Technical Appendix for "Consumption and Labor Supply with Partial Insurance: An Analytical Framework"

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January 2012

This Technical Appendix is organized as follows. Section A contains an extended proof of Proposition 1 (existence of no-trade competitive equilibrium and characterization of equilibrium allocations, asset prices and asset purchases). Section B develops in detail the two household models discussed in Section 3.1 of the paper which provide a foundation for equivalizing the data. Section C contains identification proofs for Proposition 3 (when consumption data are not available) and an extension for the case where data are biannual. Section D states the additional identifying assumption about end-points we make in the estimation of the model.

A Extended Proof of Proposition 1

The proof is in two parts. In the first part we describe a planner's problem, and show that the allocations for consumption and hours described in Proposition 1, part (ii) are the solution to this problem. In the second part, we decentralize these allocations in a competitive equilibrium, and show that the asset prices described in Proposition 1, part (iii) and the no-inter-island-trade result described in part (i) form part of this decentralization.

Planner's Problem (allocations): We first solve for equilibrium allocations for consumption and hours worked by solving a set of static planning problems. Each island-level planner maximizes equally weighted period utility for a set of agents that share a common age a, a common preference weight φ, and a common wage component α_t. Let $x_t = (a, \varphi, \alpha_t)$ denote these island-level components of the individual state. Each island-level planner controls a set of agents with the age-specific population distributions $F^a_{\kappa,t}$ and $F_{\theta,t}$. Let $F^a_{\varepsilon,t}$ denote the implied age-specific distribution over $\varepsilon_t = \kappa_t + \theta_t$. The planner's problem on an island defined by x_t is to choose functions $c_t(x_t, \varepsilon_t)$, $h_t(x_t, \varepsilon_t)$ to solve

$$\max_{\{c_t(x_t,\cdot),h_t(x_t,\cdot)\}} \int \left[\frac{c_t(x_t,\varepsilon_t)^{1-\gamma}-1}{1-\gamma} - \exp(\varphi) \frac{h_t(x_t,\varepsilon_t)^{1+\sigma}}{1+\sigma} \right] dF^a_{\varepsilon,t}$$

subject to the island-level resource constraint

$$\int \left[\lambda \left(\exp(\alpha_t + \varepsilon_t) \, h_t(x_t,\varepsilon_t) \right)^{1-\tau} - c_t(x_t,\varepsilon_t) \right] dF^a_{\varepsilon,t} = 0.$$

The first-order conditions with respect to $c_t(x_t, \varepsilon_t)$ and $h_t(x_t, \varepsilon_t)$ are, respectively,

$$\begin{aligned} c_t(x_t,\varepsilon_t)^{-\gamma} &= \chi_t(x_t), \\ \exp(\varphi) \, h_t(x_t,\varepsilon_t)^{\sigma} &= \chi_t(x_t) \lambda \exp(\alpha_t(1-\tau)) \exp(\varepsilon_t(1-\tau)) (1-\tau) h_t(x_t,\varepsilon_t)^{-\tau}, \end{aligned}$$

where $\chi_t(x_t)$ is the multiplier on the date t resource constraint. Note that $c_t(x_t, \varepsilon_t) = \chi_t(x_t)^{-\frac{1}{\gamma}}$, and thus does consumption does not depend on ε_t. Combining the two FOCs gives

$$h_t(x_t,\varepsilon_t) = ((1-\tau)\,\lambda)^{\frac{1}{\sigma+\tau}} c_t(x_t)^{-\frac{\gamma}{\sigma+\tau}} \exp\left(\frac{1-\tau}{\sigma+\tau} (\alpha_t+\varepsilon_t) - \frac{1}{\sigma+\tau} - \frac{\varphi}{\sigma+\tau} \right). \tag{A1}$$

Substituting (A1) into the resource constraint gives

$$\begin{aligned} c_t(x_t,\varepsilon_t) &= \lambda((1-\tau)\,\lambda)^{\frac{1-\tau}{\sigma+\tau}} \exp(\alpha_t(1-\tau)) \, c_t(x_t,\varepsilon_t)^{-\frac{\gamma(1-\tau)}{\sigma+\tau}} \exp\left(-\frac{1-\tau}{\sigma+\tau}\right) \\ &\times \exp\left(-\frac{\varphi(1-\tau)}{\sigma+\tau} + \alpha_t \frac{(1-\tau)^2}{\sigma+\tau} \right) \int \exp((1-\tau)\varepsilon_t) \exp\left(\frac{(1-\tau)^2}{\sigma+\tau} \varepsilon_t \right) dF^a_{\varepsilon t}. \end{aligned}$$

Taking logs and simplifying yields

$$\begin{aligned} &\log c_t(x_t,\varepsilon_t) \\ &= \frac{1+\sigma}{\sigma+\tau+\gamma(1-\tau)} \log\lambda + \frac{1-\tau}{\sigma+\tau+\gamma(1-\tau)} \log(1-\tau) - \frac{1-\tau}{\sigma+\tau+\gamma(1-\tau)} \varphi \\ &+ \frac{(1-\tau)(1+\sigma)}{\sigma+\tau+\gamma(1-\tau)} \alpha_t + \frac{\sigma+\tau}{\sigma+\tau+\gamma(1-\tau)} \log \int \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau} \varepsilon_t \right) dF^a_{\varepsilon t}. \end{aligned}$$

By using the definition for the tax-modified Frisch elasticity $\widehat{\sigma} = (\sigma + \tau)/(1 - \tau)$, the above expression simplifies to:

$$\log c_t(x_t, \varepsilon_t) = -\frac{\varphi}{\widehat{\sigma} + \gamma} + \frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma} + \gamma}\alpha_t + \mathcal{C}_t^a \tag{A2}$$

which is the expression in Proposition 1, part (ii), where $\mathcal{C}_t^a$ is a constant common to all agents of age a in year t given by

$$\begin{aligned}
\mathcal{C}_t^a &= \frac{1}{\widehat{\sigma} + \gamma}\left((1+\widehat{\sigma})\log\lambda + \log(1-\tau)\right) + \mathcal{M}_t^a, \\
\mathcal{M}_t^a &= \frac{\widehat{\sigma}}{\widehat{\sigma} + \gamma}\log\int \exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\varepsilon_t\right) dF_{\varepsilon,t}^a.
\end{aligned}$$

Note that if we were to assume, for example, that $\log \varepsilon_t^a \sim N\left(-\frac{v_{\varepsilon t}^a}{2}, v_{\varepsilon t}^a\right)$, then we could solve out the integral in the expression for $\mathcal{M}_t^a$:

$$\mathcal{M}_t^a = \frac{1}{\widehat{\sigma} + \gamma}\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\left(1 - \tau\left(1+\widehat{\sigma}\right)\right)\frac{v_{\varepsilon t}}{2}\right).$$

We now substitute the expression for $\log c_t(x_t, \varepsilon_t)$ in (A2) into (A1) to solve for $\log h_t(x_t, \varepsilon_t)$:

$$\log h_t(x_t, \varepsilon_t) = -\frac{1}{(1-\tau)(\widehat{\sigma} + \gamma)}\varphi + \left(\frac{1-\gamma}{\widehat{\sigma} + \gamma}\right)\alpha_t + \frac{1}{\widehat{\sigma}}\varepsilon_t + \mathcal{H}_t^a$$

which is the expression in Proposition 1, part (ii), where

$$\mathcal{H}_t^a \equiv \frac{1}{(1-\tau)(\widehat{\sigma} + \gamma)}\left((1-\gamma)\log\lambda + \log(1-\tau)\right) - \frac{\gamma}{\widehat{\sigma}(1-\tau)}\mathcal{M}_t^a.$$

Decentralization (prices): We now turn to the second part of the proof of Proposition 1, namely the decentralization of the solution to the above planner's problem. We begin by conjecturing prices in this equilibrium. We set pre-tax wages equal to individual labor productivity:

$$w_t(x_t, \varepsilon_t) = \exp(\alpha_t + \varepsilon_t).$$

At this wage, the intratemporal FOC from the agent's problem (2.1) described in the main text is identical to the intratemporal FOC for the planner described in eq.($A1$). Thus at competitive wages and the conjectured allocations (eqs. 7 and 8) agents are optimizing on the intratemporal margin. At first blush this might seem surprising, given the presence of progressive earnings taxation in the economy. Recall, however, that individual agents (in the competitive equilibrium) and island-level planners (in the problem described above) are atomistic and hence both take the tax system parameters as exogenous.

We next conjecture equilibrium prices for intertemporal insurance claims. At this point it is convenient to revert to history-dependent notation, so we will write $c_t(s^t)$ rather than $c_t(x_t, \varepsilon_t)$. We begin with the price of within-island insurance $Q_t\left(S; s^t\right)$. The intertemporal FOC from the agent's problem (2.1) defines the price at which an agent of age a with history s^t is willing, on the margin, to buy or sell a set of insurance contracts $B_t(S; s^t)$ that pay δ^{-1} units of consumption if and only if $s_{t+1} = (\omega_{t+1}, \eta_{t+1}, \theta_{t+1}) \in S \subseteq \mathbb{S}$. This price is simply the average marginal rate of substitution in those states:[2]

$$Q_t\left(S; s^t\right) = \beta\delta\delta^{-1} \int_S \frac{c_{t+1}(s^t, s_{t+1})^{-\gamma}}{c_t(s^t)^{-\gamma}} dF_{s,t+1}. \tag{A3}$$

Substituting in the expression for consumption (A2) we have

$$Q_t\left(S; s^t\right) = \beta \exp\left(-\gamma\left(\mathcal{C}_{t+1}^{a+1} - \mathcal{C}_t^a\right)\right) \int_S \exp\left(-\gamma(1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\omega_{t+1}\right) dF_{s,t+1}, \tag{A4}$$

which is the expression in Proposition 1, part (iii), where $\mathcal{C}_t^a$ is defined above, and

$$\begin{aligned}
\mathcal{C}_{t+1}^{a+1} - \mathcal{C}_t^a &= \frac{\widehat{\sigma}}{\widehat{\sigma}+\gamma}\left[\log\int \exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\varepsilon_{t+1}\right) dF_{\varepsilon,t+1}^{a+1} - \log\int \exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\varepsilon_t\right) dF_{\varepsilon,t}^{a}\right] \\
&= \frac{\widehat{\sigma}}{\widehat{\sigma}+\gamma}\log\left(\frac{\int \exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\eta_{t+1}\right) dF_{\eta,t+1} \int \exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\theta_{t+1}\right) dF_{\theta,t+1}}{\int \exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\theta_t\right) dF_{\theta,t}}\right)
\end{aligned}$$

is independent of a. Thus the prices $Q_t(S; s^t)$ are consistent with optimization on the consumer side.

Note that $Q_t\left(S; s^t\right) = Q_t(S)$: insurance prices are independent of the individual history s^t and age a. From eq. (A4) there are two pieces to this result. First, $F_{s,t+1}$, the joint distribution over $s_{t+1} = (\omega_{t+1}, \eta_{t+1}, \theta_{t+1})$ at $t+1$, is independent of s^t and thus the second term in eq. (A4) is independent of s^t. Second, insurance prices are also independent of age a, because while average consumption $\mathcal{C}_t^a$ is age-dependent, *growth* in average consumption $\mathcal{C}_{t+1}^{a+1} - \mathcal{C}_t^a$ is independent of age, reflecting the permanent-transitory model for individual productivity dynamics. Note also that the price of insurance against η_{t+1} and θ_{t+1} simply reflects probabilities, while the price of insurance against ω_{t+1} also reflects the conditional marginal rate of substitution, with insurance against low ω_{t+1} realizations being more expensive than equally likely high ω_{t+1} realizations. This asymmetry reflects the fact that η_{t+1}

[2] Note that the agent effectively discounts at rate $\beta\delta$, while mortality insurance ensures payment of δ^{-1} units of consumption in the event that the agent survives to the next period and $s_{t+1} \in S$.

and θ_{t+1} are perfectly insured in equilibrium, while ω_{t+1} remains uninsured. The price of a risk-free bond $Q_t(\mathbb{S})$ is

$$Q_t\left(\mathbb{S};s^t\right)=\beta\exp\left(-\gamma\left(\mathcal{C}_{t+1}^{a+1}-\mathcal{C}_t^a\right)\right)\int_{\mathbb{S}}\exp\left(-\gamma(1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\omega_{t+1}\right)dF_{s,t+1}=Q_t\left(\mathbb{S}\right).$$

We now turn to the price function for insurance claims traded across islands. Because any contract that can be traded between islands can also be traded within an island, the inter-island price for a claim that pays δ^{-1} units of consumption iff $s_{t+1}\in Z$ must, by arbitrage, equal the corresponding within-island price, for any Z. This implies

$$Q_t^*\left(Z;s^t\right)=\Pr\left((\eta_{t+1},\theta_{t+1})\in Z\right)\times Q_t(\mathbb{S})=Q_t^*\left(Z\right).$$

Thus these prices are just probabilities times the price of a risk-free bond.[3]

Assuming log-normal distributions for ω_{t+1}, η_{t+1} and θ_{t+1} allows us to solve out the integral in the expression for the risk-free rate $Q_t(\mathbb{S})$. In this case,

$$\mathcal{C}_{t+1}^{a+1}-\mathcal{C}_t^a=\frac{(1-\tau)(1+\widehat{\sigma})\left(1-\tau\left(1+\widehat{\sigma}\right)\right)}{\left(\widehat{\sigma}+\gamma\right)\widehat{\sigma}}\left(\frac{v_{\eta,t+1}+v_{\theta,t+1}-v_{\theta,t}}{2}\right)$$

and thus

$$\begin{aligned}Q_t\left(\mathbb{S}\right) &= \beta\exp\left(-\gamma\frac{(1-\tau)(1+\widehat{\sigma})\left(1-\tau\left(1+\widehat{\sigma}\right)\right)}{\left(\widehat{\sigma}+\gamma\right)\widehat{\sigma}}\left(\frac{v_{\eta,t+1}+v_{\theta,t+1}-v_{\theta,t}}{2}\right)\right)\\ &\quad\times\exp\left(-\gamma(1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\left(-\gamma(1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}-1\right)\frac{v_{\omega,t+1}}{2}\right).\end{aligned}\tag{A5}$$

Expression (10) in the main text is a special case when $\tau=0$.

Decentralization (asset purchases): We now derive expressions for insurance contract purchases, $B_t(s_{t+1};s^t)$ and $B_t^*(\eta_{t+1},\theta_{t+1};s^t)$ and verify that, given all conjectured prices and quantities, agents' budget constraints are satisfied.

Given that any available inter-island insurance contract can be purchased at the same price on the within-island market, $B_t^*(\eta_{t+1},\theta_{t+1};s^t)=0$ for all $(\eta_{t+1},\theta_{t+1})$ is consistent with individual optimization (Proposition 1, part (iii)). Thus, agents optimize when purchasing all their insurance on the island on which they are located. At the same time, because $Q_t^*\left(Z;s^t\right)=Q_t^*\left(Z\right)$, no agent has an incentive to try to sell insurance to an agent located on another island. To understand this, note that the price at which one agent (say agent i_1) with

[3] If we allowed insurance contracts to be traded across islands contingent on ω_{t+1} then agents would pool ω_{t+1} risk and insurance prices would be $\Pr\left((\omega_{t+1},\eta_{t+1},\theta_{t+1})\in S\right)\times\beta\neq Q_t\left(S\right)$.

history $s^t_{i_1}$ is willing to buy, on the margin, a set of claims that pay if and only if $(\eta_{t+1}, \theta_{t+1}) \in Z$ is the probability of that event times agent i_1's expected marginal rate of substitution, i.e. $\Pr((\eta_{t+1}, \theta_{t+1}) \in Z) \times Q_t(\mathbb{S}; s^t_{i_1})$. The price at which a second agent on a different island (agent i_2 with history $s^t_{i_2}$) is willing to sell this insurance to agent i_1 is the same probability times agent i_2's expected marginal rate of substitution, $\Pr((\eta_{t+1}, \theta_{t+1}) \in Z) \times Q_t(\mathbb{S}; s^t_{i_2})$. If agents i_1 and i_2 did not share the same marginal rate of substitution (i.e., if $Q_t(\mathbb{S}; s^t_{i_1}) \neq Q_t(\mathbb{S}; s^t_{i_2})$), then there could be no equilibrium without inter-island trade, because any such equilibrium would feature unexploited gains from trade. Thus $Q_t(\mathbb{S}, s^t) = Q_t(\mathbb{S})$ is the crucial result supporting an absence of inter-island trade.

Finally, we now derive an expression for purchases of state-contingent claims, $B_t(s_{t+1}; s^t)$, and verify budget balance. Given $B^*_t(Z; s^t) = 0 \ \forall Z, \forall s^t$, realized wealth at s^t implicitly defines insurance purchases:

$$B_{t-1}(s_t; s^{t-1}) = \delta d_t(s^t).$$

We will now guess and verify the following solution for $d_t(s^t)$:

$$d_t(s^t) = \hat{d}_t(s^t) + T_t(s^t)$$

where

$$\begin{aligned} T_t(s^t) &= c_t(s^t) - \lambda \left(w_t(s^t) h_t(s^t)\right)^{1-\tau}, \\ \hat{d}_t(s^t) &= \mathbb{E}_{s^t}\left[\sum_{j=1}^{\infty} \frac{(\beta\delta)^j c_{t+j}(s^{t+j})^{-\gamma}}{c_t(s^t)^{-\gamma}} T_{t+j}(s_{t+j})\right]. \end{aligned}$$

The logic for this guess is that insurance payouts must deliver the appropriately discounted present value of lifetime differences between consumption and after-tax earnings.

We now need to check that the agent's budget constraint is satisfied. Given $B^*_t(Z; s^t) = 0$ $\forall Z, \forall s^t$ this amounts to checking that

$$c_t(s^t) + \int\int\int Q_t(\omega, \eta, \theta)\ B_t((\omega, \eta, \theta); s^t)\ d\omega\, d\eta\, d\theta = \lambda \left(w_t(s^t) h_t(s^t)\right)^{1-\tau} + \hat{d}_t(s^t) + T_t(s^t).$$

Given the conjecture for $T(s^t)$ this simplifies to

$$\int\int\int Q_t(\omega, \eta, \theta)\ B_t((\omega, \eta, \theta); s^t)\ d\omega\, d\eta\, d\theta = \hat{d}_t(s^t). \tag{A6}$$

To verify that this equation is in fact satisfied, we will write the functions $Q_t(\omega, \eta, \theta)$, $B_t((\omega, \eta, \theta); s^t)$ and $\hat{d}_t(s^t)$ all in terms of the decision rule for consumption $c_t(s^t)$. The ratio of after-tax earnings to consumption is

$$\frac{\lambda\left(w_t\left(s^t\right)h_t\left(s^t\right)\right)^{1-\tau}}{c_t\left(s^t\right)}=\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\varepsilon_t-\frac{\gamma+\widehat{\sigma}}{\widehat{\sigma}}\mathcal{M}_t^a\right),$$

so

$$\begin{aligned}T_t\left(s^t\right) &= \left(1-\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_t+\theta_t)-\frac{\gamma+\widehat{\sigma}}{\widehat{\sigma}}\mathcal{M}_t^a\right)\right)c_t\left(s^t\right)\\ &= c_t(s^t)\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_t+\theta_t)\right)}{\int\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_t+\theta_t)\right)\,dF_{\kappa,t}^a dF_{\theta,t}}\right),\end{aligned}$$

where the second line uses

$$\mathcal{M}_t^a=\frac{\widehat{\sigma}}{\widehat{\sigma}+\gamma}\log\int\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\left(\kappa_t+\theta_t\right)\right)\,dF_{\kappa,t}^a dF_{\theta.t}.$$

Substituting the definition for $T_{t+j}\left(s_{t+j}\right)$ into the one for $\hat{d}_t\left(s^t\right)$, and multiplying and dividing by $c_t(s^t)$, gives

$$\begin{aligned}\hat{d}_t\left(s^t\right) &= c_t(s^t)\mathbb{E}_{s^t}\left[\sum_{j=1}^{\infty}\frac{(\beta\delta)^j c_{t+j}(s^{t+j})^{-\gamma}}{c_t(s^t)^{-\gamma}}\frac{c_{t+j}(s^{t+j})}{c_t(s^t)}\times\right.\\ &\quad\left.\times\left(1-\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\left(\kappa_t+\sum_{i=1}^{j}\eta_{t+i}+\theta_{t+j}\right)-\frac{\gamma+\widehat{\sigma}}{\widehat{\sigma}}\mathcal{M}_{t+j}^{a+j}\right)\right)\right]\\ &= c_t(s^t)\mathbb{E}_{s^t}\left[\sum_{j=1}^{\infty}\frac{(\beta\delta)^j c_{t+j}(s^{t+j})^{1-\gamma}}{c_t(s^t)^{1-\gamma}}\times\right.\\ &\left.\times\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\left(\kappa_t+\sum_{i=1}^{j}\eta_{t+i}+\theta_{t+j}\right)\right)}{\int....\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\left(\kappa_t+\sum_{i=1}^{j}\eta_{t+i}+\theta_{t+j}\right)\right)\,dF_{\kappa,t}^a\,dF_{\eta,t+1}\,...\,dF_{\eta,t+j}\,dF_{\theta,t+j}}\right)\right]\\ &= c_t(s^t)\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF_{\kappa,t}^a}\right)\mathbb{E}_{s^t}\left[\frac{\sum_{j=1}^{\infty}(\beta\delta)^j c_{t+j}(s^{t+j})^{1-\gamma}}{c_t(s^t)^{1-\gamma}}\right],\end{aligned}\tag{A7}$$

where the second equation uses

$$\mathcal{M}_{t+j}^{a+j}=\frac{\widehat{\sigma}}{\widehat{\sigma}+\gamma}\log\int....\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\left(\kappa_t+\sum_{i=1}^{j}\eta_{t+i}+\theta_{t+j}\right)\right)\,dF_{\kappa,t}^a\,dF_{\eta,t+1}\,...\,dF_{\eta,t+j}\,dF_{\theta,t+j}.$$

Thus

$$\begin{aligned} & B_{t-1}((\omega_t,\eta_t,\theta_t)\,;s^{t-1}) \qquad\qquad (A8)\\ = \;& \delta\left(\hat{d}_t\left(s^t\right)+T_t\left(s^t\right)\right)\\ = \;& \delta c_t(s^t)\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF^a_{\kappa,t}}\right)\mathbb{E}_{s^t}\left[\sum_{j=1}^{\infty}\frac{(\beta\delta)^j\,c_{t+j}(s^{t+j})^{1-\gamma}}{c_t(s^t)^{1-\gamma}}\right]\\ & +\delta c_t(s^t)\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_t+\theta_t)\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_t+\theta_t)\right)\,dF^a_{\kappa,t}dF_{\theta,t}}\right). \end{aligned}$$

Substituting eq. (A7) and eq. (A3) into eq. (A6) gives

$$\begin{aligned} & \beta c_t\left(s^t\right)^{\gamma}\mathbb{E}_{s^t}\left[c_{t+1}(s^t,s_{t+1})^{-\gamma}\,B_t\left(s_{t+1};s^t\right)\right] \qquad\qquad (A9)\\ = \;& c_t\left(s^t\right)\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF^a_{\kappa,t}}\right)\mathbb{E}_{s^t}\left[\sum_{j=1}^{\infty}\frac{(\beta\delta)^j\,c_{t+j}(s^{t+j})^{1-\gamma}}{c_t(s^t)^{1-\gamma}}\right]. \end{aligned}$$

Let $LHS(s^t)$ denote the left-hand side of eq. (A9), substitute in eq. (A8) and simplify

$$\begin{aligned} LHS(s^t) \;=\; & \beta c_t\left(s^t\right)^{\gamma}\mathbb{E}_{s^t}\left[c_{t+1}(s^t,s_{t+1})^{-\gamma}\,\delta c_{t+1}(s^{t+1})\times\right.\\ & \times\left\{\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_{t+1}\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_{t+1}\right)\,dF^{a+1}_{\kappa,t+1}}\right)\mathbb{E}_{s^{t+1}}\left[\sum_{j=1}^{\infty}\frac{(\beta\delta)^j\,c_{t+1+j}(s^{t+1+j})^{1-\gamma}}{c_{t+1}(s^{t+1})^{1-\gamma}}\right]+\right.\\ & \left.\left.+\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_{t+1}+\theta_{t+1})\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}(\kappa_{t+1}+\theta_{t+1})\right)\,dF^{a+1}_{\kappa,t+1}dF_{\theta,t+1}}\right)\right\}\right] \end{aligned}$$

$$\begin{aligned} = \;& \beta\delta c_t\left(s^t\right)^{\gamma}\mathbb{E}_{s^t}\left[c_{t+1}(s^t,s_{t+1})^{1-\gamma}\times\right.\\ & \times\left\{\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF^a_{\kappa,t}}\right)\mathbb{E}_{s^{t+1}}\left[\sum_{j=1}^{\infty}\frac{(\beta\delta)^j\,c_{t+1+j}(s^{t+1+j})^{1-\gamma}}{c_{t+1}(s^{t+1})^{1-\gamma}}\right]+\right.\\ & \left.\left.+\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF^a_{\kappa,t}}\right)\right\}\right]\\ = \;& \beta\delta c_t\left(s^t\right)^{\gamma}\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF^a_{\kappa,t}}\right)\mathbb{E}_{s^t}\left[\mathbb{E}_{s^{t+1}}\left[\sum_{k=1}^{\infty}(\beta\delta)^{k-1}\,c_{t+k}(s^{t+k})^{1-\gamma}\right]\right]\\ = \;& c_t\left(s^t\right)\left(1-\frac{\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)}{\int\exp\left((1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}}\kappa_t\right)\,dF^a_{\kappa,t}}\right)\mathbb{E}_{s^t}\left[\sum_{k=1}^{\infty}(\beta\delta)^k\,\frac{c_{t+k}(s^{t+k})^{1-\gamma}}{c_t\left(s^t\right)^{1-\gamma}}\right], \end{aligned}$$

which is the same as the right-hand side of eq. (A9). We conclude that the budget constraint is satisfied when state-contingent bond purchases are given by eq. (A8).

B Household Models

We begin with the household model of Section 3 where household composition is insurable (an abbreviated version is contained in Appendix A.2). Next, we present the alternative model, also briefly discussed in Section 3, where demographics are uninsurable.

B.1 The household model of Section 3

Suppose that utility for individual i in a household of g adult workers ("g" for "grownups") and k children ("k" for kids) is given by

$$u(c, h_i, g, k) = \frac{1}{1-\gamma}\left(\frac{c}{e(g,k)}\right)^{1-\gamma} - \frac{\exp(\varphi)}{1+\sigma} h_i^{1+\sigma},$$

where c is household consumption and h_i is i's hours worked. The equivalence scale is given by e and satisfies $e_g \in (0,1]$, $e_{gg} < 0$, $e_k \in (0,1]$, and $e(1,0) = 1$ for all $g \geq 1$ and $k \geq 0$. Assume that the household utility function attaches equal weights to all adults (and no weight to the children), so total utility is given by

$$U = \sum_{i=1}^{g} u(c, h_i, g) = \frac{g}{1-\gamma}\left(\frac{c}{e(g,k)}\right)^{1-\gamma} - \sum_{i=1}^{g}\left(\frac{\exp(\varphi)}{1+\sigma} h_i^{1+\sigma}\right) \tag{B10}$$

As in Section A, let $x_t = (a, \varphi, \alpha_t)$ denote the island-level components of the individual state. Each island-level planner can insure realizations of ε_t, g, and k. The planner's problem is to choose functions $c_t(x_t, g, k)$, $h_{it}(x_t, \varepsilon_t, g, k)$ for $i = 1, ..., g$ to solve

$$\max_{\{c(x_t,\cdot), h_{it}(x_t,\cdot)\}} \int \left[\frac{g}{1-\gamma}\left(\frac{c_t(x_t,g,k)}{e(g,k)}\right)^{1-\gamma} - \sum_{i=1}^{g} \int \frac{\exp((\gamma+\sigma)\varphi)}{1+\sigma} h_{it}(x_t,\varepsilon_t,g,k)^{1+\sigma}\, dF^a_{\varepsilon t}\right] dF_t(g,k)$$

subject to the after-tax resource constraint

$$\int \left[\sum_{i=1}^{g} \int \lambda \left[\exp(\alpha_t + \varepsilon_{it})\, h_{it}(x_t, \varepsilon_t, g, k)\right]^{1-\tau} dF^a_{\varepsilon t} - c_t(x_t, g, k)\right] dF_t(g,k) = 0,$$

where the objective function and the constraint recognize that there is a non-degenerate within-island distribution $F_t(g,k)$ of household workers and children. Moreover, in light of the result of Section A, we have imposed that consumption is independent of ε_t.

The first-order conditions with respect to $c_t(x_t, g, k)$ and $h_{it}(x_t, \varepsilon_t, g, k)$ are, respectively,

$$ge(g,k)^{\gamma-1} c_t(x_t, g, k)^{-\gamma} = \chi_t \tag{B11}$$

$$\exp(\varphi)\, h_{it}(x_t, \varepsilon_t, g, k)^{\sigma+\tau} = \chi_t \lambda (1-\tau) \exp((1-\tau)\alpha_t) \exp((1-\tau)\varepsilon_t), \tag{B12}$$

where χ_t is the multiplier on the date t island-level resource constraint.

Let $c_t(x_t, 1, 0)$ denote household consumption for a one-person household. Then equation (B11) implies

$$c_t(x_t, g, k) = c_t(x_t, 1, 0) \left(\frac{g}{e(g,k)^{1-\gamma}} \right)^{\frac{1}{\gamma}}.$$

Combining the two first-order conditions (B11)-(B12) gives

$$h_{it}(x_t, \varepsilon_t, g, k) = \left(\frac{g c_t(x_t, g, k)^{-\gamma}}{e(g,k)^{1-\gamma}} \right)^{\frac{1}{\sigma+\tau}} (\lambda(1-\tau))^{\frac{1}{\sigma+\tau}} \exp\left(-\frac{\varphi}{\sigma+\tau} + \left(\frac{1-\tau}{\sigma+\tau}\right)\alpha_t + \left(\frac{1-\tau}{\sigma+\tau}\right)\varepsilon_t \right).$$

Substitute in the consumption expression for the one-person households:

$$h_{it} = c_t(x_t, 1, 0)^{-\frac{\gamma}{\sigma+\tau}} (\lambda(1-\tau))^{\frac{1}{\sigma+\tau}} \exp\left(-\frac{\varphi}{\sigma+\tau} + \left(\frac{1-\tau}{\sigma+\tau}\right)\alpha_t + \left(\frac{1-\tau}{\sigma+\tau}\right)\varepsilon_t \right),$$

so individual hours are insensitive to household size.

Finally we can solve for $c_t(x_t, 1, 0)$ from the island resource constraint:

$$\begin{aligned}
0 &= \int \left\{ \sum_{i=1}^{g} \int \lambda \left[\exp(\alpha_t + \varepsilon_t) h_{it}(x_t, \varepsilon_t, g, k)\right]^{1-\tau} dF^a_{\varepsilon t} - c_t(x_t, g, k) \right\} dF_t(g,k) \\
&= \int \left\{ \sum_{i=1}^{g} \int \int \lambda \Big[\exp(\alpha_t + \kappa_{it} + \theta_{it}) c_t(x_t, 1, 0)^{-\frac{\gamma}{\sigma+\tau}} (\lambda(1-\tau))^{\frac{1}{\sigma+\tau}} \right. \\
&\quad \times \exp\left(-\frac{\varphi}{\sigma+\tau} + \frac{1-\tau}{\sigma+\tau}\alpha_t + \left(\frac{1-\tau}{\sigma+\tau}\right)\varepsilon_t \right) \Big]^{1-\tau} dF^a_{\varepsilon t} \\
&\quad \left. - c_t(x_t, 1, 0) \left(\frac{g}{e(g,k)^{1-\gamma}} \right)^{\frac{1}{\gamma}} \right\} dF_t(g,k).
\end{aligned}$$

Collecting terms:

$$\begin{aligned}
& c_t(x_t, 1, 0)^{1+\left(\frac{1-\tau}{\sigma+\tau}\right)\gamma} \\
= \;& \bar{g} \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\alpha_t - \frac{1-\tau}{\sigma+\tau}\varphi \right) \times \int \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\varepsilon_t \right) dF^a_{\varepsilon t} \\
& \times (1-\tau)^{\frac{1-\tau}{\sigma+\tau}} (\lambda)^{\frac{1+\sigma}{\sigma+\tau}} \left(\int \left(\frac{g}{e(g,k)^{1-\gamma}} \right)^{\frac{1}{\gamma}} dF_t(g,k) \right)^{-1},
\end{aligned}$$

where $\bar{g} = \int g \, dF_t(g,k)$. Substitute out for $\widehat{\sigma} = (\sigma+\tau)/(1-\tau)$ and simplify the expression as:

$$\log c_t(x_t, 1, 0) = (1-\tau) \frac{1+\widehat{\sigma}}{\gamma+\widehat{\sigma}}\alpha_t - \frac{\varphi}{\gamma+\widehat{\sigma}} + \mathcal{C}^a_t,$$

where the constant $\mathcal{C}_t^a$ is defined as

$$\exp \mathcal{C}_t^a = (\bar{g})^{\frac{\widehat{\sigma}}{\gamma+\widehat{\sigma}}} \left(\int \left(\frac{g}{e(g,k)^{1-\gamma}} \right)^{\frac{1}{\gamma}} dF_t(g,k) \right)^{-\frac{\widehat{\sigma}}{\gamma+\widehat{\sigma}}} (1-\tau)^{\frac{1}{\gamma+\widehat{\sigma}}} (\lambda)^{\frac{1+\widehat{\sigma}}{\gamma+\widehat{\sigma}}}$$
$$\times \left[\int \exp \left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau} \varepsilon_t \right) dF_{\varepsilon t}^a \right]^{\frac{\widehat{\sigma}}{\gamma+\widehat{\sigma}}}.$$

The implied allocations for household consumption and individual labor supply are then

$$\log c_t^a(x_t, g, k) = D(g,k) - (1-\tau)\widehat{\varphi} + (1-\tau)\left(\frac{1+\widehat{\sigma}}{\hat{\sigma}+\gamma}\right)\alpha_t + \mathcal{C}_t^a$$
$$\log h_{it}(x_t, \varepsilon_t, g, k) = -\widehat{\varphi} + \frac{(1-\gamma)}{(\widehat{\sigma}+\gamma)}\alpha_t + \frac{1}{\widehat{\sigma}}\varepsilon_t + \mathcal{H}_t^a,$$

where $\widehat{\varphi} = \varphi / ((1-\tau)(\widehat{\sigma}+\gamma))$ is the rescaled preference weight,

$$D(g,k) \equiv \log g + \frac{(1-\gamma)}{\gamma} \log \left(\frac{g}{e(g,k)} \right)$$

and

$$\mathcal{H}_t^a = -\frac{\gamma}{\sigma+\tau}\mathcal{C}_t^a + \frac{1}{\sigma+\tau}\log(\lambda(1-\tau)).$$

Note that the individual hours allocation is independent of (g,k), and the household consumption allocation is independent of ε_t.

B.2 Alternative household model with uninsurable demographics

Utility of a household with g adults and k children is still given by equation (B10). The island-level components of the individual state are now $x_t = (a, \varphi, \alpha_t, g, k)$. The planner can insure only against realizations of ε_t. The planner chooses functions $c_t(x_t)$ and $h_{it}(x_t, \varepsilon_t)$ for $i = 1, ..., g$ to solve

$$\max_{c_t(x_t), \{h_{it}(x_t, \cdot)\}} \left\{ \frac{g}{1-\gamma} \left(\frac{c_t(x_t)}{e(g,k)} \right)^{1-\gamma} - \sum_{i=1}^{g} \int \frac{\exp(\varphi)}{1+\sigma} h_{it}(x_t, \varepsilon_t)^{1+\sigma} dF_{\varepsilon t}^a \right\}$$

subject to the island-level after-tax resource constraint

$$c_t(x_t) - \sum_{i=1}^{g} \int \lambda \left[\exp(\alpha_t + \varepsilon_t) h_{it}(x_t, \varepsilon_t) \right]^{1-\tau} dF_{\varepsilon t}^a = 0,$$

where, in light of the result of Section A, we have imposed that consumption is independent of ε_t.

The first-order conditions with respect to $c_t(x_t)$ and $h_{it}(x_t, \varepsilon_t)$ are, respectively,

$$\begin{aligned}
\frac{g}{e(g,k)^{1-\gamma}} c_t(x_t)^{-\gamma} &= \chi_t \\
\exp(\varphi) h_{it}(x_t, \varepsilon_t)^{\sigma+\tau} &= \chi_t \lambda (1-\tau) \exp((1-\tau)\alpha_t) \exp((1-\tau)\varepsilon_t).
\end{aligned}$$

Combining the two conditions and simplifying terms yields the expression for individual hours

$$\begin{aligned}
h_{it}(x_t, \varepsilon_t) &= \left(\frac{g\, c_t(x_t)^{-\gamma}}{e(g,k)^{1-\gamma}}\right)^{\frac{1}{\sigma+\tau}} (\lambda(1-\tau))^{\frac{1}{\sigma+\tau}} \exp\left(-\frac{\varphi}{\sigma+\tau} + \frac{1-\tau}{\sigma+\tau}\alpha_t + \frac{1-\tau}{\sigma+\tau}\varepsilon_t\right) \quad \text{(B13)} \\
&= g^{\frac{1}{\widehat{\sigma}(1-\tau)}} e(g,k)^{\frac{\gamma-1}{\widehat{\sigma}(1-\tau)}} (\lambda(1-\tau))^{\frac{1}{\widehat{\sigma}(1-\tau)}} \exp\left(\frac{1}{\widehat{\sigma}}\left(\alpha_t + \varepsilon_t - \frac{\varphi}{1-\tau}\right)\right) c_t(x_t)^{\frac{-\gamma}{\widehat{\sigma}(1-\tau)}}
\end{aligned}$$

and the expression for household consumption

$$\begin{aligned}
c_t(x_t) &= \sum_{i \in 1}^{g} \int \lambda \left[\exp(\alpha_t + \varepsilon_t) h_{it}(x_t, \varepsilon_t)\right]^{1-\tau} dF_{\varepsilon t}^{a} \\
&= g \exp((1-\tau)\alpha_t) \int \lambda \exp((1-\tau)\varepsilon_t) h_{it}(x_t, \varepsilon_t)^{1-\tau} dF_{\varepsilon t}^{a} \\
&= (g)^{1+\frac{1-\tau}{\sigma+\tau}} e(g,k)^{(\gamma-1)\frac{1-\tau}{\sigma+\tau}} c_t(x_t)^{-\frac{\gamma(1-\tau)}{\sigma+\tau}} \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\alpha_t - \frac{1-\tau}{\sigma+\tau}\varphi\right) \\
&\quad \times (1-\tau)^{\frac{1-\tau}{\sigma+\tau}} \lambda^{\frac{1+\sigma}{\sigma+\tau}} \int \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\varepsilon_t\right) dF_{\varepsilon t}^{a}
\end{aligned}$$

Taking logs:

$$\begin{aligned}
&\left(1 + \gamma\frac{1-\tau}{\sigma+\tau}\right) \log c_t(x_t) \\
&= \left(1 + \frac{1-\tau}{\sigma+\tau}\right) \log g + (\gamma-1)\frac{1-\tau}{\sigma+\tau} \log e(g,k) + \frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\alpha_t - \frac{1-\tau}{\sigma+\tau}\varphi \\
&\quad + \frac{1-\tau}{\sigma+\tau} \log(1-\tau) + \log\frac{1+\sigma}{\sigma+\tau}\lambda + \log \int \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\varepsilon_t\right) dF_{\varepsilon t}^{a}
\end{aligned}$$

and rearranging

$$\begin{aligned}
\log c_t(x_t) &= \frac{\frac{\sigma+\tau}{1-\tau}}{\frac{\sigma+\tau}{1-\tau}+\gamma} \log g + \frac{1}{\frac{\sigma+\tau}{1-\tau}+\gamma} \log g - \frac{1-\gamma}{\frac{\sigma+\tau}{1-\tau}+\gamma} \log e(g,k) + \frac{1+\sigma}{\frac{\sigma+\tau}{1-\tau}+\gamma}\alpha_t \\
&\quad - \frac{\varphi}{\frac{\sigma+\tau}{1-\tau}+\gamma} + \frac{1}{\frac{\sigma+\tau}{1-\tau}+\gamma} \log(1-\tau) + \frac{\frac{1+\sigma}{1-\tau}}{\frac{\sigma+\tau}{1-\tau}+\gamma} \log \lambda \\
&\quad + \frac{\frac{\sigma+\tau}{1-\tau}}{\frac{\sigma+\tau}{1-\tau}+\gamma} \log \int \exp\left(\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}\varepsilon_t\right) dF_{\varepsilon t}^{a}.
\end{aligned}$$

Using the $\widehat{\sigma} = (\sigma+\tau)/(1-\tau)$ notation:

$$\log c_t(x_t) = \frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma} \log g - \frac{1-\gamma}{\widehat{\sigma}+\gamma} \log e(g,k) + \frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}+\gamma}\alpha_t - \frac{\varphi}{\widehat{\sigma}+\gamma} + \mathcal{C}_t^{a}, \quad \text{(B14)}$$

where $\mathcal{C}_t^a$ is defined as

$$\mathcal{C}_t^a \equiv \frac{1}{\widehat{\sigma}+\gamma}\left[\log(1-\tau)+(1+\widehat{\sigma})\log\lambda+\widehat{\sigma}\log\int\exp\left(\frac{(1-\tau)(1+\widehat{\sigma})}{\widehat{\sigma}}\varepsilon_t\right)dF^a_{\varepsilon t}\right].$$

Now substitute the expression for $c_t(x_t)$ of eq. (B14) into eq. (B13) to derive an expression for $h_{it}(x_t,\varepsilon_t)$,

$$\begin{aligned}\log h_{it}(x_t,\varepsilon_t) &= \frac{1}{1-\tau}\frac{1}{\widehat{\sigma}}\log g+\frac{\gamma-1}{1-\tau}\frac{1}{\widehat{\sigma}}\log e(g,k)-\frac{1}{\widehat{\sigma}}\frac{\varphi}{1-\tau}+\frac{1}{\widehat{\sigma}}\alpha_t \\ &\quad +\frac{1}{\widehat{\sigma}}\varepsilon_t+\frac{1}{\widehat{\sigma}}\frac{1}{1-\tau}\log(\lambda(1-\tau))-\frac{1}{\widehat{\sigma}}\frac{\gamma}{1-\tau}\log c_t(x_t) \\ &= \frac{1}{1-\tau}\frac{1-\gamma}{\widehat{\sigma}+\gamma}(\log g-\log e(g,k))+\frac{1}{\widehat{\sigma}}\varepsilon_t+\frac{1-\gamma}{\widehat{\sigma}+\gamma}\alpha_t-\frac{1}{\widehat{\sigma}+\gamma}\frac{\varphi}{1-\tau}+\mathcal{H}_t^a,\end{aligned}$$

where

$$\mathcal{H}_t^a \equiv -\frac{1}{\widehat{\sigma}}\frac{1}{1-\tau}\log(\lambda(1-\tau))-\frac{1}{\widehat{\sigma}}\frac{\gamma}{1-\tau}\mathcal{C}_t^a.$$

We conclude that household consumption and individual hours are given by

$$\begin{aligned}\log c_t(x_t) &= D^c(g,k)-(1-\tau)\widehat{\varphi}+(1-\tau)\frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\alpha_t+\mathcal{C}_t^a \\ \log h_{it}(x_t,\varepsilon_t) &= D^h(g,k)-\widehat{\varphi}+\frac{1-\gamma}{\widehat{\sigma}+\gamma}\alpha_t+\frac{\varepsilon_t}{\widehat{\sigma}}+\mathcal{H}_t^a,\ i=1,...,g\end{aligned}$$

where $\widehat{\varphi}=\varphi/((1-\tau)(\widehat{\sigma}+\gamma))$ is the rescaled preference weight, and the equivalization dummies $D^c(g,k)$ and $D^h(g,k)$ are given by

$$\begin{aligned}D^c(g,k) &= \frac{1+\widehat{\sigma}}{\widehat{\sigma}+\gamma}\log g-\frac{1-\gamma}{\widehat{\sigma}+\gamma}\log e(g,k) \\ D^h(g,k) &= \frac{1}{1-\tau}\frac{1-\gamma}{\widehat{\sigma}+\gamma}[\log g-\log e(g,k)].\end{aligned}$$

To sum up, in this case both household consumption and individual hours depend in general on the vector of household type (g,k). In the special case $\gamma=1$ hours are again independent of (g,k) and consumption is proportional to the number of adults g.

C Identification

This appendix contains proofs of Propositions 3 and a new Corollary 3.1 that extends Proposition 3 to allow for biannual panel data. Finally, this appendix also contains the additional identification assumptions made in the estimation of the model (see Section 4.3).

C.1 Proof of Proposition 3

Proposition 3 [identification with no consumption data] *With an unbalanced panel on wages and hours from $t = 1, ..., T$, and an external estimate of measurement error in earnings $v_{\mu y}$, the same parameters as in Proposition 2 are identified.*

Proof The proof is organized in three sequential steps.

1. Given foreknowledge of $v_{\mu y}$, we identify $\widehat{\sigma}$, $v_{\mu h}$, and the sequence $\{v_{\theta,t}\}_{t=1}^{T-1}$ off moments involving (co-)variance of changes minus changes in (co-)variances:

   (a) The Frisch elasticity $1/\widehat{\sigma}$ is identified by $1/\widehat{\sigma}$ equal to

$$\frac{cov_t^a(\Delta \log \hat{w}, \Delta \log \hat{h}) + var_t^a(\Delta \log \hat{h}) - \Delta cov_t^a(\log \hat{w}, \log \hat{h}) - \Delta var_t^a(\log \hat{h})}{cov_t^a\left(\Delta \log \hat{w}, \Delta \log \hat{h}\right) + var_t^a\left(\Delta \log \hat{w}\right) - \Delta cov_t^a\left(\log \hat{w}, \log \hat{h}\right) - \Delta var_t^a\left(\log \hat{w}\right) - 2v_{\mu y}}.$$

   This expression can equivalently be formulated as

$$\frac{1}{\widehat{\sigma}} = \frac{cov_t^a(\Delta \log \hat{y}, \Delta \log \hat{h}) - \Delta cov_t^a(\log \hat{y}, \log \hat{h})}{cov_t^a(\Delta \log \hat{y}, \Delta \log \hat{w}) - \Delta cov_t^a(\log \hat{y}, \log \hat{w}) - 2v_{\mu y}}.$$

   (b) The sequence $\{v_{\theta,t}\}_{t=1}^{T-1}$ is then identified by panel data available from $t = 2, ..., T$:

$$\begin{aligned} & cov_t^a(\Delta \log \hat{w}, \Delta \log \hat{h}) + var_t^a(\Delta \log \hat{h}) - \Delta cov_t^a(\log \hat{w}, \log \hat{h}) - \Delta var_t^a(\log \hat{h}) \\ = \; & 2\frac{(1+\widehat{\sigma})}{\widehat{\sigma}^2} v_{\theta,t-1}. \end{aligned}$$

   (c) Measurement error in hours is then identified from e.g.

$$var_t^a\left(\Delta \log \hat{h}\right) - \Delta var_t^a\left(\log \hat{h}\right) = \frac{2}{\widehat{\sigma}^2} v_{\theta,t-1} + 2v_{\mu h}.$$

2. The parameter γ and the two sets of parameters $\{v_{\eta t}\}_{t=2}^{T}$ and $\{v_{\omega t}\}_{t=2}^{T}$ are then identified from within-cohort changes in the macro moments, $\Delta var_t^a\left(\log \hat{w}\right)$, $\Delta var_t^a\left(\log \hat{h}\right)$, and $\Delta cov_t^a\left(\log \hat{w}, \log \hat{h}\right)$, all available from $t = 2, ..., T$. These parameters can be identified recursively as follows:

3. Combine (24)-(25) to get

$$\frac{\left(\Delta cov_t^a\left(\log \hat{w}, \log \hat{h}\right) - \frac{1}{\widehat{\sigma}}\left(v_{\eta t} + \Delta v_{\theta t}\right)\right)^2}{\left(\Delta var_t^a\left(\log \hat{h}\right) - \frac{1}{\widehat{\sigma}^2}\left(v_{\eta t} + \Delta v_{\theta t}\right)\right)} = \frac{\left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)^2 \left(v_{\omega t}\right)^2}{\left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)^2 v_{\omega t}} = v_{\omega t}$$

Combine this with (23) to get an equation in $(v_{\eta t}+\Delta v_{\theta t})$,

$$\frac{\left(\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)-\frac{1}{\widehat{\sigma}}\left(v_{\eta t}+\Delta v_{\theta t}\right)\right)^2}{\left(\Delta var_t^a\left(\log\hat{h}\right)-\frac{1}{\widehat{\sigma}^2}\left(v_{\eta t}+\Delta v_{\theta t}\right)\right)}=\Delta var_t^a\left(\log\hat{w}\right)-\left(v_{\eta t}+\Delta v_{\theta t}\right).$$

Therefore, each element of the sequence $\{v_{\eta t}+\Delta v_{\theta t}\}_{t=2}^{T}$ is identified by[4]

$$\left(v_{\eta t}+\Delta v_{\theta t}\right)=\frac{\left(\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)\right)^2-\Delta var_t^a\left(\log\hat{w}\right)\cdot\Delta var_t^a\left(\log\hat{h}\right)}{\frac{\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)}{\widehat{\sigma}}-\frac{\Delta var_t^a\left(\log\hat{w}\right)}{\widehat{\sigma}^2}-\Delta var_t^a\left(\log\hat{h}\right)},$$

which, given $\{v_{\theta,t}\}_{t=1}^{T-1}$, pins down $\{v_{\eta,t}\}_{t=2}^{T-1}$.

(a) Given $v_{\eta t}+\Delta v_{\theta t}$, each element of the sequence $\{v_{\omega t}\}_{t=2}^{T}$ is identified by (23),

$$v_{\omega t}=\Delta var_t^a\left(\log\hat{w}\right)-\left(v_{\eta t}+\Delta v_{\theta t}\right).$$

(b) Given $v_{\eta t}+\Delta v_{\theta t}$ and $v_{\omega t}$, the risk aversion parameter γ is determined by (25) as the solution to the following equation:

$$\frac{1-\gamma}{\widehat{\sigma}+\gamma}=\frac{\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)}{v_{\omega t}}-\frac{1}{\widehat{\sigma}}\frac{\left(v_{\eta t}+\Delta v_{\theta t}\right)}{v_{\omega t}}.$$

4. Given values for γ, $\widehat{\sigma}$, $\{v_{\theta t}\}_{t=1}^{T-1}$ and $\{v_{\mu h},v_{\mu y}\}$, the following macro moments, available for all $t=1,...,T$ and evaluated for the youngest age group, identify the sequence of cohort effects $\{v_{\widehat{\varphi}t},v_{\alpha^0 t}\}_{t=1}^{T}$, $\{v_{\kappa^0 t}\}_{t=2}^{T}$, and $(v_{\kappa^0 T}+v_{\theta T})$. We do it in two steps

   (a) First, the following two linearly independent macro moments, available for all $t=1,...,T$ and evaluated for the youngest age group, identify the sequence of

[4] To see this, note that

$$\begin{aligned}
&\left(\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)-\frac{1}{\widehat{\sigma}}\left(v_{\eta t}+\Delta v_{\theta t}\right)\right)^2\\
=\;&\left(\Delta var_t^a\left(\log\hat{w}\right)-\left(v_{\eta t}+\Delta v_{\theta t}\right)\right)\left(\Delta var_t^a\left(\log\hat{h}\right)-\frac{1}{\widehat{\sigma}^2}\left(v_{\eta t}+\Delta v_{\theta t}\right)\right)\\
&\left(\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)\right)^2-\frac{\Delta cov_t^a\left(\log\hat{w},\log\hat{h}\right)}{\widehat{\sigma}}\left(v_{\eta t}+\Delta v_{\theta t}\right)+\frac{1}{\widehat{\sigma}^2}\left(v_{\eta t}+\Delta v_{\theta t}\right)^2\\
=\;&\Delta var_t^a\left(\log\hat{w}\right)\cdot\Delta var_t^a\left(\log\hat{h}\right)-\frac{\Delta var_t^a\left(\log\hat{w}\right)}{\widehat{\sigma}^2}\left(v_{\eta t}+\Delta v_{\theta t}\right)\\
&-\left(v_{\eta t}+\Delta v_{\theta t}\right)\cdot\Delta var_t^a\left(\log\hat{h}\right)+\frac{1}{\widehat{\sigma}^2}\left(v_{\eta t}+\Delta v_{\theta t}\right)^2
\end{aligned}$$

cohort effects in insurable- and uninsurable initial wages, $\{v_{\alpha^0 t}\}_{t=1}^T$, $\{v_{\kappa^0 t}\}_{t=2}^T$, and $(v_{\kappa^0 T} + v_{\theta T})$,

$$\begin{aligned} var_t^0(\log \hat{w}) &= v_{\alpha^0 t} + (v_{\kappa^0 t} + v_{\theta t}) + v_{\mu y} + v_{\mu h} \\ cov_t^0\left(\log \hat{w}, \log \hat{h}\right) &= \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right) \cdot v_{\alpha^0 t} + \frac{1}{\widehat{\sigma}}(v_{\kappa^0 t} + v_{\theta t}) - v_{\mu h}. \end{aligned}$$

(b) Finally, the following macro moments, available for all $t = 1, ..., T$ and evaluated for the youngest age group, identify the sequence of cohort effects in preference heterogeneity, $\{v_{\widehat{\varphi} t}\}_{t=1}^T$,

$$cov_t^0\left(\log \hat{w}, \log \hat{h}\right) + var_t^0\left(\log \hat{h}\right) = v_{\widehat{\varphi} t} + \frac{(1-\gamma)(1+\widehat{\sigma})}{(\widehat{\sigma}+\gamma)^2} v_{\alpha^0 t} + \frac{1+\widehat{\sigma}}{\widehat{\sigma}^2}(v_{\kappa^0 t} + v_{\theta t}),$$

since every other parameter in those three moments is already known. This concludes the proof.

C.2 Extending Proposition 3 to biannual data

It is possible to extend Proposition 3 to allow for biannual panel data towards the end of the sample, so the proposition can be applied directly to the PSID. This amounts to combining Proposition 3 with Corollary 2.2. We state this formally as the following corollary.

Corollary 3.1 [extending Proposition 3 to biannual panel data] *Suppose one has access to an unbalanced panel on wages and hours, but no data on consumption. The panel data are available annually until year $\hat{t}$ and biannually thereafter, i.e. available for the years $t = 1, 2, ..., \hat{t}$ and $t = \hat{t}+2, \hat{t}+4, ..., T-2, T$. Suppose further that one has an exogenous estimate of measurement error in earnings, $v_{\mu y}$. Then, one can identify $\{\widehat{\sigma}, \gamma, v_{\mu h}, v_{\mu c}\}$, the sequences $\{v_{\widehat{\varphi} t}\}_{t=1}^T$, $\{v_{\omega t}, v_{\eta t}\}_{t=2}^{\hat{t}}$, $\{v_{\theta t}, v_{\kappa^0 t}, v_{\alpha^0 t}\}_{t=1}^{\hat{t}}$, and $\{v_{\theta t}, v_{\kappa^0 t}, v_{\alpha^0 t}, v_{\omega,t-1} + v_{\omega t}, v_{\eta,t-1} + v_{\eta t}\}$ for $t = \hat{t}+2, \hat{t}+4, ..., T-2$, as well as the sums $\{v_{\eta,T-1} + v_{\eta,T} + v_{\theta,T}\}$ and $\{v_{\kappa^0,T} + v_{\theta,T}\}$.*

Proof Start by following the proof of Proposition 3 for the years $t = 1, 2, ..., \hat{t}$. Consider then the time-varying parameters for the biannual sample, i.e., for $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2, T$. These parameters are identified in five sequential steps.

1. Identify $\{v_{\theta,t}\}$ for the biannual years $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2$, as well as $v_{\eta \hat{t}}$ and $v_{\kappa^0 \hat{t}}$, by combining the following moments,

$$var_t^a\left(\Delta^2 \log \hat{w}\right) - \Delta^2 var_t^a(\log \hat{w}) = 2v_{\theta,t-2} + 2(v_{\mu y} + v_{\mu h}).$$

which is available for $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2, T$. Note that, since $v_{\theta,\hat{t}}$ is identified, so are $v_{\eta,\hat{t}}$ and $v_{\kappa^0 \hat{t}}$.

2. Identify $\{v_{\eta t} + v_{\eta,t-1}\}$ for the biannual years $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2$, and the sum $(v_{\eta,T} + v_{\eta,T-1} + v_{\theta T})$. Start by combining the biannual versions of (24), (25), and (23) to get an equation where $(v_{\eta t} + v_{\eta,t-1} + \Delta^2 v_{\theta t})$ is the only unknown:

$$\begin{aligned} & \frac{\left(\Delta^2 cov_t^a\left(\log \hat{w}, \log \hat{h}\right) - \frac{1}{\hat{\sigma}}\left(v_{\eta t} + v_{\eta,t-1} + \Delta^2 v_{\theta t}\right)\right)^2}{\Delta^2 var_t^a\left(\log \hat{h}\right) - \frac{1}{\hat{\sigma}^2}\left(v_{\eta t} + v_{\eta,t-1} + \Delta^2 v_{\theta t}\right)} \\ = & \frac{\left(\left(\frac{1-\gamma}{\hat{\sigma}+\gamma}\right)\left(v_{\omega t} + v_{\omega,t-1}\right)\right)^2}{\left(\frac{1-\gamma}{\hat{\sigma}+\gamma}\right)^2\left(v_{\omega t} + v_{\omega,t-1}\right)} = \left(v_{\omega t} + v_{\omega,t-1}\right) \\ = & \Delta^2 var_t^a\left(\log \hat{w}\right) - \left(v_{\eta t} + v_{\eta,t-1} + \Delta^2 v_{\theta t}\right). \end{aligned}$$

This gives a linear equation in $(v_{\eta t} + v_{\eta,t-1} + \Delta^2 v_{\theta t})$,

$$\begin{aligned} & \left(v_{\eta t} + v_{\eta,t-1} + v_{\theta t} - v_{\theta,t-2}\right) \\ = & \frac{\left(\Delta^2 cov_t^a\left(\log \hat{w}, \log \hat{h}\right)\right)^2 - \Delta^2 var_t^a\left(\log \hat{w}\right)\cdot \Delta^2 var_t^a\left(\log \hat{h}\right)}{\frac{1}{\hat{\sigma}}\cdot \Delta^2 cov_t^a\left(\log \hat{w}, \log \hat{h}\right) - \frac{1}{\hat{\sigma}^2}\cdot \Delta^2 var_t^a\left(\log \hat{w}\right) - \Delta^2 var_t^a\left(\log \hat{h}\right)}. \end{aligned}$$

Since $\{v_{\theta t}\}$ is known for the years $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2$, this equation identifies $\{v_{\eta,t} + v_{\eta,t-1}\}$ for the biannual years $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2$, as well as the sum $(v_{\eta,T} + v_{\eta,T-1} + v_{\theta,T})$.

3. Given $\{v_{\eta,t} + v_{\eta,t-1}\}$ for the biannual years $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2$ and $(v_{\eta,T} + v_{\eta,T-1} + v_{\theta,T})$, the sequence of variances of uninsurable shocks $\{v_{\omega,t} + v_{\omega,t-1}\}$ for the biannual years $t = \hat{t}, \hat{t}+2, \hat{t}+4, ..., T-2$ is identified from the growth in wage inequality:

$$\Delta^2 var_t^a\left(\log \hat{w}\right) = \left(v_{\omega t} + v_{\omega,t-1}\right) + \left(v_{\eta t} + v_{\eta,t-1} + v_{\theta t}\right) - v_{\theta,t-2}.$$

4. Consider now the cohort effects $\{v_{\alpha^0 t}, v_{\kappa^0 t}\}$ for the biannual years $t = \hat{t}, \hat{t}+2, ..., T$. The uninsurable component $v_{\alpha^0 t}$ is identified as

$$\begin{aligned} & \left(cov_t^0\left(\log \hat{w}, \log \hat{h}\right) + v_{\mu h}\right) - \frac{1}{\hat{\sigma}}\left(var_t^0\left(\log \hat{w}\right) - \left(v_{\mu y} + v_{\mu h}\right)\right) \\ = & \frac{1-\gamma}{\hat{\sigma}+\gamma} v_{\alpha^0 t} + \frac{1}{\hat{\sigma}}\left(v_{\kappa^0 t} + v_{\theta t}\right) - \frac{1}{\hat{\sigma}}\left(v_{\alpha^0 t} + \left(v_{\kappa^0 t} + v_{\theta t}\right)\right) \\ = & \left(\frac{1-\gamma}{\hat{\sigma}+\gamma} - \frac{1}{\hat{\sigma}}\right) v_{\alpha^0 t}, \end{aligned}$$

which is available for $t = \hat{t}, \hat{t}+2, ..., T$. The wage inequality for new cohorts then identify the variance of the insurable cohort effect $\{v_{\kappa^0,t}\}$:

$$var_t^0 (\log \hat{w}) = v_{\alpha^0 t} + (v_{\kappa^0 t} + v_{\theta t}) + v_{\mu y} + v_{\mu h},$$

which is available for $t = \hat{t}, \hat{t}+2, ..., T-2$ since the other components on the right-hand side are known those years. For the final year $t = T$ we can only identify the sum $(v_{\kappa^0,T} + v_{\theta T})$.

5. Finally, the cohort effects $\{v_{\widehat{\varphi},t}\}_{t=\hat{t}}^{T}$ are identified by

$$var_t^0 \left(\log \hat{h}\right) = v_{\widehat{\varphi} t} + \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)^2 v_{\alpha^0 t} + \frac{1}{\widehat{\sigma}^2} (v_{\kappa^0 t} + v_{\theta t}) + v_{\mu h}$$

for the biannual years $t = \hat{t}, \hat{t}+2, ..., T$, and by

$$var_t^1 \left(\log \hat{h}\right) = v_{\widehat{\varphi},t-1} + \left(\frac{1-\gamma}{\widehat{\sigma}+\gamma}\right)^2 (v_{\alpha^0,t-1} + v_{\omega t}) + \frac{1}{\widehat{\sigma}^2} (v_{\kappa^0,t-1} + v_{\eta t} + v_{\theta t}) + v_{\mu h},$$

available at $t = \hat{t}, \hat{t}+2, ..., T-2, T$, to identify $\{v_{\widehat{\varphi},t}\}_{t=\hat{t}}^{T}$ in the in-between years.

D Additional identifying assumptions

In the estimation, we make the following additional identifying assumptions:

1. Assume $v_{\kappa^0,T} = v_{\kappa^0,T-2}$. Given that $\{v_{\kappa^0,T} + v_{\theta,T}\}$ and $\{v_{\eta,T-1} + v_{\eta,T} + v_{\theta,T}\}$ are already identified from Corollary 2.2, this assumption identifies $v_{\theta,T}$ and $\{v_{\eta,T-1} + v_{\eta,T}\}$.

2. For $t = \hat{t}+1,\ \hat{t}+3, ..., T-1$, assume $v_{\kappa^0,t} = \frac{v_{\kappa^0,t-1} + v_{\kappa^0,t+1}}{2}$. Given this "smooth cohort effects" assumption, the moment

$$var_t^1 (\log \hat{w}) - var_t^0 (\log \hat{w}) = (v_{\alpha^0,t-1} + v_{\omega t}) + (v_{\kappa^0,t-1} + v_{\eta t}) - v_{\alpha^0 t} - v_{\kappa^0 t} \qquad \text{(D15)}$$

   for $t = \hat{t}+2,\ \hat{t}+4, ..., T$ identifies the corresponding values for $v_{\eta,t}$. Given that $\{v_{\eta,t-1} + v_{\eta t}\}$ is already identified for these years from Corollary 2.2 and Assumption 1, the corresponding values for $v_{\eta,t-1}$ are also identified.

3. For $t = \hat{t}+1,\ \hat{t}+3, ..., T-1$, assume $v_{\theta t} = \frac{v_{\theta,t-1} + v_{\theta,t+1}}{2}$.

4. Assume $v_{\omega 1} = v_{\omega 2}$.

5. Assume $v_{\eta 1} = v_{\eta 2}$.

Table 5: Parameter Estimates: Baseline Model

	$v_{\alpha^0 t}$	$v_{\kappa^0 t}$	$v_{\varphi t}$	$v_{\omega t}$	$v_{\eta t}$	$v_{\theta t}$
1967	0.127	0.023	0.445	0.003	0.001	0.024
	[0.106, 0.141]	[0.009, 0.042]	[0.399, 0.522]	[0.002, 0.004]	[0.000, 0.002]	[0.017, 0.030]
1968	0.110	0.000	0.563	0.003	0.001	0.008
	[0.080, 0.133]	[0.000, 0.000]	[0.487, 0.683]	[0.002, 0.004]	[0.000, 0.002]	[0.004, 0.013]
1969	0.112	0.000	0.538	0.000	0.007	0.010
	[0.085, 0.132]	[0.000, 0.002]	[0.466, 0.657]	[0.000, 0.000]	[0.002, 0.012]	[0.006, 0.015]
1970	0.111	0.000	0.541	0.000	0.007	0.017
	[0.085, 0.126]	[0.000, 0.020]	[0.465, 0.659]	[0.000, 0.000]	[0.002, 0.011]	[0.012, 0.021]
1971	0.099	0.009	0.507	0.000	0.005	0.015
	[0.079, 0.115]	[0.000, 0.029]	[0.439, 0.622]	[0.000, 0.005]	[0.000, 0.009]	[0.009, 0.019]
1972	0.107	0.029	0.518	0.004	0.002	0.021
	[0.087, 0.121]	[0.009, 0.053]	[0.448, 0.635]	[0.000, 0.009]	[0.000, 0.007]	[0.014, 0.026]
1973	0.106	0.029	0.537	0.000	0.004	0.025
	[0.088, 0.121]	[0.006, 0.052]	[0.474, 0.646]	[0.000, 0.006]	[0.000, 0.007]	[0.020, 0.031]
1974	0.101	0.036	0.481	0.000	0.002	0.031
	[0.084, 0.116]	[0.013, 0.060]	[0.422, 0.576]	[0.000, 0.004]	[0.000, 0.006]	[0.023, 0.037]
1975	0.097	0.024	0.455	0.005	0.003	0.026
	[0.079, 0.111]	[0.003, 0.050]	[0.397, 0.546]	[0.000, 0.011]	[0.000, 0.009]	[0.019, 0.033]
1976	0.096	0.032	0.468	0.004	0.003	0.034
	[0.079, 0.108]	[0.009, 0.057]	[0.412, 0.560]	[0.000, 0.009]	[0.000, 0.009]	[0.028, 0.041]
1977	0.089	0.021	0.506	0.001	0.008	0.029
	[0.075, 0.103]	[0.001, 0.043]	[0.446, 0.600]	[0.000, 0.008]	[0.001, 0.012]	[0.022, 0.035]
1978	0.086	0.017	0.490	0.003	0.005	0.021
	[0.072, 0.099]	[0.000, 0.038]	[0.431, 0.588]	[0.000, 0.009]	[0.000, 0.010]	[0.016, 0.027]
1979	0.090	0.013	0.499	0.008	0.008	0.024
	[0.074, 0.098]	[0.000, 0.035]	[0.443, 0.597]	[0.000, 0.014]	[0.003, 0.016]	[0.017, 0.030]
1980	0.090	0.036	0.508	0.000	0.023	0.016
	[0.077, 0.101]	[0.016, 0.057]	[0.450, 0.607]	[0.000, 0.006]	[0.016, 0.027]	[0.011, 0.022]
1981	0.089	0.045	0.493	0.000	0.014	0.021
	[0.076, 0.101]	[0.025, 0.064]	[0.435, 0.595]	[0.000, 0.000]	[0.008, 0.020]	[0.015, 0.026]
1982	0.089	0.056	0.452	0.000	0.013	0.022
	[0.075, 0.101]	[0.033, 0.078]	[0.398, 0.546]	[0.000, 0.000]	[0.007, 0.019]	[0.015, 0.029]
1983	0.104	0.065	0.453	0.022	0.004	0.026
	[0.090, 0.116]	[0.041, 0.086]	[0.399, 0.547]	[0.014, 0.029]	[0.000, 0.013]	[0.019, 0.033]
1984	0.120	0.072	0.463	0.023	0.000	0.030
	[0.105, 0.131]	[0.049, 0.096]	[0.405, 0.567]	[0.014, 0.030]	[0.000, 0.008]	[0.021, 0.038]
1985	0.119	0.081	0.491	0.008	0.007	0.035
	[0.106, 0.132]	[0.058, 0.104]	[0.425, 0.592]	[0.000, 0.018]	[0.000, 0.014]	[0.028, 0.043]
1986	0.124	0.078	0.518	0.008	0.004	0.034
	[0.108, 0.137]	[0.057, 0.100]	[0.448, 0.625]	[0.000, 0.016]	[0.000, 0.011]	[0.027, 0.043]
1987	0.123	0.079	0.523	0.008	0.004	0.035
	[0.110, 0.134]	[0.058, 0.099]	[0.458, 0.628]	[0.000, 0.018]	[0.000, 0.010]	[0.029, 0.042]
1988	0.122	0.078	0.488	0.007	0.000	0.039
	[0.109, 0.132]	[0.059, 0.099]	[0.424, 0.593]	[0.000, 0.013]	[0.000, 0.003]	[0.031, 0.046]
1989	0.112	0.072	0.496	0.000	0.000	0.039
	[0.099, 0.123]	[0.053, 0.093]	[0.431, 0.599]	[0.000, 0.003]	[0.000, 0.000]	[0.032, 0.045]

Table 5: (Continued) Parameter Estimates: Baseline Model

	$v_{\alpha^0 t}$	$v_{\kappa^0 t}$	$v_{\varphi t}$	$v_{\omega t}$	$v_{\eta t}$	$v_{\theta t}$
1990	0.114	0.052	0.506	0.000	0.000	0.050
	[0.100, 0.125]	[0.030, 0.076]	[0.440, 0.612]	[0.000, 0.000]	[0.000, 0.000]	[0.041, 0.058]
1991	0.110	0.056	0.475	0.001	0.002	0.048
	[0.098, 0.123]	[0.033, 0.080]	[0.410, 0.578]	[0.000, 0.008]	[0.000, 0.007]	[0.039, 0.055]
1992	0.126	0.042	0.396	0.022	0.000	0.057
	[0.110, 0.136]	[0.020, 0.066]	[0.339, 0.488]	[0.010, 0.027]	[0.000, 0.007]	[0.048, 0.066]
1993	0.126	0.035	0.468	0.000	0.000	0.072
	[0.110, 0.138]	[0.015, 0.060]	[0.398, 0.573]	[0.000, 0.000]	[0.000, 0.000]	[0.061, 0.083]
1994	0.127	0.029	0.451	0.002	0.004	0.051
	[0.111, 0.140]	[0.006, 0.053]	[0.377, 0.558]	[0.000, 0.009]	[0.000, 0.009]	[0.042, 0.062]
1995	0.120	0.046	0.484	0.002	0.000	0.064
	[0.106, 0.135]	[0.018, 0.069]	[0.404, 0.600]	[0.000, 0.009]	[0.000, 0.000]	[0.052, 0.073]
1996	0.107	0.034	0.541	0.000	0.000	0.050
	[0.092, 0.122]	[0.005, 0.058]	[0.455, 0.666]	[0.000, 0.000]	[0.000, 0.000]	[0.041, 0.059]
1997	0.143	0.033	0.573	0.032	0.000	0.053
	[0.120, 0.158]	[0.006, 0.059]	[0.484, 0.712]	[0.011, 0.043]	[0.000, 0.005]	[0.042, 0.062]
1998	0.131	0.033	0.587	0.004	0.002	0.055
	[0.115, 0.145]	[0.006, 0.060]	[0.508, 0.713]	[0.000, 0.022]	[0.000, 0.007]	[0.044, 0.064]
1999	0.117	0.029	0.611	0.006	0.004	0.067
	[0.099, 0.131]	[0.003, 0.055]	[0.521, 0.740]	[0.000, 0.013]	[0.000, 0.009]	[0.055, 0.077]
2000	0.110	0.024	0.595	0.000	0.000	0.079
	[0.094, 0.127]	[0.000, 0.049]	[0.514, 0.726]	[0.000, 0.009]	[0.000, 0.004]	[0.067, 0.090]
2001	0.121	0.031	0.575	0.021	0.005	0.086
	[0.097, 0.142]	[0.007, 0.057]	[0.491, 0.717]	[0.000, 0.034]	[0.000, 0.013]	[0.073, 0.098]
2002	0.114	0.039	0.653	0.013	0.000	0.093
	[0.094, 0.131]	[0.014, 0.064]	[0.552, 0.809]	[0.000, 0.028]	[0.000, 0.009]	[0.078, 0.107]
2003	0.079	0.050	0.662	0.000	0.000	0.090
	[0.058, 0.102]	[0.022, 0.081]	[0.528, 0.847]	[0.000, 0.010]	[0.000, 0.000]	[0.078, 0.104]
2004	0.111	0.061	0.655	0.043	0.000	0.087
	[0.080, 0.134]	[0.030, 0.098]	[0.498, 0.868]	[0.028, 0.050]	[0.000, 0.000]	[0.077, 0.101]
2005	0.100	0.061	0.500	0.007	0.008	0.059
	[0.100, 0.100]	[0.030, 0.098]	[0.500, 0.500]	[0.000, 0.021]	[0.005, 0.009]	[0.048, 0.069]
2006	0.100	0.061	0.500	0.000	0.008	0.031
	[0.100, 0.100]	[0.030, 0.098]	[0.500, 0.500]	[0.000, 0.013]	[0.005, 0.009]	[0.020, 0.036]

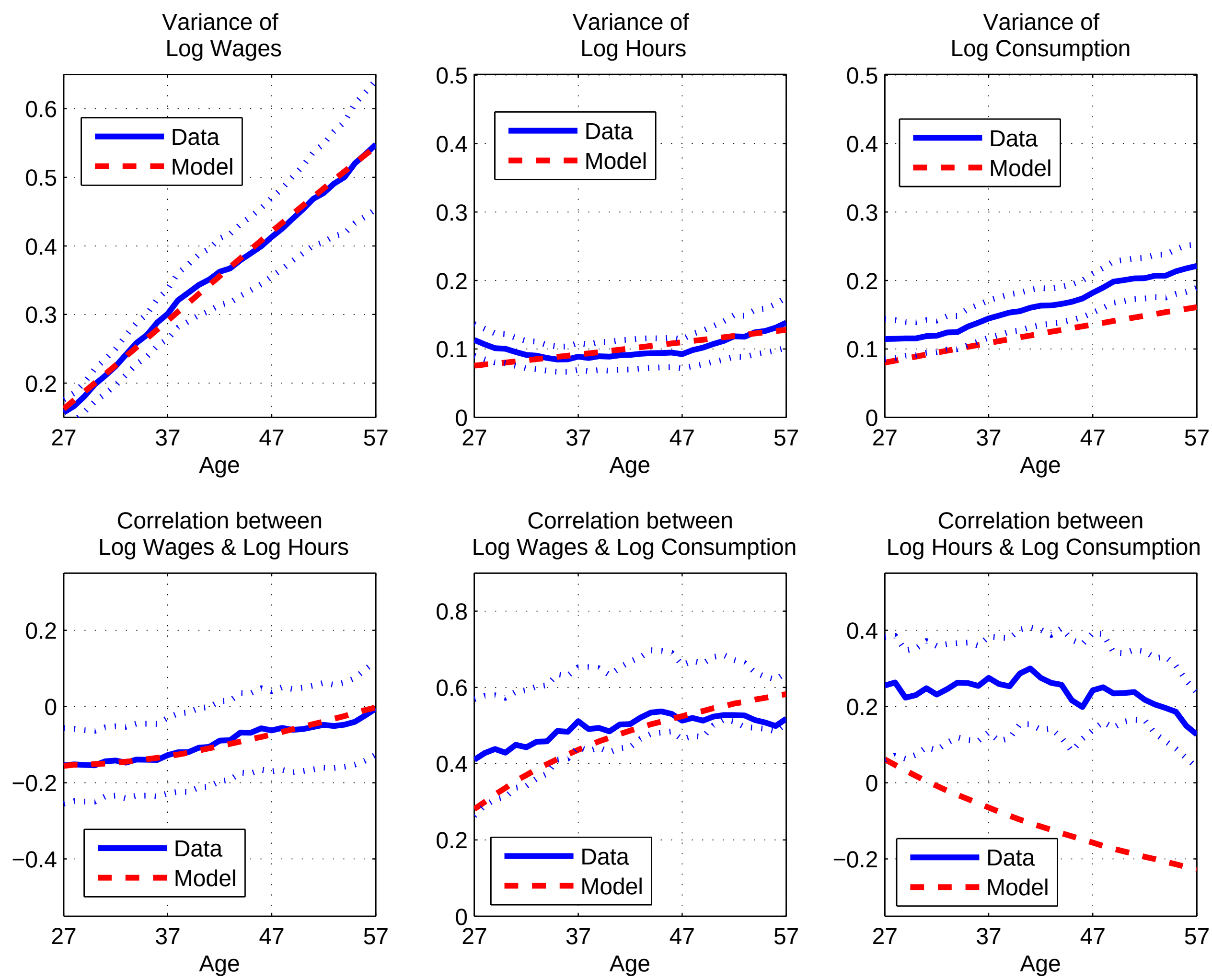

Figure 7: Estimation without CEX data. Data and model fit for moments in levels along the age dimension. Dotted lines denote 90–10 bootstrapped confidence intervals for the empirical moments.

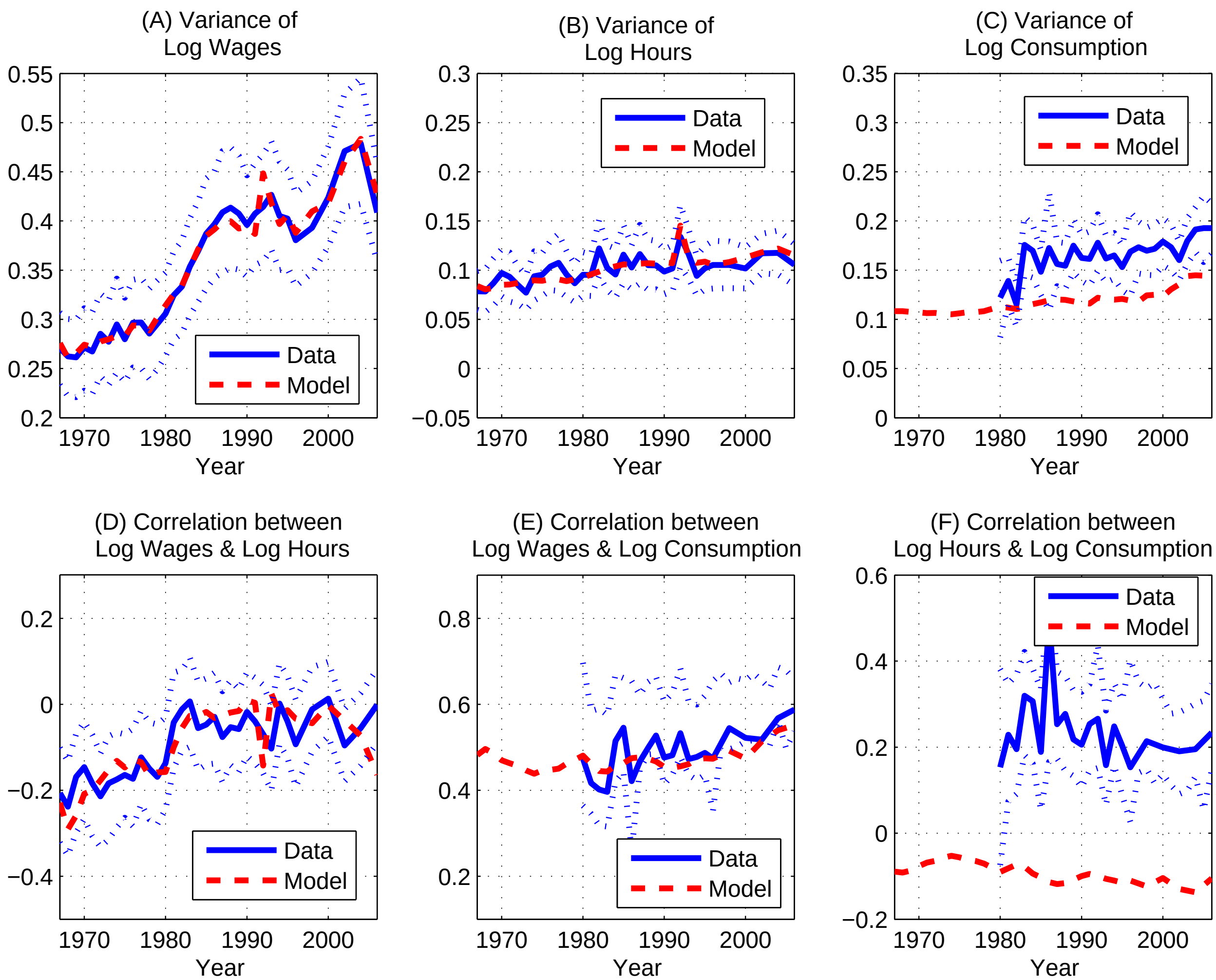

Figure 8: Estimation without CEX data. Data and model fit for moments in levels along the time dimension. Dotted lines denote 90–10 bootstrapped confidence intervals for the empirical moments.