International Finance Exam

November 8, 2005

1. Consider the following representative-agent small-open-economy endowment economy.

   The representative household receives a stochastic endowment y_t each period that can take one of two values, $y_t \in [0, 1]$. The endowment is drawn from the same distribution each period, where $pr(y = 0) = pr(y = 1) = \frac{1}{2}$.

   The representative household trades with a representative competitive (ie price-taking) risk-neutral lender. The lender can earn a safe return $r > 0$ on world capital markets.

   Suppose that the assets traded are two one-period state-contingent bonds. One of the bonds pays one unit of consumption at $t + 1$ if and only if $y_{t+1} = 0$, and the other pays one unit iff $y_{t+1} = 1$. Neither the household nor the lender is allowed to play Ponzi games.

   Suppose initial household wealth at the start of period 0 before any shocks are drawn is equal to zero. Let $y^t = (y_0, y_1, ..., y_t)$ denote the history up to and including t. Let $\pi(y^t)$ denote the time zero probability of some history y^t.

   Suppose the period utility function for the household is

   $$u(c) = (1 - \beta)\sqrt{c}$$

   and that households discount at rate $\beta = 1/(1 + r)$. Let $q^0(y^t)$ and $q^1(y^t)$ denote the price of the two assets at y^t and $b^0(y^t)$ and $b^1(y^t)$ denote the quantities purchased

   (a) Define the representative household's problem and the describe the equations that characterize the solution to this problem

   (b) Define a competitive equilibrium for this economy

   (c) What are the equilibrium values for $c(y^t)$, $b^0(y^t)$, $b^1(y^t)$, $q^0(y^t)$ and $q^1(y^t)$ for any y^t?

2. Now suppose that there is a single risk-averse agent (household) facing the income process described above, and a single risk-neutral agent (lender) with no income. Assume now that the lender does not have access to world capital markets. Assume that the household discounts future utility at rate β and that the lender discounts future income (profits) at the same rate. Suppose allocations are chosen by a planner. Let $c(y^t)$ denote the resources given to the household in y^t and $p(y^t)$ denote resources given to the lender. Suppose the planner faces the constraint $c(y^t) \geq 0$, while there is no non-negativity constraint on $p(y^t)$. Suppose that at any point either the household or the lender can reject the planner's proposed allocation. In the event of default, in the current and all future periods the household will get y_t and the lender will get zero

   (a) Write down the planner's problem, assuming that the planner puts relative weight λ on welfare of the household and $1-\lambda$ on the lender. Be sure to include any constraints that must be satisfied.

   (b) Is $c(s^t) = \frac{1}{2}$ for all t and s^t a feasible allocation for all $\beta \in (0,1)$ for some $\beta \in (0,1)$, or for no $\beta \in (0,1)$? Explain your answer.

   (c) For what values for β does there exist a $\overline{c}$ such that $c(y^t) = \overline{c}$ for all t and y^t is a feasible choice?

   (not very helpful hint: In which state is the lender most tempted to default, and in which state is the borrower most tempted to default?)