Dollarization and Financial Integration

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Abstract

This paper builds a simple theoretical model designed to study the effects of dollarization on financial conditions. We compare allocations under two regimes: a flexible exchange rate regime and a dollarized regime. Under flexibility, each period a benevolent government decides how much to borrow or lend on an international bond market and the devaluation rate. Under dollarization the government decides only its debt policy. In equilibrium, international borrowing is limited endogenously such that the government always chooses to repay when the penalty for default is permanent future exclusion from financial markets. Dollarization implies the loss of the devaluation rate as a policy instrument, but may still be optimal. The reason is that floating defaulters can use the devaluation rate as a substitute for debt in responding to country-specific shocks while dollarized economies in default find themselves in a more uncomfortable situation. Thus dollarization reduces a government's incentives to default, and thereby increases a country's ability to borrow in equilibrium. We quantify these gains by calibrating the model to three countries for which dollarization has been implemented or considered: Mexico, El Salvador and Ecuador, and find that El Salvador features the greatest gains from dollarization.

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1 Introduction

The recurrence of currency and financial crises in emerging markets has generated an intense debate on the appropriate exchange rate regime. Dollarization has attracted special attention, in part because of the recent official dollarization of Ecuador and El Salvador.[1]

The most important difference between dollarizing and simply pegging the exchange rate is that dollarizing represents more permanent restrictions on domestic monetary policy. Thus thinking about dollarization leads one to thinking about what a government might have to gain by tying its hands more tightly with regards to monetary policy, which in turn leads to the issue of credibility. In discussing papers in a conference volume on the topic of dollarization, Sargent (2001) writes

> *"In their papers and verbal discussions, proponents of dollarization often appealed to commitment and information problems that somehow render dollarization more credible and more likely to produce good outcomes. Those proponents presented no models of how dollarization was connected with credibility. We need some models."*

In this paper we explore one avenue via which dollarization may increase credibility. In particular, we explore a model in which dollarization enhances a borrowing government's credibility in international financial markets, and thereby increases international financial integration.

The basic idea is as follows. Emerging-markets economies are typically subject to big shocks, large fractions of government revenue are linked to volatile commodity prices, and raising tax rates often increases evasion and substitution towards the informal sector. Thus traditional sources of government revenue are often volatile and difficult to adjust. In this context seigniorage is a valuable fiscal instrument, since extra money can rapidly be printed as required.

At the same time, emerging-markets economies also issue debt to smooth fluctuations and to ease temporary liquidity problems. Dollarization can help strengthen fragile sovereign debt markets by increasing borrowers' incentives to repay loans. The reason is that debt and seigniorage are partial substitutes as revenue sources, so that an economy with an independent monetary policy is prone to default, even if this means future exclusion from credit markets. Dollarization makes the ability to access international debt markets more valuable, reducing the likelihood of default and thereby loosening credit constraints.

We study a small open endowment economy populated by consumers, firms and a government. Consumers work for firms, and each period they produce a stochastic quantity of goods that can be used for private or public consumption. Firms sell these goods in exchange for cash. Once the goods market has closed, firms pay their workers. Thus the cash that consumers spend on goods in the current period must be carried over from the previous period. The government

[1] Dollarization in the broad sense of unilaterally adopting a stronger foreign currency such as the U.S. Dollar, Euro, or Yen.

is not subject to a cash-in-advance constraint: as long it has control over monetary policy it can print new money after observing firm output and spend it immediately to purchase goods that will be provided publicly. The government is benevolent and rational, and seeks to maximize the utility of a representative consumer. In addition to revenue from seigniorage, the government also trades one period bonds in international financial markets at a constant real interest rate. However, the government cannot commit to repay international debts; contracts must be self-enforcing. Thus foreign creditors set borrowing limits such that the government always has the incentive to honor its obligations, where the penalty in case of default is permanent financial autarky.

In addition to output shocks, the economy is also subject to taste shocks in the form of changes to the relative taste for publicly versus privately-provided consumption. Taking as given the process for shocks, the government chooses one of two possible exchange rate regimes: flexibility or dollarization. Under flexibility, the government sets the money growth rate and implicitly the inflation and devaluation rate period-by-period. Under dollarization, the inflation rate is constant and equal to zero, and the government receives no revenue from seigniorage. Thus once dollarized the government's only policy instrument is its international debt position. The dollarization decision is assumed to be irreversible [2] For simplicity, the government does all the international borrowing and lending in the economy.

To assess quantitatively the effect that dollarization has on financial integration, we calibrate our model to three countries: Mexico, El Salvador and Ecuador so that the flexible exchange rate economy replicates some key features the economies. We calibrate to these countries because they have been frequently been discussed as a candidates for dollarization with El Salvador and Ecuador having in fact implemented dollarization. We find that large taste shocks are required to account for the fact that government expenditure (and government consumption) is much more volatile than GDP. The model does a good job in terms of replicating some key empirical correlations in all three countries, such as the countercyclicality of inflation and the positive correlation between government consumption and inflation.

By comparing the flexible regime to the dollarized regime, we find that the dollarized economy exhibits looser borrowing constraints and less frequent debt crises, identified as periods in which the borrowing constraint is binding. We find that El Salvador features the greatest gains from dollarization in terms of better financial conditions. In our model dollarization has the greatest impact on financial integration when the economy's fluctuations come in large part by varying government needs as opposed to varying output. The reason is that debt policy is a good substitute for the inflation policy when inflation is being used to finance government shocks and thus dollarization has the least costs in this case. We find evidence that El Salvador

[2] In reality, it is presumably possible, though costly to de-dollarize a dollarized economy. However, the key difference between dollarization and a fixed exchange rate regime is precisely that under dollarization it is more costly to allow the exchange rate to float.

faces larger fluctuations coming from government shocks and thus dollarization has its greatest net benefit.

>In order to address welfare issues, we then move to consider the version of the model in which the economy starts out under a floating regime, and each period the government decides whether or not to dollarize. We find that dollarization is the optimal regime choice when taste shocks are relatively large to productivity shocks, and when the government's international debt becomes sufficiently large.

1.1 Historical experience

Our model predicts that dollarizing economies should exhibit greater international borrowing ability - which may by manifested either as an increase in international borrowing or as a decline in risk-premia - in addition to declines in the level and volatility of inflation. There is not too much direct historical evidence which can be used to test the model, but the experience of the small number of countries that have dollarized appears to be broadly consistent with the predictions of our theory.

Ecuador dollarized in 2000 in the midst of a severe economic crisis with a collapsing banking system, a sliding local currency, and after defaulting on its Brady bonds late in 1999. The regime was implemented in an attempt to reduce inflation, bring stability to the economy, and gain credibility with international investors. Since dollarization, Ecuador's inflation has been significantly reduced to single digits and the country has been able to renegotiate its debts at somewhat lower interest rates.

Figure 1 shows the yields on Ecuadorian government bonds denominated in dollars that are traded internationally , (JP Morgan Emerging Market Bond Index for Ecuador), together with the yield on US 20 year government bonds. The difference in these two yields is a common measure of default risk. The figure shows that default risk increased significantly in 1998 precluding the 1999 crisis and default. In July 2000 the yields came down again after Ecuador dollarized and renegotiated its debt. Figure 2 shows the yields on domestic Ecuadorian government bonds denominated in dollars and in sucres (Ecuador's domestic currency before dollarization). The difference between these two yields is a broad measure of devaluation risk because both types of bonds are traded locally, have similar terms and conditions, and the difference is the currency denomination.[3] The figure shows that devaluation risk increased dramatically in 1998, and it collapsed to zero when the economy dollarized. A key feature from these figures is that devaluation risk and default risk move together as periods with high default probabilities are associated with expectations of devaluations.

El Salvador implemented its dollarization plan in 2001 more in an effort to attract foreign

[3]Sucre and Dollar Bond yields are on 4 to10 year maturity bonds. In Ecuador, the domestic market for government bonds is thin, and due to the lack of data we grouped the longer maturity bonds available to report the time series of yields.

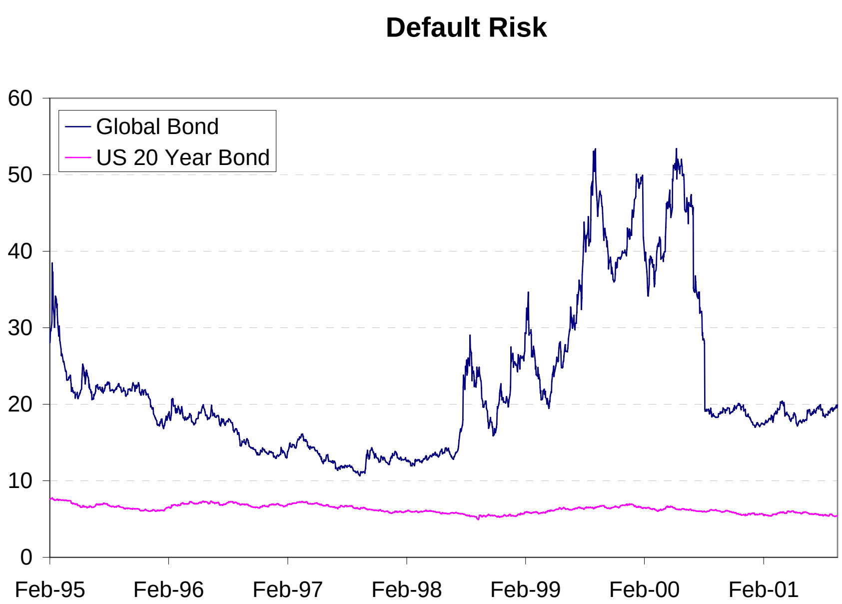

Figure 1:

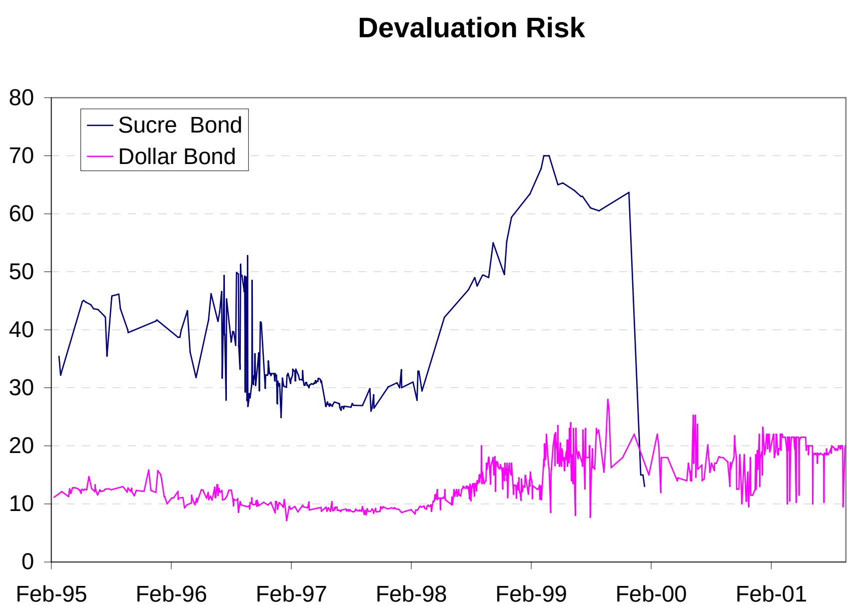

Figure 2:

investors than to stabilize inflation; the Colon had been pegged to the dollar since 1994. In January 2001, the currency began to be phased out and today the dollar is the only unit of account. The most notable benefit that Salvadoreans have enjoyed post-dollarization is lower interest rates. In fact the day after El Salvador adopted the new currency, the interest rate on consumer loans and mortgages fell from 17 to 11 percent. Consumer credit is growing, and companies and the government have been helped by cheaper international financing.

Panama is the nation that has been dollarized the longest, since 1904. Goldfajn and Olivares (2000) document that Panama has had lower inflation rates than other countries in Latin America. Panama is also strongly integrated in international financial markets, with many foreign banks operating in the country. They report that foreign banks charge lower interest rates on loans than domestic banks.[4]

1.2 Related literature

Proponents of dollarization argue that dollarizing offers other benefits, in addition to greater credibility in international financial markets. One is the greater integration in goods and asset markets that may follow from reducing exchange rate uncertainty and transaction costs. For example, Mendoza (2000) argues that dollarization can deliver substantial welfare gains by eliminating distortionary uncertainty over the duration of stabilization policies, though he does not model the source of this lack of policy credibility explicitly (see also Calvo 1999 and Berg and Borensztein 2000). A second argument in favor of dollarization is that it eliminates the possibility of currency crises. In particular, dollarization solves the 'fear of floating' problem (Calvo 2000) which arises when international liabilities are denominated in dollars and currency devaluations can therefore precipitate debt crises. Third, there is empirical evidence that dollarization brings lower and less volatile inflation to countries adopting a stronger currency (Edwards 2001).

One interpretation of the high and volatile inflation rates in some emerging-markets economies is that these countries face more severe time-consistency problems in setting monetary policy than countries whose currencies are being adopted (see, for example, Cooper and Kempf 2001). Another view is that high inflation rates emerge when countries perceive control of the printing press as an opportunity for beggar-thy-neighbor policy. Thus Cooper and Kempf (2003) build a model in which inflation acts as a tax on foreigners wishing to purchase domestic goods, prompting competitive governments to choose inefficiently-high inflation rates in equilibrium. Similarly, Cooley and Quadrini (2000) argue that Mexico may prefer a higher inflation rate than the U.S. because higher nominal interest rates can have favorable effects on the terms of trade.

In each of these three cases, dollarization (supposing it can somehow be achieved) might be

[4] Other dollarized economies include East Timor, Guatemala, Kosovo and other small countries, and nations such as Bulgaria, Costa Rica and Honduras are also considering this option.

viewed as increasing monetary policy credibility, in the sense of lowering the equilibrium inflation rate. In this paper we pursue a different explanation for inflation dynamics in emerging-markets economies: episodes of high and volatile inflation simply reflect periods during which less distorting sources of revenue are not available. Thus in our model, policy is always time-consistent and the devaluation rate does not have any beggar-thy-neighbor effects, but dollarizing still reduces the mean and the variance of the inflation rate and the devaluation rate.

Among the costs of dollarization that are often cited are the loss of seigniorage revenues and the inability to respond to external shocks with monetary policy (see, for example, Schmitt-Grohe and Uribe 2001). For developed economies, the advantage of monetary independence is usually expressed in terms of the ability to adjust the short-run real interest rate or the real exchange rate in response to country-specific shocks. For emerging-markets economies, there is often a simpler reason to want to retain the ability to print one's own currency - seigniorage can be an important and flexible source of government revenue. We will document the importance of seigniorage revenue for Mexico and other emerging-markets economies later in the paper. Canzoneri and Rogers (1990) explore the importance of seigniorage in the European Union, and find that the optimal inflation rate is country-specific depends on differences in the efficiency of tax collection systems across countries.

Sims (2002) argues that dollarization is costly because it prevents the economy from issuing (state-contingent) nominal debt, without affecting dollar interest rates. However, governments in emerging markets are largely unable to issue external debt in their own currency, no matter what exchange rate regime they have, so it is not clear that this constitutes a strong argument against dollarization in practice.

2 Model

The economy is populated by a large number of identical, infinitely-lived consumers, a representative firm, and a government. In each period $t = 0, 1, ...$ the economy experiences one of finitely-many events s_t. We denote by $s^t = (s_0, ...s_t)$ the history of events up to and including period t. The probability, as of date 0, of a particular history s^t is $\phi(s^t)$. The initial realization s_0 is given.

2.1 Firms

The representative firm in the economy produces a stochastic amount of output $y(s^t)$ at date t given history s^t. This good can be converted into a private consumption good $c(s^t)$ or a public consumption good, $g(s^t)$. Output is produced at the start of the period, and then allocated between consumers and the government in a cash market in the middle of the period. At the end of the period, firms pay their workers (consumers) nominal wages $w(s^t)$.

2.2 Consumers

Consumers are infinitely-lived, discount at rate β, and derive utility from both privately and publicly provided consumption goods. Expected lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \phi(s^t) u(c(s^t), g(s^t), \lambda(s^t)) \tag{1}$$

Period utility takes the following separable form:

$$u(c(s^t), g(s^t), \lambda(s^t)) = \frac{1}{1-\gamma} \left[\lambda(s^t) c(s^t)^{1-\gamma} + (1-\lambda(s^t)) g(s^t)^{1-\gamma}\right] \qquad 0 < \lambda(s^t) < 1,$$

where $\lambda(s^t)$ is a stochastic taste shock. We view $\lambda(s^t)$ as capturing changes through time in household preferences for public versus private goods, or changes in the taste for the allocation mechanism (government provision versus market provision). One possible manifestation of these changes in taste would be electoral cycles in which populist free-spending governments and more fiscally conservative market-oriented governments take turns in power. We will assume that $s_t = (y_t, \lambda_t)$ evolves according to a first-order Markov process.

The representative consumer enters the period with money savings from the previous period $n^s(s^{t-1})$ and wages from the previous period $w(s^{t-1})$. He observes the endowment shock $y(s^t)$, the taste shock $\lambda(s^t)$, and the price level $P(s^t)$. He then decides how much of his money to spend, subject to the cash-in-advance constraint and the budget constraint:

$$P(s^t) c(s^t) \leq n^s(s^{t-1}) + w(s^{t-1}) \equiv n(s^{t-1}) \tag{2}$$

$$n^s(s^t) = n(s^{t-1}) - P(s^t) c(s^t) \tag{3}$$

where $n(s^{t-1})$ denotes total nominal balances carried into period t. Note that these two constraints jointly imply $n^s(s^t) \geq 0$. Let

$$n^p(s^t) \equiv n(s^{t-1}) - n^s(s^t)$$

denote money used for purchases by the household at date t.

2.3 Household Problem

The household problem is to choose the sequence of money savings $n^s(s^t)$ and consumptions $c(s^t)$ to maximize expected lifetime utility (eq 1) subject to the cash-in-advance constraint (eq. 2), the budget constraint (eq. 3) and a non-negativity constraint on consumption, $c(s^t) \geq 0$, taking as a given a complete set of date and state contingent endowments $y(s^t)$, taste shocks

$\lambda(s^t)$, wages $w(s^t)$, prices $P(s^t)$, probabilities $\phi(s^t)$, and initial money holdings $n(s_{-1})$:

The inter-temporal first order condition for the household's problem is

$$\begin{aligned} \phi(s^t)u_1(c(s^t),g(s^t),\lambda(s^t)) &\geq \beta \sum_{s_{t+1}} \left[\phi(s^t,s_{t+1})\left(\frac{u_1(c(s^t,s_{t+1}),g(s^t,s_{t+1}),\lambda(s^t,s_{t+1}))}{\pi(s^t,s_{t+1})}\right)\right] \quad (4) \\ \text{with equality if } n^s(s^t) &> 0 \end{aligned}$$

where $\pi(s^t,s_{t+1})$ denotes the gross inflation rate when next period's state is s_{t+1}.

The transversality condition is

$$\lim_{s^t \to s^\infty} \beta^t \sum_{s^t} \phi(s^t)u_1(c(s^t),g(s^t),\lambda(s^t))\frac{n^s(s^t)}{P(s^t)} = 0. \quad (5)$$

2.4 Government

The government is the only actor in the economy with access to a competitive international bond market.[5] In the bond market the government can borrow and lend one-period real bonds. We assume that international lenders can decide whether to lend, how much to lend, and at what price to lend. However they cannot make the price of loans contingent on the borrowing government's current net bond position, on net bond purchases, or on the shocks that will hit the economy in the next period. Thus asset markets are far from complete. However, the assumed market structure seems appropriate for emerging-markets economies, for whom international borrowing must typically be repaid at non-contingent dates in non-contingent numbers of U.S. dollars.

International debt contracts are not externally enforceable. We assume that lenders can commit to honor their debt contracts, but the domestic government cannot commit not to default on its debt obligations if it is an international net debtor. If it defaults, creditors are assumed to credibly punish the government by permanently excluding it from the bond market: a government that has defaulted in the past can neither buy nor sell bonds.

International lenders in the bond market can earn a safe real return r on the world market. Given the assumptions that bond prices must be non-contingent and that the market for international loans is competitive, all lenders sell bonds at the same price $1/(1+r)$. The only way that lenders can assure repayment is by rationing credit.[6] Thus lenders impose endogenous borrowing constraints on the government such that no borrowing occurs beyond the point at

[5] In reality, non-government international trade in financial assets is growing, but it is still the case that most external debt in countries where dollarization is considered a possibility represents government borrowing. For example, as of March 2003, Argentina government debt accounts for 67% of the total stock of foreign debt; in Mexico, it accounts for 56% and in Ecuador it accounts for 73%.

[6] One could imagine an alternative market structure in which lenders offer a menu of contracts, each of which specifies a loan amount and an interest rate. Contracts for greater loan amounts would then be associated with higher interest rates to compensate for greater risk of default. In equilibrium the unconditional expected return to the lender would be equalized across contracts. This market structure is adopted in Arellano, 2003.

which the probability of default in the subsequent period is positive.

2.5 Government Problem when Floating

At time zero, the government in the flexible regime decides on a policy $\Lambda = \{g(s^t), B(s^t), \mu(s^t)\}$ which defines government consumption, asset holdings $B(s^t)$, and the gross money growth rate $\mu(s^t)$ for all t and for all s^t given some initial assets B_{-1}.

The government is not subject to a cash-in-advance constraint. Thus it can print new money $D(s^t)$ within the period after observing $y(s^t)$ and $\lambda(s^t)$ and use this money immediately to help finance public consumption, $g(s^t)$. Let $M(s^t)$ denote the aggregate stock of money in circulation at the end of period t. Thus the money growth rate $\mu(s^t)$ is equal to

$$\mu(s^t) = \frac{M(s^t)}{M(s^{t-1})} = \frac{M(s^{t-1}) + D(s^t)}{M(s^{t-1})}.$$

In addition to seigniorage and revenue from international borrowing, the government can also seize a constant fraction of the endowment τ directly. This can be interpreted as a constant tax on private-sector output, or as the government producing fraction τ of output. Thus the government nominal budget constraint, prior to default, is given by:

$$P(s^t)g(s^t) = \tau P(s^t)y(s^t) + D(s^t) - P(s^t)B(s^t) + (1+r)P(s^t)B(s^{t-1}) \tag{6}$$

where $B(s^t)$ are real riskless foreign assets purchased at s^t

In addition, the state-contingent borrowing constraint requires bonds purchased to exceed some (typically negative) state-contingent limit:

$$B(s^t) \geq \underline{B}(s^t) \tag{7}$$

The government is allowed to default at any date. If the government chooses to default at s^t, for all histories s^r consistent with s^t the government budget constraint becomes

$$P(s^t)g(s^t) = \tau P(s^t)y(s^t) + D(s^t). \tag{8}$$

However, as noted above the lenders will not lend beyond the point at which the default probability becomes positive. We assume that the constraints $\underline{B}(s^t)$ are tight enough to deter the government from ever defaulting in equilibrium, but "not too tight". They are not too tight in the sense that if the history is s^t and the start-of-period assets are $\underline{B}(s^t)$, the government is exactly indifferent between repaying and defaulting. We will return to the not-too-tight condition when we define an equilibrium.

2.6 Government Problem when Dollarized

The problem for the government in a dollarized economy differs from the one described above in two respects. First, the money growth rate is not a domestic policy instrument. We assume that in the dollarized economy, a foreign government (the United States) conducts monetary policy to target some path for the foreign inflation rate. Assuming the law of one price holds, the domestic price and inflation rate will equal their foreign counterparts at every date. We assume that the foreign government retains any seigniorage it collects from supplying cash to the domestic economy. Since printing new cash is not a source of revenue for the domestic government, the term $D(s^t)$ drops out of the government budget constraints pre and post default, eqs. 6 and 8. When we compare the flexible and dollarized economies we will assume that the foreign government targets a constant inflation rate equal to zero.

The second difference is that the maximum amount of borrowing allowed at a point in time, $\underline{B}(s^t)$, will differ across the flexible and dollarized regimes. This is because default incentives will vary with the exchange rate regime. In particular, if debt and seigniorage are good substitutes for financing temporary spending needs, then borrowing constraints will likely be looser in the dollarized economy. The reason is simply that the punishment for default (loss of the debt instrument) is painful in the dollarized economy when monetary policy cannot be used to substitute for debt, but less painful in the flexible regime, where post-default seigniorage can be used in place of debt in periods when the government needs revenue.

2.7 Some Equilibrium Relationships

The relationships described below apply to both the flexible and dollarized economies. The difference between the two regimes is that under flexibility new money in local circulation $D(s^t)$ is controlled by the domestic government and the associated seigniorage appears in the domestic government budget constraint. In the dollarized economy, $D(s^t)$ is controlled by the foreign government, and associated seigniorage does not appear in the domestic government budget constraint.

At the end of the period, firms pay as wages all the cash they hold. Thus

$$w(s^t) = P(s^t)(1-\tau)y(s^t)$$

The market clearing condition for the cash goods market is

$$P(s^t)(1-\tau)y(s^t) = n^p(s^t) + D(s^t) \tag{9}$$

If $D(s^t)$ is negative, the interpretation under flexibility is that the government is borrowing goods abroad, selling them on the domestic market, and taking the money it receives in exchange out of circulation. The interpretation under dollarization is that the foreign government is selling goods in exchange for dollars.

In equilibrium, all the money in the economy at the end of the period (after firms have paid wages) is held by households, so

$$M(s^t) = M(s^{t-1}) + D(s^t) = n(s^t)$$

Note that if households do no money saving ($n^s(s^t) = 0$) we get the standard quantity equation with velocity equal to one:

$$P(s^t)(1-\tau)y(s^t) = n(s^{t-1}) + D(s^t) = M(s^t).$$

>From the goods market clearing condition, the price level is given by

$$P(s^t) = \frac{n^p(s^t) + D(s^t)}{(1-\tau)y(s^t)} = \frac{M(s^t) - n^s(s^t)}{(1-\tau)y(s^t)}. \tag{10}$$

Substituting $P(s^t)$ into the consumer's budget constraint, eq. 3, gives

$$\begin{aligned} n^s(s^t) &= n(s^{t-1}) - \left(\frac{n^p(s^t) + D(s^t)}{(1-\tau)y(s^t)}\right) c(s^t) \\ c(s^t) &= (1-\tau)y(s^t)\left(\frac{n^p(s^t)}{n^p(s^t) + D(s^t)}\right) = (1-\tau)y(s^t)\left(\frac{M(s^{t-1}) - n^s(s^t)}{M(s^t) - n^s(s^t)}\right) \end{aligned}$$

Note that if the household is not doing any money saving ($n^s(s^t) = 0$), then $c(s^t) = (1-\tau)y(s^t)/\mu(s^t)$.

By printing money, the government (domestic or foreign) controls:

$$\widehat{\mu}(s^t) \equiv \frac{M(s^t) - n^s(s^t)}{M(s^{t-1}) - n^s(s^t)} \tag{11}$$

where $\widehat{\mu}(s^t)$ is the effective gross tax rate on consumption.

The inflation rate is given by

$$\pi(s^{t+1}) = \frac{P(s^{t+1})}{P(s^t)} = \left(\frac{M(s^{t+1}) - n^s(s^{t+1})}{M(s^t) - n^s(s^t)}\right)\frac{y(s^t)}{y(s^{t+1})} = \frac{\widehat{\mu}(s^{t+1})}{\widehat{\mu}(s^t)}\frac{n^p(s^{t+1})}{n^p(s^t)}\frac{y(s^t)}{y(s^{t+1})} \tag{12}$$

Thus inflation is related to the growth rates of the effective consumption tax rate, the growth rate of cash used to purchase goods, and the growth rate of output.

At the end of period t there are some nominal dollars in the hands of households, $M(s^t)$. It is clear, however, that the dollar face value of this money is not going to be allocative. In particular, let $x(s^t)$ denote the fraction of cash on hand that agents save at s^t :

$$x(s^t) = \frac{n^s(s^t)}{n(s^{t-1})}$$

We can now express all real variables in terms of the sequences $y(s^t)$, $\lambda(s^t)$, $B(s^t)$, $\mu(s^t)$ and $x(s^t)$, with no reference to nominal variables $M(s^t)$, $P(s^t)$ or $D(s^t)$.

In particular, from 12 the real return to money saving is given by

$$\pi(s^{t+1}) = \frac{P(s^{t+1})}{P(s^t)} = \left(\frac{\mu(s^{t+1}) - x(s^{t+1})}{\mu(s^t) - x(s^t)}\right)\mu(s^t)\frac{y(s^t)}{y(s^{t+1})} \tag{13}$$

Note that if the money savings rate $x(s^t) = 0$ then inflation does not depend on the current money growth rate $\mu(s^t)$. If $x(s^t) > 0$ then increasing $\mu(s^t)$ (while holding constant future money growth rates) will have a weaker impact on inflation, since households will respond to faster money growth by reducing savings, driving up the price level at date t (see eq. 10) and thereby reducing inflation between t and $t + 1$.

$>$From 11 the effective gross tax rate on consumption is

$$\widehat{\mu}(s^t) = \frac{\mu(s^t) - x(s^t)}{1 - x(s^t)}$$

$>$From the household's budget constraint 3 and the goods market clearing condition 9 we can express consumption, $c(s^t)$, real balances and real money savings as

$$c(s^t) = \left(\frac{1 - x(s^t)}{\mu(s^t) - x(s^t)}\right)(1-\tau)y(s^t) \tag{14}$$

$$\frac{M(s^t)}{P(s^t)} = \left(\frac{\mu(s^t)}{\mu(s^t) - x(s^t)}\right)(1-\tau)y(s^t) \tag{15}$$

$$\frac{n^s(s^t)}{P(s^t)} = \left(\frac{x(s^t)}{\mu(s^t) - x(s^t)}\right)(1-\tau)y(s^t) \tag{16}$$

Note that the money growth rate has a direct effect on consumption and an indirect effect via $x(s^t)$. The direct effect is that faster money growth reduces purchasing power and reduces consumption. The indirect effect is that if $x(s^t)$ is positive, then agents will partially compensate by reducing savings.[7]

$$\frac{dc}{dx} = 1 - \mu$$

The real value of seigniorage (received by the domestic government under flexibility, and by

[7] Note that faster money growth in period t (higher $\mu(s^t)$) does not reduce the savings rate in period t because agents want to avoid seeing their money devalued; their money will be devalued whether they spend it or save it.

the foreign under dollarization) is:

$$\frac{D(s^t)}{P(s^t)} = \frac{M(s^t) - M(s^{t-1})}{P(s^t)} = \left(\frac{\mu(s^t) - 1}{\mu(s^t) - x(s^t)}\right)(1-\tau)y(s^t) \tag{17}$$

Note that $x(s^t) \in [0,1]$ .Setting $\mu(s^t) = 1$ implies zero seigniorage. As $\mu(s^t) \to \infty$, $\frac{D(s^t)}{P(s^t)} \to (1-\tau)y(s^t)$ and thus $c(s^t) \to 0$. Note that for $\mu(s^t) \geq 1$ seigniorage is (weakly) positive. For $\mu(s^t) < 1$, seigniorage is negative.

Note that consumption plus seigniorage is equal to the fraction of domestic output that is not taxed directly:

$$c(s^t) + \frac{D(s^t)}{P(s^t)} = (1-\tau)y(s^t)$$

Substituting this equation into the government budget constraint for the flexible economy, we see that real domestic absorption is independent of the money growth rate under flexibility:

$$c(s^t) + g(s^t) = y(s^t) - B(s^t) + (1+r)B(s^{t-1}) \tag{18}$$

Note that there is no analogue to eq. 18 in the dollarized economy.

3 Definition of Equilibrium

We first define equilibrium for two economies that are not of direct interest, but that are useful for defining borrowing constraints that are not too tight. The first economy is one in which: (i) the borrowing constraints $\{\underline{B}(s^t)\}$ are exogenous, and (ii) the government must respect the constraints and is not allowed to default. The second economy is one that has defaulted in the past and has no access to the international debt market. The third economy, which is the economy of interest, features borrowing constraints that are not too tight. In this economy default is permitted but never observed in equilibrium.

Definition 1 *Equilibrium with exogenous borrowing constraints. Consider a set of constraints* $\widetilde{B} = \left\{\widetilde{B}(s^t)\right\}$ $\forall t \geq 0$ *and for all* s^t*. A competitive continuation equilibrium given history* s^r *and initial assets* B_{r-1} *is a policy* Λ_r *and an associated allocation rule mapping policies into private savings choices* $x_r(\Lambda_r)$ *such that for all* $t \geq r$ *and for all* s^t *consistent with* s^r *: (i) the household's intertemporal first order condition 4 and transversality condition 5 are satisfied when consumption is given by 14, inflation is given by 13, and real money savings are given by 16, (ii) the policy is feasible: eqs. 6 and 7 are satisfied given initial assets* B_{r-1} *and the constraints* $\widetilde{B}$*, and* $c(s^t)$*,* $g(s^t) \geq 0$ *when seigniorage* $D(s^t)/P(s^t)$ *is given by 17 in the flexible economy, and is equal to zero in the dollarized economy.*

Definition 2 *Post-default equilibrium. A post default equilibrium is defined in exactly the same* *way, except that feasibility for the government requires $B(s^t) = 0$ for all t, s^t. Thus eqs. 6 and 7 are replaced by 8*

Definition 3 *Ramsey problem. The time zero Ramsey problem is to choose a policy Λ_0 that maximizes expected lifetime utility 1 when the allocation rule $x_0(\Lambda_0)$ satisfies the conditions for competitive equilibrium. The Ramsey equilibrium is the solution to the Ramsey problem.*

Let $W(B_{r-1}, s^r; \widetilde{B})$ denote the value of Ramsey equilibrium with exogenous borrowing constraints $\widetilde{B}$, given assets B_{r-1} and history s^r. Let $V(s^r)$ denote the value of the Ramsey post-default equilibrium.

Definition 4 *Borrowing constraints that are not too tight. A set of borrowing constraints $\underline{B} = \underline{B}(s^t)$ for all $t \geq r$, s^t such that for all s_{t+1} in which $\phi(s^t, s_{t+1}) > 0$*

$$\min_{s_{t+1}\ st.\ \phi(s^t, s_{t+1})>0} W(\underline{B}(s^t), \left(s^t, s_{t+1}\right); \underline{B}) - V\left(s^t, s_{t+1}\right) = 0 \tag{19}$$

Definition 5 *Monetary equilibrium with competitive riskless lending. This is defined in exactly the same way as the economy with exogenous borrowing constraints, except that (i) the borrowing constraints are defined by the solution to eq. 19 (i.e. they are not too tight), and (ii) at each t and s^t the government has the option of defaulting, in which case from t onwards eqs. 6 and 7 are replaced by the constraint 8.*

Note that for a given set of constraints $\underline{B}$, the value $W(a, s^t; \underline{B})$ is strictly increasing in a for any s^t, while the value of default is independent of the quantity of debt defaulted on. It follows immediately that if for any (s^t, s_{t+1}) the government weakly prefers not to default given inherited assets $\underline{B}(s^t)$ the government will strictly prefer to repay for any $a > \underline{B}(s^t)$. Conversely, if the government is indifferent about default for some s_{t+1} given s^t, then if $a < \underline{B}(s^t)$ and s_{t+1} is realized, the government will strictly prefer to default. Note that if all lenders but one were in total willing to lend an amount strictly less than $\underline{B}(s^t)$ in s^t, then the last lender could make a positive profit on a marginal additional loan by charging a real interest rate greater than r and bearing no default risk (assuming the government is borrowing constrained at s^t). Thus the only equilibrium in the lending market in which no excess profits remain is one in which lenders are willing in aggregate to lend up to $\underline{B}(s^t)$ at the safe world interest rate r.

3.1 Ramsey Equilibria

We now describe how we solve for the Ramsey equilibrium in our economies. Solving the Ramsey problem in the dollarized economy is simpler, because monetary policy in this case is exogenous, and the planner only needs to decide on the optimal debt policy.

3.1.1 Dollarized Economy

The Ramsey equilibrium in the dollarized monetary economy with riskless lending can be characterized by solving the following planner's problem. Consider a planner who maximizes expected lifetime utility (eq. 1) subject to budget constraints

$$g(s^t) = \tau y(s^t) - B(s^t) + (1+r)B(s^{t-1}) \tag{20}$$

and a set of borrowing constraints of the form eq. 7.

Sufficient conditions for a solution to this problem are the optimality conditions for bonds:

$$\begin{aligned} \phi(s^t)(1-\lambda(s^t))g(s^t)^{-\gamma} &\geq \beta(1+r)\sum_{s_{t+1}}\left[\phi(s^t, s_{t+1})(1-\lambda(s^{t+1}))g(s^{t+1})^{-\gamma}\right] \qquad (21) \\ \text{with equality if } B(s^t) &> \underline{B}(s^t) \end{aligned}$$

$$\lim_{s^t \to s^\infty} \beta^t \sum_{s^t} \phi(s^t)(1-\lambda(s^t))g(s^t)^{-\gamma}\left(B(s^t) - \underline{B}(s^t)\right) = 0. \tag{22}$$

Note that in the dollarized economy, separability between private and public consumption in preferences implies that consumers and the government end up solving completely separate problems. Consumers use money savings to smooth the marginal utility of private consumption through time, taking as given inflation rates. The government uses debt to smooth the marginal utility of public consumption through time, taking as given the world interest rate and state-contingent borrowing constraints.

3.1.2 Flexible Economy

We will show that the Ramsey equilibrium in the flexible monetary economy with riskless lending can be characterized by solving the following planner's problem.

Planner's Problem Without Money Consider a planner who maximizes expected lifetime utility (eq. 1) subject to a set of borrowing constraints of the form eq. 7 and an aggregate resource constraint 18.

Sufficient conditions for a solution to this planner's problem are the optimality conditions for bonds (eqs. 21 and 22) described above and an intra-temporal first-order condition

$$\lambda(s^t)c(s^t)^{-\gamma} = (1-\lambda(s^t))g(s^t)^{-\gamma} \tag{23}$$

which says that the planner wants to equate the marginal utilities of privately and publicly provided goods at each date.

Combining 18 and 23 gives

$$c(s^t) = \frac{R(s^t)}{(1+\kappa)} \tag{24}$$

where

$$\begin{aligned} R(s^t) &= y(s^t) - B(s^t) + (1+r)B(s^{t-1}) \\ \kappa(s^t) &= \left(\frac{\lambda(s^t)}{(1-\lambda(s^t))}\right)^{-\frac{1}{\gamma}} \end{aligned} \tag{25}$$

Thus

$$g(s^t) = \kappa(s^t)c(s^t) \tag{26}$$

Note that because the marginal utilities of private and public consumption are equated state-by-state, the inter-temporal first order condition 21 can be expressed in terms of $c(s^t)$, $g(s^t)$ or total resources available for domestic consumption $R(s^t)$. For example, in the case $\gamma = 1$, the first order condition simplifies to

$$\frac{\phi(s^t)}{R(s^t)} \geq \beta(1+r)\sum_{s_{t+1}}\left[\frac{\phi(s^t, s_{t+1})}{R(s^t, s_{t+1})}\right].$$

Thus in this case, the planner simply wants to smooth fluctuations in the endowment through time, irrespective of the process for taste shocks: a floating, credit-worthy government will typically issue when the endowment is relatively low, and repay when the endowment is high.

Decentralization Proposition Suppose the borrowing constraints faced by this planner are "not too tight" in the same sense that we defined not-too-tight constraints in the flexible monetary equilibrium with competitive riskless lending. Suppose $\gamma = 1$. Then the allocations $c(s^t)$, $g(s^t)$ and $B(s^t)$ that solve the planner's problem without money also solve the Ramsey problem in the monetary economy with riskless lending, and the values of the not-too-tight borrowing constraints are also the same in both economies.

Note that the Ramsey planner in the monetary economy maximizes the same objective as in the planner's problem above, faces the same aggregate resource constraint period-by-period (the consumers and government budget constraints, eqs. 14 and 6 imply eq. 18), and will have the same optimality conditions for debt. Thus it remains to show that the government in the monetary economy, in which control of the money growth rate is the only way to reallocate resources between the public and private sectors, can achieve the same allocations as a planner who can effectively use lump-sum taxes and transfers to redistribute freely period by period. In particular, we need to show that there exist sequences for money growth rates $\mu(s^t)$, inflation rates $\pi(s^{t+1})$ and savings rates $x(s^t)$ such that:

(i) The consumer's budget constraint is satisfied: given the target values for consumption (eq. 24), $\mu(s^t)$ and $x(s^t)$ satisfy the budget constraint (eq. 14). This implies that the savings rate can be expressed as

$$x(s^t) = \frac{c(s^t)\mu(s^t) - (1-\tau)y(s^t)}{c(s^t) - (1-\tau)y(s^t)} \tag{27}$$

(ii) The government's budget constraint is satisfied: given the target values for consumption (eq. 24), $\mu(s^t)$ and $x(s^t)$ satisfy the budget constraint (eq. 6).

(iii) The optimality conditions for money savings are satisfied: given the target values for consumption, $\mu(s^t)$, $\pi(s^{t+1})$ and $x(s^t)$ are such that if

$$\phi(s^t)\lambda(s^t)c(s^t)^{-\gamma} > \beta \sum_{s_{t+1}} \left[\phi(s^t, s_{t+1})\lambda(s^{t+1})c(s^{t+1})^{-\gamma}\frac{1}{\pi(s^{t+1})}\right] \tag{28}$$

where the inflation rate $\pi(s^{t+1})$ is given by

$$\pi(s^{t+1}) = \left(\mu(s^{t+1}) - x(s^{t+1})\right)\frac{y(s^t)}{y(s^{t+1})} \tag{29}$$

then today's money growth rate is given by[8]

$$\mu(s^t) = \frac{(1-\tau)y(s^t)}{c(s^t)}. \tag{30}$$

Otherwise, if the strict inequality in 28 does not hold, money growth rates are defined by the difference equation

$$\phi(s^t)\lambda(s^t)c(s^t)^{-\gamma} = \sum_{s_{t+1}} \left[\phi(s^t, s_{t+1})\lambda(s^{t+1})c(s^{t+1})^{-\gamma}\frac{1}{\pi(s^{t+1})}\right] \tag{31}$$

and the transversality condition

$$\lim_{s^t \to s^\infty} \beta^t \sum_{s^t} \phi(s^t)\lambda(s^t)c(s^t)^{-\gamma}\frac{M(s^t)}{P(s^t)} = 0. \tag{32}$$

where $\pi(s^{t+1})$ is given by 13, and real balances $M(s^t)/P(s^t)$ are given by 15.

Proof of Decentralization Proposition First note that if savings rates are given by 27 then the consumer's budget constraint is satisfied, and since the aggregate resource and borrowing constraints must be satisfied - they appear in the constraint set of the planner's problem without money - the government's budget constraint is satisfied by Walras' Law. Thus it only remains to show that there exists a set of money growth rates $\mu(s^t)$ that can implement the target values for $c(s^t)$ while satisfying the optimality conditions for money savings.

Suppose that borrowing constraints are not too tight according to the planner's problem, and that the sequence for government debt solves the planner's problem. We will show that for any

[8]Note that from 27 this implies that $x(s^t) = 0$.

possible monetary policy implemented after some date T, we can decentralize the target values for $c(s^t)$ and $g(s^t)$ between periods 0 and T in a monetary equilibrium with riskless lending. Total resources $R(s^t)$ in the planner's solution are given by eq. 25. A sufficient condition for proving that the planner's choices for $c(s^t)$ and $g(s^t)$ can be implemented is that the planner can implement any value for $c(s^t) \in (0, R(s^t))$ for all s^t and for all $t \leq T$. This amounts to showing that the monetary authority effectively has access to a lump-sum tax instrument. To verify that the condition is satisfied we need to understand the response of prices and quantities to changes in the money growth rate.

Note first, that because this is an endowment economy, neither money growth nor inflation have any distortionary effects on factor supplies. Suppose that from period $T+1$ onwards, the government will follow a particular policy defined by Λ_{T+1}. The money growth rate at s^t, $\mu(s^t)$, will affect $c(s^t)$, $x(s^t)$ and $\pi(s^{t+1})$. However it will not affect future savings rates and consumption values, which will depend solely on Λ_{T+1}. Past monetary policy does not restrict the set of feasible allocations that can be achieved looking forward, because current and future policy determine the real value of the dollars consumers carry into the period, which is what matters for real allocations.[9] Given arbitrary choices for T, s^T and the tax rate τ we will show first that the government can implement any value for $c(s^T) \in \big(0, R(s^T)\big)$ with an appropriate choice for $\mu(s^T)$, and thus we can also implement the $c(s^T)$ that solves the planner's problem without money Then we can work backwards to compute the value for $\mu(s^{T-1})$ that delivers the appropriate $c(s^{T-1})$, exploiting once again the fact that changes in $\mu(s^{T-1})$ do not impact $c(s^{T-1}, s_T)$. In this fashion we can work backwards all the way to period 0, along the way deriving sequences for $\mu(s^t)$, $\pi(s^{t+1})$ and $x(s^t)$ that decentralize the planner's solution.

We guess, and will verify, that given a particular monetary policy from tomorrow onwards, there will be a critical money growth rate $\overline{\mu}(s^t)$ such that for any $\mu(s^t) \geq \overline{\mu}(s^t)$ the money savings rate $x(s^t)$ is constant and equal to zero, while for $\mu(s^t) < \overline{\mu}(s^t)$ the savings rate $x(s^t)$ is continuous and decreasing in $\mu(s^t)$, with the property that $x(s^t) \to 0$ as $\mu(s^t) \to \overline{\mu}(s^t)$.

If the target value for $c(s^t)$ is less than or equal to

$$\overline{c}(s^t) = \frac{(1-\tau)y(s^t)}{\overline{\mu}(s^t)}.$$

then it can be implemented with a money growth rate $\mu(s^t)$ defined by 30, where $\mu(s^t) \geq \overline{\mu}(s^t)$ and $x(s^t) = 0$. In this case, the lower is the target value for $c(s^t)$ the higher is the required $\mu(s^t)$. From 14, as $\mu(s^t) \to \infty$, $c(s^t) \to 0$.

[9] The real purchasing power of consumers' money balances entering the goods market at (s^t, s_{t+1}) is given by

$$\frac{M(s^t)}{P(s^t, s_{t+1})} = \left(\frac{1}{\mu(s^t, s_{t+1}) - x(s^t, s_{t+1})}\right)(1-\tau)y(s^t, s_{t+1})$$

and thus does not depend on $\mu(s^t)$ or $x(s^t)$.

If the target value for $c(s^t)$ is greater than $\overline{c}(s^t)$ then it will not be possible to implement in a monetary economy without money savings. In this case, the required money growth rate will be low or negative, savings $x(s^t)$ will be positive, and the inter-temporal first order condition for money saving will be an equality, eq. 31. Since $c(s^t, s_{t+1}) > 0$, $\mu(s^t, s_{t+1}) > x(s^t, s_{t+1})$ for all s_{t+1}, and using the expressions 14 and 13 for current consumption and the inflation rate between t and $t+1$ the inter-temporal first order condition equation implicitly defines $x(s^t)$ as a continuous function of $\mu(s^t)$:

In particular, assuming $\gamma = 1$ and substituting in the expression for inflation from 13 the inter-temporal first order condition may be written

$$\frac{\lambda(s^t)}{c(s^t)} = \beta \sum_{s_{t+1}} \frac{\phi(s^t, s_{t+1})}{\phi(s^t)} \frac{\lambda(s^t, s_{t+1})}{c(s^t, s_{t+1}; \Lambda_{t+1})} \frac{(\mu(s^t) - x(s^t))\, y(s^{t+1})}{\mu(s^t)\,(\mu(s^t, s_{t+1}) - x(s^t, s_{t+1}; \Lambda_{t+1}))\, y(s^t)} \tag{33}$$

Using 14 to express consumption as a function of $x(s^t)$ and $\mu(s^t)$ gives

$$\frac{\lambda(s^t)\mu(s^t)}{(1 - x(s^t))\,(1-\tau)} = \beta \sum_{s_{t+1}} \frac{\phi(s^t, s_{t+1})}{\phi(s^t)} \frac{\lambda(s^t, s_{t+1})}{c(s^t, s_{t+1}; \Lambda_{t+1})} \frac{y(s^{t+1})}{(\mu(s^t, s_{t+1}) - x(s^t, s_{t+1}; \Lambda_{t+1}))} \tag{34}$$

It is immediate from this expression that the savings rate $x(s^t)$ is everywhere decreasing in $\mu(s^t)$.[10] The critical money growth rate $\overline{\mu}(s^t)$ is the value of $\mu(s^t)$ that solves 34 when $x(s^t) = 0$. For $\mu(s^t) > \overline{\mu}(s^t)$ the inter-temporal first order condition will be a strict inequality with $x(s^t) = 0$, confirming the guess that for money growth rates exceeding $\overline{\mu}(s^t)$, household maximization will imply no money saving.

The important point relating to our decentralization result is that when $\gamma = 1$ the savings rate $x(s^t)$ is uniformly decreasing in the money growth rate $\mu(s^t)$. The implication is that if the government has infinite resources it can make seigniorage arbitrarily small and consumption arbitrarily large by reducing $\mu(s^t)$ towards the point at which $x(s^t) = \mu(s^t)$ (see eq. 14). In practice, of course the government does not have infinite resources, but it always has a least $R(s^t) - (1-\tau)y(s^t)$ resources from direct taxation and international borrowing. So it can reduce the money growth rate to the point at which seigniorage is equal to, the negative of this number, in which case $c(s^t) = R(s^t)$. Thus we have shown that, for the case $\gamma = 1$, the monetary authority can implement any value for $c(s^t) \in (0, R(s^t))$ with an appropriate choice

[10] Here is some intuition for the response of $x(s^t)$ to $\mu(s^t)$. Absent a change in the savings rate, a reduction in the money growth rate $\mu(s^t)$ reduces the current price level $P(s^t)$ and increases expected inflation $\pi(s^{t+1})$, which tends to reduce savings. It also increases current consumption, and reduces the marginal utility of consumption, making consumers want to save more. With no change in the savings rate the second effect would dominate, leaving the marginal utility of consumption at s^t too low (see 34). Of course, in equilibrium prices and decisions adjust so that the household's inter-temporal first order condition is satisfied. When $\gamma = 1$ the equilibrium adjustment mechanism is that the expected inflation rate rises by more than under the no-savings-adjustment hypothsis, and the savings rate rises. This increase in the savings rate is consistent with the inflation dynamic (see 13), and the reduced return to saving reduces the right hand side of the intertemporal first order condition. At the same time, a higher savings rate actually increases equilibrium consumption (see eq. 14), reducing the left hand side of the first order condition, but for $\gamma = 1$ the first effect dominates.

for $\mu(s^t)$.

Since the monetary authority effectively has access to lump-sum taxes, it can and will set $\mu(s^t)$ at each date to equalize the marginal utilities of private and public consumption state-by-state, thereby achieving the same allocations as in the planner's problem without money. Recall that the planner uses debt to smooth fluctuations in the endowment over time. Thus monetary policy will be used primarily to adjust the mix between private and public consumption in response to taste shocks. When $\lambda(s^t)$ is high, indicating a preference for private consumption, the money growth rate $\mu(s^t)$ and thus seigniorage will be relatively low.

Decentralization When $\gamma > 1$ When $\gamma > 1$ the solution to the planner's problem can still be decentralized in a monetary equilibrium. However, there are multiple monetary equilibria, and only one of them is constrained efficient.

[TO BE COMPLETED]

3.2 Three Questions

Here we briefly address three questions. First, for a given government policy in the monetary economy, is there a unique equilibrium? Second, assuming the government can commit to monetary policy under flexibility, is the welfare-maximizing monetary policy the same when the government takes borrowing constraints as given versus when the government recognizes that the position of constraints that are not-too-tight will depend in principle on the announced policy? Third, is the policy that solves the Ramsey problem time consistent? We will argue that for each of the questions, the answer is "yes".

First, for a given monetary policy there is a unique monetary equilibrium when $\gamma = 1$. This follows immediately from the fact that the savings rate $x(s^t)$ is everywhere decreasing in $\mu(s^t)$. In particular, for any policy Λ_{T+1} defining policy from period $T+1$ and onwards, each possible money growth rate $\mu(s^T)$ at s^T implies a unique value for $x(s^T)$ and thus for $c(s^T)$ and $\pi(s^T, s_{t+1})$. A similar argument can be applied, recursively, at each date $t \leq T$. When $\gamma > 1$, the savings rate $x(s^t)$ is not everywhere decreasing in $\mu(s^t)$. Furthermore, for a given Λ_{T+1}, there are typically ranges for the money growth rate within which one value for $\mu(s^T)$ supports more than one combination of $c(s^T)$, $x(s^T)$ and $\pi(s^T, s_{t+1})$ satisfying all the equilibrium conditions. Thus for $\gamma > 1$ multiple equilibria can arise.

Regarding the second question, suppose that the government could credibly commit to a future money growth rate policy prior to default, but cannot commit to repay foreign lending, and cannot commit to monetary policy post-default. Is it possible that the government can increase welfare beyond the value of the solution to the Ramsey problem, in which the government takes the position of the borrowing constraints as given? The answer is no. The reason is as follows. First, taking as given borrowing constraints that are not-too-tight, the solution to the Ramsey problem described above maximizes value. Thus the only way a different policy could potentially increase value would be if it implied looser borrowing constraints. However, we can argue that not-too-tight constraints can only be tighter under alternative policies.

First, note that post default, the government will always set the money growth rate to equalize the marginal utilities of private and public consumption. Thus changing monetary policy prior to default cannot change the value of default. Second, prior to default, for any given set of borrowing constraints, value inside the contract is maximized under the policy described above, where monetary policy is used to equate the marginal utilities of private and public consumption. Since value inside the contract could only be lower for any alternative monetary policy, not-too-tight borrowing constraints could only be tighter under any alternative policy.

The third question was whether the policy that solves the Ramsey problem is time-consistent, or whether the policy that maximizes expected value at time zero no longer looks optimal if the government is allowed to re-optimize in some future date and state. The policy described above is in fact time-consistent, because the optimal policy from s^t onwards is independent of policy prior to date t. Potential deviations from the pre-announced policy would be to default on outstanding debts, or to deviate from pre-announced debt or monetary policy. Default will reduce welfare, given borrowing constraints that are not-too-tight. Deviating from pre-announced debt policy will reduce welfare, since the pre-announced policy smooths government consumption as effectively as possible. Deviating from pre-announced monetary policy will be welfare-reducing, because such a change can only change the mix between private and public consumption (in a welfare-reducing fashion) without changing the total resources $R(s^t)$ in the economy.

4 Quantitative Analysis

In this section we solve numerically the two monetary economies: one with a flexible exchange rate regime, and the other one with a dollarized regime. The objective of the quantitative analysis is to compare across regimes the degree of financial integration in terms of borrowing constraints, probabilities of crisis, and debt policy. Our strategy is to calibrate the flexible regime economy to countries in which dollarization has been discussed or implemented as their exchange rate policy. We calibrate the model to three countries: El Salvador and Ecuador, that dollarized in 2001 and 2000, and Mexico for which dollarization has been considered as possible policy. With a given calibration for each country, we then solve the economies assuming a dollarized regime and compare then in terms of business cycles and financial conditions.

4.1 Calibration

Data for real output, government consumption and household consumption in the three countries is per capita annual data taken from the World Development Indicators for 1960-2002.[11] To study the dynamics of foreign public debt in our model we use the series for Government Foreign Financing as a percentage of GDP taken from the International Financial Statistics for 1980-2002. This series measures the change in net foreign assets held by the government. Inflation is the percentage change in consumer prices. We log the series of output, private and public consumption and filter all the data with a 15 year Band-Pass Filter.[12] Figure 2 shows the raw statistics and the filtered series of output for the three countries.

The stochastic structure for the shocks are calibrated for each country. We assume λ and y follow are AR(1) jointly distributed log normal process, where their innovations are allowed to be correlated. Allowing for correlation between shocks is important for giving the model enough flexibility to match the business cycle statistics for the three countries. In particular we assume the following stochastic structure:

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t^y$$

$$\log(\lambda_t) = \rho \log(\lambda_{t-1}) + \varepsilon_t^\lambda$$

$$E[\varepsilon^y] = E[\varepsilon^\lambda] = 0$$

$$E[\varepsilon^{y\prime}\varepsilon^\lambda] = \Omega = \begin{bmatrix} \sigma_y^2 & \sigma_{y\lambda} \\ \sigma_{y\lambda} & \sigma_\lambda^2 \end{bmatrix}$$

The volatility and covariance of shocks are estimated jointly such that the model under the flexible regime matches the volatility of government consumption, volatility of output, and the correlation between government consumption and output in each of the countries. Thus σ^y, $\sigma^{y\lambda}$, and σ^λ are calibrated country by country to match the data. We assume a common persistence parameter ρ across shocks and across countries of 0.8, which is the average persistence of output across the three countries. Shocks are discretized into a 25 state Markov process using quadrature as in Tauchen and Hussey.

The annual interest rate r is set to 4% which is a standard value for real business cycle models. We assume that utility is log separable with $\gamma = 1$. We choose the time preference parameter β to match the volatility of public foreign assets in Mexico. The mean value for λ is

[11] The specific series we used from WDI are: General government final consumption expenditure, Household final consumption expenditure, GDP, (denominated in real local domestic currency and divided by Population) and Inflation, consumer prices (annual %).

[12] We use a longer filter to keep some of the lower frequency movements that have been documented by Aguiar and Gopinath (2004) to be important for emerging countries.

set to 0.88 to match the mean government consumption to GDP ratio across the three countries. We assume that default entails an extra cost of lowering the countries endowment by θ, which we set to 1%. Table 1 summarizes the parameters values in the model.

Table 1.

Estimation of Parameters per Country			
Target Statistics per Country	Mexico	El Salvador	Ecuador
Volatility of output	$\sigma_y = 0.0257$	$\sigma_y = 0.0344$	$\sigma_y = 0.0281$
Volatility of government consumption	$\sigma_\lambda = 0.0031$	$\sigma_\lambda = 0.0092$	$\sigma_\lambda = 0.0091$
Correlation of government and output	$\rho_{\lambda y} = -0.3226$	$\rho_{\lambda y} = 0.4424$	$\rho_{\lambda y} = -0.7430$
Common Parameters Across Countries			
Persistence	$\rho = 0.8$		
Output costs post default	$\theta = 1\%$		
Mean government/output	$\overline{\lambda} = 0.88$		
Public foreign assets volatility in Mexico	$\beta = 0.95808$		

4.2 Results

Table 2 presents business cycle statistics for the data in Mexico and for the model under the flexible and dollarized regime. Consumption and output volatilities are similar in Mexico, with consumption being slightly less volatile. Government expenditures are much more volatile than output. Inflation in Mexico has been extremely volatile. In Mexico, the correlations among private consumption, public consumption and output are positive. Inflation rates are weakly negatively correlated with output, and positively correlated with government spending. In Mexico, net public foreign assets are positively correlated with output and weakly positively correlated with government consumption. Thus the government uses more foreign debt to finance its budget in recessions.[13]

Table 2.

MEXICO	Data			Flexible Economy			Dollar Economy		
	σ_x	$\rho_{x,y}$	$\rho_{x,g}$	σ_x	$\rho_{x,y}$	$\rho_{x,g}$	σ_x	$\rho_{x,y}$	$\rho_{x,g}$
Government	5.67	0.78	–	5.67	0.78	–	3.98	0.78	–
Consumption	3.84	0.94	0.67	3.40	0.93	0.71	3.94	1.0	0.91
Inflation	19.10	-0.09	0.24	2.71	-0.05	0.06	–	–	–
Public foreign assets	1.31	0.41	0.14	1.31	0.47	0.20	0.20	0.18	-0.23
Output	3.95	–	0.78	3.94	–	0.78	3.94	–	0.78
Borrowing constraint				-24.81			-25.39		
% in Crisis				39.02			25.63		

[13] It has been documented that the trade balance is on the contrary countercyclical; however our emphasis here is on the public sector debt dynamics.

Table 3 shows business cycle statistics for the benchmark model with the calibration for Mexico under flexible exchange rates and dollarization. Business cycle statistics are those of the limiting distribution of assets. Our calibration suggests that productivity shocks are the most important with the standard deviation of productivity shocks being 8 times larger than that of taste shocks.

Standard deviations of output, government, and the public foreign assets and the correlation of government and output were calibrated jointly by the choices for the standard deviations and correlation of shocks and the time preference parameter. The flexible economy replicates some other important features of the Mexican economy. The volatility of consumption is lower than output volatility, as in the data. The model can also match the contemporaneous positive correlation between government and private consumption, and the positive correlation between government and output. In the model, the taste shock makes households value highly private consumption, and simultaneously value public consumption less, which tends to make the correlation negative. But productivity shocks imply a positive correlation, because when output is high, the government optimally wants to increase both public and private consumption. Given that productivity shocks are most important in Mexico, the model can generate the positive correlations with private consumption.

The model matches the acliclycality of inflation and predicts a slightly positive correlation between inflation and government as observed in Mexico. This is because in the flexible economy, the government finances government consumption with the inflation tax revenue and debt and thus when the economy has a large taste for government consumption inflation is higher. However given that output shocks and government consumption taste are positively correlated ($\rho_{\lambda y} < 0$), periods of high taste of government consumption are also periods of high output. Thus inflation doesn't need to be adjusted as heavily as to deliver the right level of government consumption. However this same reason is also why the model misses the high volatility of inflation observed in Mexico.

The model matches the procyclicality of public foreign assets. In the model with strong productivity shocks, debt is used to smooth output fluctuations; the government runs down its assets in periods of low output, and engages in precautionary savings in periods of relatively high output.

Table 3 also presents statistics for the dollarized economy using the benchmark calibration. The labor tax τ used equals 12.87, which is the optimal labor tax assuming that post default the government adjusts one time the labor tax to the optimal post default level. In several respects, the dollarized economy exhibits a slightly higher degree of international financial integration than the flexible exchange rate economy. The dollarized economy can sustain more borrowing in equilibrium as the endogenous borrowing constraint in the dollarized economy is looser than in the flexible economy and the probability of being in crisis is smaller.[14] The government of the

[14]We have assumed that all elements in the transition matrix are non-zero, such that the model generates a

dollarized economy has less incentives to default because debt is the only policy available for smoothing government fluctuations, and thus the government's credibility in financial markets is increased. Figure 4 presents the distribution of assets in the limiting distribution for the flexible and dollarized economies. Both economies have a distribution of assets that have a probability mass at the constraint. The dollarized economy features a lower probability of being in a financial crises: the probability mass at the respective constraint is 39% in the flexible economy compared to 25% in the dollarized case. However the dollarized economy uses debt less strongly as seen in the lower volatility of foreign assets and the smaller range for asset positions in the limiting distribution.

Business cycle statistics in the dollarized economy, are qualitatively similar to those in the flexible economy. Consumption volatility is exactly equal to output volatility, because in the dollarized economy with constant taxes there is no way to affect the time path of private consumption. Government consumption volatility is much smaller in the dollarized economy. This suggests that for Mexico, debt in the dollarized regime is not as good of a substitute policy for the inflation rate in the flexible regime. In our model dollarization changes the correlation between foreign assets and government consumption by making it negative. In the dollarized regime debt is used to finance volatile government consumption needs; the government borrows when it faces a high taste for government consumption, and saves when it faces a low taste.

We next analyze the case of El Salvador. This country dollarized its economy in 2000 and here we use our model to analyze the effect of such policy on its international financial integration.

Table 3.

EL SALVADOR	Data			Flexible Economy			Dollar Economy		
	σ_x	$\rho_{x,y}$	$\rho_{x,g}$	σ_x	$\rho_{x,y}$	$\rho_{x,g}$	σ_x	$\rho_{x,y}$	$\rho_{x,g}$
Government	9.73	-0.05	–	9.73	-0.04	–	8.27	-0.05	
Consumption	7.58	0.90	-0.38	5.22	0.90	-0.22	5.27	1.0	0.12
Inflation	4.46	-0.29	0.19	3.96	-0.19	0.34	–	–	–
Public net foreign assets	0.48	0.03	-0.19	2.17	0.53	-0.18	1.19	0.45	-0.83
Output	5.27	–	-0.05	5.27	–	-0.04	5.27	–	-0.04
Borrowing constraint				-24.91			-25.56		
% in Crisis				22.18			9.54		

Business cycle statistics for El Salvador are quite different than for Mexico. Government consumption is almost twice as volatile than output and private consumption is substantially more volatile than output. Inflation in El Salvador features a negative correlation with output and a positive correlation with government consumption. Public government consumption present a negative correlation with both output and private consumption which is opposite from the Mexican data. Lastly, public debt is acyclical and negatively correlated with government

single borrowing constraint for each regime.

consumption. Thus periods of high government expenditures needs are associated with periods of larger debt levels.

Our calibration suggests that taste shocks are more important for El Salvador than for Mexico, although productivity shocks are also very important. Productivity shocks are 3.7 times bigger than taste shocks for El Salvador calibration. In addition the calibration suggests that the taste for government consumption is high in periods of low output. The sources of shocks are important for the effect of dollarization on financial integration. As we show below the increase in financial integration with dollarization is the largest when taste shocks are strong.

The flexible model calibrated to El Salvador matches many of the features of the data. The model features the negative correlations of private and public consumption as in the data, the countercyclicality of inflation, and the positive correlation of inflation and government consumption. The model predicts that in El Salvador inflation is used heavily in periods of high government consumption needs, not only due to a high taste for government consumption but also because output is exactly lower in those periods. Thus this calibration generates a larger volatility in inflation that is countercyclical.

In addition the model matches the positive correlation of public foreign assets and output and the negative correlation of public foreign assets and government consumption. In the flexible economy debt is used to smooth output fluctuations and taste shocks. The model generates a negative correlation of foreign assets and government consumption because conditional on the productivity shock, the government engages in more borrowing to finance the larger need for government consumption. The model underestimates a bit the volatility of both inflation and private consumption, but overall it provides a good fit to the Salvadorean data.

The table presents business cycle statistics for the dollarized regime, where we use a labor tax τ of 0.1317 estimated in the same way as for Mexico. The dollarized economy calibrated to El Salvador presents a higher degree of financial integration. The economy is able to borrow a larger proportion of output, and the probability of being in crisis drops by half from 22% to 9% of the time. In addition with this calibration, we find that debt in the dollarized regime is a good substitute for the inflation policy in the flexible regime, as seen by the similar volatility of government consumption across regimes. The reason why debt is a good substitute to inflation for this calibration and not for the Mexican calibration is because a larger proportion of the volatility of government consumption in El Salvador is coming from the volatility of taste shocks. In the dollarized economy, borrowing is the largest when output is low, and when the taste for government consumption is large and this states happen simultaneously for El Salvador. In Mexico, taste shocks are small and are negatively correlated with output, and thus the economy faces a trade-off such that debt cannot be used as effectively for the two purposes.

We now analyze the case of Ecuador which dollarized its economy in 2000. Business cycle statistics in Ecuador have a similar flavor than those in Mexico in various aspects. Government consumption is very strongly correlated with output and private consumption, and private consumption is slightly less volatile than output; pointing to evidence of strong productivity shocks. However in Ecuador government consumption is very volatile, more than 3 times as volatile as output and inflation is countercyclical, pointing to evidence of taste shocks.

Our calibration suggests that taste shocks are quite strong in Ecuador but that they are strongly positively correlated with productivity shocks. This suggests that government needs increase when output increases but by much more. The reason for this relation is the oil dependence that Ecuador has in terms of government consumption and output. Oil constitutes a large source of government revenues and a large fraction of the economy as a whole and thus both tend to move together.

The model under the flexible regime can match various aspects of the Ecuadorian economy, but does a less good of a job than for the calibration of the other two countries. In particular it predicts a lower volatility of private consumption, and a strong positive correlation of inflation and output which is at odds with the data. The reason for the procyclicality of inflation is that in booms government consumption needs are even higher than the increase in output, and so inflation is high to finance it. Government consumption in the model is financed mainly by inflation and not debt, because the government finds it better to save in those periods for precautionary reasons given the high output.

Table 4.

ECUADOR	Data			Flexible Economy			Dollarized Economy		
	σ_x	$\rho_{x,y}$	$\rho_{x,g}$	σ_x	$\rho_{x,y}$	$\rho_{x,g}$	σ_x	$\rho_{x,y}$	$\rho_{x,g}$
Government	13.37	0.85	–	13.37	0.85	–	7.05	0.85	–
Consumption	4.29	0.90	0.81	2.97	0.84	0.61	4.32	1.00	0.83
Inflation	11.62	-0.21	-0.02	3.26	0.30	0.40	–	–	–
Public net foreign assets	0.75	0.10	0.06	1.54	0.49	0.33	0.50	-0.36	-0.82
Output	4.32	–	0.85	4.32	–	0.85	4.32	–	0.85
Borrowing Constraint				-24.82			-25.36		
% in Crisis				25.08			18.78		

Our model predicts that dollarization in Ecuador will increase financial integration but not significantly. The endogenous borrowing constraint is a bit looser, allowing the economy to borrow more, and the probability of being in crisis decreases from 25% to 19%. However debt is not as effective to finance government consumption needs in the dollarized regime as inflation is in the flexible regime. The volatility of government consumption is much smaller in the dollarized regime because debt is not used as heavily as observed by the low volatility of foreign assets. It is interesting that with this calibration the model can generate a negative correlation between foreign assets and output even in this insurance model of debt. The reason is that in Ecuador what drives the use of debt in the dollarized regime is the high volatility of government expenditure relative to output fluctuations. The economy borrows when the taste for government consumption is high regardless of the fact that the economy might be in a boom.

The labor tax for this calibration is of 12.86.

Our main findings are that the effect dollarization has on financial integration depends crucially on which type of shocks economies face. If economies fluctuations are mainly due to productivity disturbances, as we found in Mexico and Ecuador, dollarization does not increase financial integration substantially. On the other hand, if economies fluctuations are strongly due to shocks affecting government expenditures needs, as we found in El Salvador, then dollarization provides a mechanism for the government to gain credibility in financial markets, and thus it increases financial integration.

4.3 Is dollarization welfare improving?

[TO BE COMPLETED]

5 Conclusion

This paper presents a simple model designed to study the interaction between dollarization and credibility in international financial markets. In our model dollarization is costly because seigniorage is lost as a policy instrument. At the same time dollarization is potentially beneficial precisely because eliminating the seigniorage instrument strengthens incentives to repay debt and thereby increases access to international credit. The overall effects of dollarization on financial integration depend crucially on which type of shocks economies face. Economies for which shocks to government revenue requirements are important will likely experience the greatest gains from relinquishing control of monetary policy.

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