

Basic Debt Dynamics

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Standard Debt Dynamics Equation (everything real)

$$\begin{aligned}B_{t+1} &= (1+r)B_t + G_t - T_t \\ \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} &= \frac{B_t}{Y_t} + \frac{rB_t}{Y_t} + \frac{PD_t}{Y_t} \\ \frac{B_{t+1}}{Y_{t+1}} &= \frac{1}{1+g} \left(\frac{B_t}{Y_t} + \frac{rB_t}{Y_t} + \frac{PD_t}{Y_t} \right)\end{aligned}$$

Let lower case letters denote variables relative to GDP

$$\begin{aligned}b_{t+1} &= \frac{1}{1+g} (b_t + rb_t + p_t) \\ \Delta b_{t+1} &= \frac{1}{1+g} (b_t + rb_t + p_t) - b_t \\ &= \left(\frac{1+r}{1+g} \right) b_t + \frac{p_t}{1+g} - b_t \\ &= \left(\frac{r-g}{1+g} \right) b_t + \frac{p_t}{1+g} \\ &\simeq (r-g) b_t + p_t\end{aligned}$$

Suppose we want a stable debt to GDP ratio b . What is the sustainable deficit $p = PD/Y$?

$$\Delta b = 0 \Rightarrow (g-r)b = p$$

If $r > g$ then a bigger b requires a more negative p

But if $g > r$ then a positive b requires a positive deficit and a bigger b requires a bigger deficit! (perverse)

Suppose $g = 0.02$ and $r = 0.0$ and $b = 1$. Then debt to GDP stable when $p = 0.02$ (2% primary deficit)

Same g and r but $p = 0.04 \Rightarrow b$ stable at 2!

$$\Delta b_{t+1} = \frac{1}{1+g} (b_t + rb_t + p_t) - b_t$$

$$\frac{1}{1+0.02} (b + 0b + 0.03) - b \text{ (black)}$$

$$\frac{1}{1+0.02} (b + 0b + 0.04) - b \text{ (red)}$$

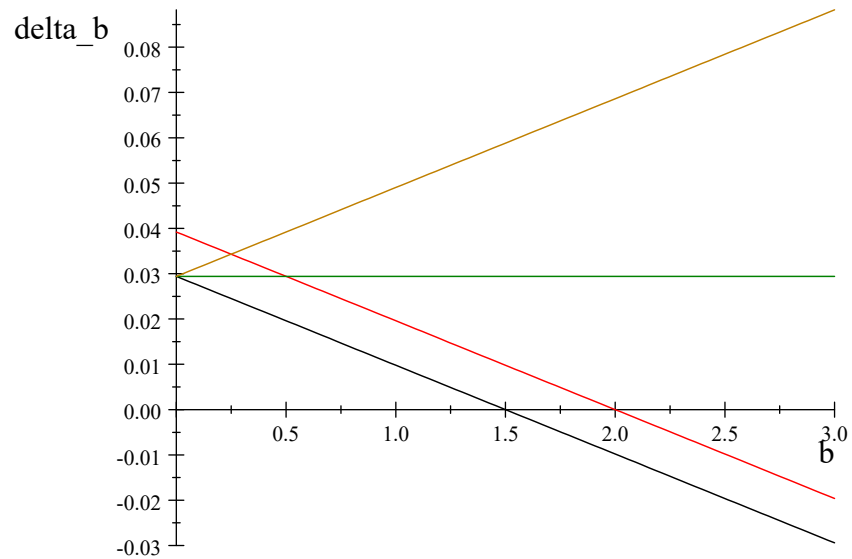
$$\frac{1}{1+0.02} (b + 0.02 * b + 0.03) - b \text{ (green)}$$

$$\frac{1}{1+0.02} (b + 0.04b + 0.03) - b \text{ (brown)}$$

If $b = 1.5$ and $r = 0$ we can keep running a 3% of GDP deficit

If r rises to 2% we need to eliminate the deficit

If r rises to 4% we need to switch to a 3% of GDP surplus



Papers on this topic:

Mehrotra and Sergeyev

<https://sites.google.com/site/neilrmehrotra/working-papers>

Furman and Summers

<https://www.brookings.edu/wp-content/uploads/2020/11/furman-summers-fiscal-reconsideration-discussion-draft.pdf>

https://www.wsj.com/articles/the-debt-question-facing-janet-yellen-how-much-is-too-much-11610993908?mod=hp_lead_pos7