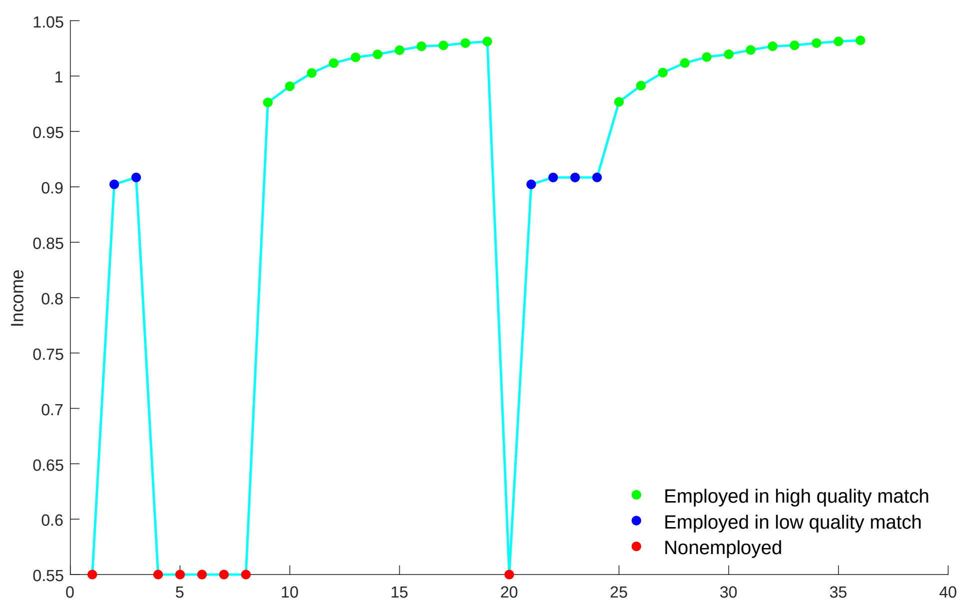

Figure 7: Sample Path for Disposable Income (worker in sector with $Y_n = 1$)