Interest Rates in a General Equilibrium Baumol-Tobin Model

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Abstract

This paper is a version of Romer's general equilibrium interpretation of the Baumol-Tobin model. It investigates the implications of modelling money demand as arising endogenously from costs associated with trading in asset markets for the behavior of the real interest rate. Under a particular rule for tax policy I look at the implications for real and nominal rates of an unexpected shock to inflation.

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1. Introduction

There is a class of models that generates real effects for monetary policy by supposing that only a fraction of the population is in the asset market at any point in time. Only these agents directly respond to and influence asset prices. These models are sometimes referred to as liquidity models of asset pricing, because the real effects of open market operations are a consequence of the fact that the liquidity in the asset market is (at least in part) pre-determined. A typical finding is that increases in the money supply require falls in real interest rates to induce the fraction of the population in the market to absorb all of the new cash.

The dynamics of economic variables in liquidity models tend to be highly sensitive to exactly who is trading financial assets at each date. For example, Alvarez, Atkeson and Kehoe [2] find that the smaller is the fraction of the total population in the asset market at a point in time, the larger is the response of the real interest rate to shocks to the money growth rate. It is therefore unfortunate that most of the models do not make the timing of asset market trips part of the set of choice variables for individual agents.

Grossman and Weiss [7], and Rotemberg [11] suppose that households get to trade interest bearing assets for money every second period, and that half of the population trades in one period, and half in the next. Lucas [8] and Grilli and Roubini [4] employ a construct in which a household has to decide how to split its money balances between the goods and asset markets before the size of the government bond issue is observed. By supposing that agents in and out of the asset market belong to the same representative family, the Lucas approach avoids the (potentially interesting) distributional issues that complicate analysis in Grossman and Weiss and Rotemberg models. Alvarez and Atkeson [1] use this multi-member family idea to generalize a Grossman-Weiss-Rotemberg style model to allow for shopping trips of stochastic length. Alvarez, Atkeson and Kehoe [2] keep the distribution of asset holdings tractable by supposing that there are only two types of households, one that is permanently excluded from the asset market and holds a fixed portfolio, and another that trades assets freely every period.

The models described above clearly demonstrate that open market operations may have significant real effects if only a fraction of the population is trading in the asset market. However, a common feature of all of these models is the implicit assumption that at any date at least some households face a very large cost of moving money between the goods and asset markets once new information about government policy is revealed. Because trading costs are either at a level high enough to deter all trade or else are zero, these models do not allow for the possibiliy that unexpected movements in the prices of goods or assets may induce some households to enter or leave the asset market.

Jovanovic [5] and Romer [9] construct general equilibrium models extending the work of Baumol [3] and Tobin [12] in which individuals decide how frequently to return to the asset market subject to paying a constant fixed cost of doing so. The analysis of these papers is restricted to looking at steady states. Grossman [6] and a later paper by Romer [10] go some way towards considering the effects of shocks to monetary policy when the timing of asset market trade is endogenous. In the Grossman model there is a proportional cost of converting bonds into money at all dates except paydates when transactions are free. Romer maintains the constant fixed cost assumption and considers the effects of permanent nominal interest rate changes on real variables, holding the real interest rate constant and allowing government spending to adjust to satisfy the government budget constraint. He does not, however, consider any policy experiments in which the path for either the real or the nominal interest rate is endogenous, and therefore cannot address many questions regarding the implications for interest rates of open market operations or money growth or inflation targetting rules.

This paper is a discrete-time version of Romer's model. Overlapping generations of households pay a fixed cost to trade assets at any time after birth. The government may be thought of as choosing a path for the inflation rate while being required to hold government spending constant. Its policy instruments are the quantities of money and bonds it issues. The path for the real interest rate is endogenous. After characterizing the steady state I investigate the response of the economy to an unanticipated shock to inflation policy.

I find that even when participation in the asset market is allowed to respond optimally to movements in asset prices, an unanticipated shock to monetary policy still has significant real effects in the short run. In particular, an increase in the inflation rate is associated with a large immediate rise in the real interest rate. As time passes, however, the real interest rate rapidly converges to its level in the steady state corresponding to the post-shock inflation rate.

2. The Model

This is a discrete time overlapping generations model. Each period a new generation of identical consumers are born. They live a finite life of length A periods. They derive utility from the consumption of a good, c, which can only be purchased with fiat money. There is a government which has the right to print money and bonds. At birth individuals receive an endowment of the good, E and an amount of cash from the government. There does not exist a storage technology for converting consumption goods today into consumption goods tomorrow and only the newborns receive an endowment. Thus, in equilibrium, aggregate consumption in each period must be less than or equal to E.

Individuals exchange their entire initial wealth for bonds with banks. Throughout their lives they can redeem their bonds for cash at the banks. Interest paid by the government to bondholders is in the form of additional bonds deposited in their accounts. At birth individuals get a free trip to the bank. Thereafter, each time they return they pay a direct utility cost d per trip, which does not depend on the volume of assets they trade. In the goods market individuals buy goods (sold by the banks) in exchange for cash.

Banks trade cash in exchange for bonds with the government and goods for cash in the goods market. Because they do not hold any assets or goods for more than a moment of time it must be the case that their net cash receipts from newborn depositors, sales in the goods market and sales of bonds to the government equal cash demand from bank customers. Similarly net bond sales to the private sector must equal bond purchases from the government.

Between period t and period $t+1$, the government chooses the inflation rate $\pi_t \left(= \frac{P_{t+1}-P_t}{P_t}\right)$. The government's policy instruments are the quantity of new cash to print and transfer to the newborn (or alternatively how much cash to take from the newborn via taxation and then destroy), and how many new bonds to sell to the banks. It faces the constraint that the real value of transfers to the newborn must be less than or equal to the real value of revenue from bond sales plus the real value of seignorage minus the real value of interest payments to bondholders. The real interest rate paid on bonds r_t is endogenous. The corresponding nominal interest rate is $i_t = r_t + \pi_t$.

2.1. An individual's problem

Note that the first subscript indicates the agent's age. He is born at age zero and lives to age $A-1$ (thus if the length of a lifetime is one period, he consumes only at age 0). The second subscript indicates the date. The individual born at date r solves

$$\max_{\{b\},\{m\},\{c\}} \sum_{t=r}^{r+A-1} \ln(c_{t-r,t}) - Qd \tag{2.1}$$

subject to

$$c_{0,r} \leq E_r + T_r - (m_{1,r} + b_{1,r}) \tag{2.2}$$

$$c_{0,r},\ m_{1,r},\ b_{1,r} \geq 0$$

and $\forall s \in [1, A-1]$

$$c_{s,r+s} \leq \frac{m_{s,r+s-1}}{(1+\pi_{r+s-1})} - m_{s+1,r+s} + (1+r_{r+s-1})\, b_{s,r+s-1} - b_{s+1,r+s} \tag{2.3}$$

$$c_{s,r+s} \leq \frac{m_{s,r+s-1}}{(1+\pi_{r+s-1})} + w_{s,r+s} \tag{2.4}$$

$$w_{s,r+s} = (1+r_{r+s-1})\, b_{s,r+s-1} - b_{s+1,r+s} \tag{2.5}$$

$$Q = \sum_{s=1}^{A-1} z_{s,r+s} \tag{2.6}$$

$$\begin{aligned} z_{s,r+s} &= 1 \text{ if } w_{s,r+s} \neq 0 \\ &= 0 \text{ otherwise} \end{aligned}$$

$$c_{s,r+s},\ m_{s+1,r+s},\ b_{s+1,r+s},\ w_{s,r+s} \geq 0$$

Here

$c_{s,r+s}$ is consumption at age s and date $r+s$.

$m_{s+1,r+s}$ is real balances carried out of period $r+s$ by an individual of age s.

$w_{s,r+s}$ is real balances withdrawn from the bank in period $r+s$ by an individual of age s.

$b_{s+1,r+s}$ is bonds carried out of period $r+s$ by an individual of age s in terms of the period $r+s$ consumption good.

T_r is transfers from the government given to the newborn at date r.

$z_{s,r+s}$ is an indicator which takes the value one if and only if an individual of age s goes to the bank in period $r+s$.

Q is the number of trips to the bank after birth.

Summing across the budget constraints of all individuals alive at date t

$$\sum_{s=0}^{A-1} c_{s,t} = E_t + T_t + \sum_{s=1}^{A-1} \frac{m_{s,t-1}}{(1+\pi_{t-1})} - \sum_{s=0}^{A-1} m_{s+1,t} + (1+r_{t-1}) \sum_{s=1}^{A-1} b_{s,t-1} - \sum_{s=0}^{A-1} b_{s+1,t}$$

Seignorage S_t is given by

$$S_t = \frac{N_t}{P_t} - \frac{N_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} = \sum_{s=0}^{A-1} m_{s+1,t} - \sum_{s=0}^{A-1} \frac{m_{s+1,t-1}}{(1+\pi_{t-1})} \tag{2.7}$$

where N_t is aggregate nominal money balances at the end of trading in t.

Using this expression the aggregate resource constraint at t simplifies to

$$C_t = E_t + T_t - S_t + (1+r_{t-1}) B_{t-1} - B_t$$

where upper case characters denote aggregate quantities.

2.2. Definition of equilibrium

An equilibrium for this economy is a set of infinite price sequences $\{P\}$, $\{r\}$ such that when individuals take these price sequences as given and solve 2.1 subject to 2.2 through 2.6, the markets for goods, money and bonds clear and the government's budget constraint is satisfied.

The goods market clearing condition is

$$C_t = E_t \tag{2.8}$$

The government budget constraint is

$$T_t \leq S_t + B_t - (1+r_{t-1})B_{t-1} \tag{2.9}$$

where S_t is given by 2.7.

Since there is no storage in this economy, I focus throughout on equilibria where 2.9 is an equality.

3. Steady State

In this economy the government's transfers T are equal to its net profits from issuing money and debt. In a more general economy with positive government spending and lump sum taxes, the steady state level of T would be the difference between tax and seignorage revenue on the one hand, and spending and debt interest payments on the other. We can think of a steady state government policy as being a choice of a constant value for this measure of the budget deficit, together with a constant value for the inflation rate.

3.1. An individual's problem

Solving an individual's problem is relatively straightforward if it is divided into three parts. First I characterize optimal behavior between bank trips. Then I show that ignoring integer constraints, optimal behavior involves evenly spaced bank trips. Finally I derive the optimal number of trips.

In steady state equilibrium, assuming a strictly positive inflation rate, every agent will choose a pattern of withdrawals such that, immediately prior to each trip to the asset market, he holds no real balances. Between trips to the bank an individual solves

$$\max_{\{m\},\{c\}} \sum_{a=d}^{d+\gamma-1} \ln(c_a)$$

subject to

$$c_a \leq w_a - m_{a+1}$$

$$c_a,\ m_{a+1} \geq 0$$

and $\forall f \in [1, \gamma - 1]$

$$c_{a+f} \leq \frac{m_{a+f}}{(1+\pi)} - m_{a+f+1}$$

$$c_{a+f}, m_{a+f+1} \geq 0$$

where w_a (taken as given) is real balances withdrawn by an individual of age a and γ is the length of time between trips to the bank or between the last trip to the bank and death.

Given the nature of preferences the solution to this problem has a simple form.

$$c_a = \frac{w_a}{\gamma}$$

$$c_{a+f} = \frac{w_a}{\gamma \left(1+\pi\right)^f}$$

Substituting these results into the utility function and re-arranging gives the following expression for utility between trips to the bank.

$$U_a = \sum_{a=d}^{d+\gamma-1} \ln(c_a) = \gamma \ln\left(\frac{w_a}{\gamma}\right) - \frac{\ln\left(1+\pi\right)\gamma\left(\gamma-1\right)}{2}$$

The next task is to determine the optimal spacing between bank trips and the optimal withdrawal of real balances at each visit. Following Romer, I first assume an individual returns to the bank only once during his lifetime, and show that in steady state, barring integer constraints, this will occur exactly halfway through life.

Let w_b be the quantity of real balances withdrawn at birth and w_{b+j} be the quantity withdrawn on the return trip where j is the length of time from birth till the return trip. Let A be the length of the individual's lifetime. Let $U1$ be total utility between birth and the return trip to the bank, and $U2$ utility over the remainder of his lifetime.

$$U1 = j \ln\left(\frac{w_b}{j}\right) - \frac{\ln\left(1+\pi\right) j \left(j-1\right)}{2}$$

$$U2 = \left(A-j\right) \ln\left(\frac{w_{b+j}}{\left(A-j\right)}\right) - \frac{\ln\left(1+\pi\right)\left(A-j\right)\left(A-j-1\right)}{2}$$

Thus supposing the individual is allowed only one trip to the bank he solves

$$\max_{j, w_b, w_{b+j}} U1 + U2 - d$$

subject to a lifetime budget constraint

$$w_b + \frac{w_{b+j}}{\left(1+r\right)^j} \leq E + T$$

Taking first order conditions with respect to j, w_b and w_{b+j} I get the expected result that

$$j^* = \frac{A}{2}$$

By induction it is possible to generalize this result to show that, abstracting from integer constraints, trips to the bank will always be evenly spaced, irrespective of the total number of trips. Thus for any given Q the optimal length of time between bank trips, γ^*, is given by

$$\gamma^* = \frac{A}{(Q+1)}$$

where Q is the total number of bank trips after birth.

Optimal real withdrawals are given by

$$w_{b+x\gamma^*} = (1+r)^{x\gamma^*} w \qquad x \in \{0...Q\}$$

$$w = \frac{E+T}{(Q+1)} \tag{3.1}$$

Substituting these results into the expression for lifetime utility U and rearranging we see that

$$U = A \ln \left(\frac{w(Q+1)}{A} \right) - \frac{A}{2} \left(\frac{A}{(Q+1)} - 1 \right) \ln(1+\pi) - Qd + \frac{A^2 Q}{2(Q+1)} \ln(1+r)$$

where w is given above.

Maximizing with respect to Q and ignoring integer constraints, the optimal time between bank trips γ^* is given by

$$\gamma^* = \sqrt{\frac{2d}{\ln(1+\pi) + \ln(1+r)}} = \sqrt{\frac{2d}{\ln((1+\pi)(1+r))}} \tag{3.2}$$

This is the Baumol-Tobin "square-root rule", although in this specification, where the cost of a bank trip is a direct utility cost, γ^* is independent of initial wealth.

Assume that integer constraints are not a complication in that no profile of unevenly spaced trips can be an optimal choice. For a given set of parameters, E, A, d, and policy choices, T, π, the maximum number of optimal timing sequences is two; a newborn individual may have a unique γ^*, or he may be indifferent between choosing γ^* or $\gamma^* \pm 1$.

3.2. Solving for aggregate variables

If all households choose the same γ^* then it is possible to compute the steady state real values of aggregate variables as functions of r and π just by summing over an individual's lifetime. If r and π are such that two values for γ are optimal then aggregate variables are given by the weighted sum of lifetime values for the two alternative values for γ, where the weights are the fractions of the population following the respective timing sequence. To simplify the exposition, in the equations below I assume that r and π are such that all households choose the same value for γ.

$$C = \sum_{i=0}^{Q} \left[(1+r)^{\gamma^* i} \sum_{j=0}^{\gamma^*-1} \left(\frac{1}{(1+\pi)} \right)^j \frac{w}{\gamma^*} \right]$$

$$M = \sum_{i=0}^{Q} \left[(1+r)^{\gamma^* i} \sum_{j=0}^{\gamma^*-1} \left(\frac{1}{(1+\pi)} \right)^j \left(1 - \frac{(j+1)}{\gamma^*} \right) w \right]$$

$$B = \sum_{j=0}^{Q} \left[(1+r)^{\gamma^* j} \sum_{i=0}^{\gamma^*-1} (1+r)^i (A+1-1-j) w \right]$$

where w is given in 3.1.

The expressions for M and B simplify to give[1]

$$M = \frac{Y(1+\pi)\left((1+\pi)^{-(\gamma^*-1)} - (1+\pi) + \gamma^*\pi\right)\kappa}{\pi^2 A} \tag{3.3}$$

[1] As an example of the necessary algebra, I show the steps for simplifying B.

$$\begin{aligned} B &= \sum_{j=0}^{Q} \left[(1+r)^{\gamma^* j} \sum_{i=0}^{\gamma^*-1} (1+r)^i (Q+1-1-j) w \right] \\ &= \left[\frac{(1+r)^{\gamma^*} - 1}{r} \right] w \sum_{j=0}^{Q} \left[(1+r)^{\gamma^* j} (Q+1-1-j) \right] \\ &= \left[\frac{(1+r)^{\gamma^*} - 1}{r} \right] w \left((Q+1)\kappa - \frac{\kappa\left(\kappa - (Q+1)(1+r)^A\right)}{1-(1+r)^{\gamma^*(Q+1)}} \right) \end{aligned}$$

$$= \left[\frac{(1+r)^{\gamma^*} - 1}{r} \right] w\kappa \left(\frac{(Q+1)-\kappa}{1-(1+r)^A} \right) = \frac{w}{r}(\kappa - (Q+1)) = \frac{Y}{r}\left(\frac{\gamma^*\kappa}{A} - 1 \right)$$

$$B = \frac{Y}{r}\left(\frac{\gamma^*\kappa}{A} - 1\right) \tag{3.4}$$

where

$$Y = E + T$$

$$\kappa = \left[\frac{1-(1+r)^A}{1-(1+r)^{\gamma^*}}\right]$$

Goods market clearing requires that

$$C = E \tag{3.5}$$

In steady state the real value of seignorage from the printing of currency is given by

$$S = M - \frac{M}{(1+\pi)} = \frac{\pi M}{(1+\pi)} \tag{3.6}$$

Given that aggregate bond holdings are constant in steady state, the government budget constraint is satisfied with equality if

$$T = S - rB \tag{3.7}$$

Using the expressions for M and B as functions of Y together with 3.6 and 3.7 it is possible to solve for Y as a function of the underlying parameters. This expression can then be substituted back into the expressions for M and B.

$$Y = E + T = \frac{EA\pi\,(1+\pi)^{\gamma^*-1}}{\left((1+\pi)^{\gamma^*} - 1\right)\kappa} \tag{3.8}$$

$$M = \frac{E\,(1+\pi)\left(1-(1+\pi)^{\gamma^*-1}(1+\pi-\gamma^*\pi)\right)}{\pi\left((1+\pi)^{\gamma^*}-1\right)} \tag{3.9}$$

$$B = \frac{E\pi\,(1+\pi)^{\gamma^*-1}(\gamma^*\kappa - A)}{r\left((1+\pi)^{\gamma^*}-1\right)\kappa} \tag{3.10}$$

It is interesting to note that in this economy M depends on r only indirectly via the γ^* term.

When $\gamma^* = 1$

$$M = 0$$

$$B = \frac{-E\left(rA - \left((1+r)^A - 1\right)\right)}{r\left((1+r)^A - 1\right)} \tag{3.11}$$

$$Y = \frac{EAr}{\left((1+r)^A - 1\right)} \tag{3.12}$$

Unfortunately it is not possible to derive an analytical expression for r as a function of the exogenous policy parameters, T and π. However, it is easy to show that a given choice for the (T, π) pair implies at most one equilibrium real interest rate r. To show this it suffices to show that Y (and therefore T) is strictly decreasing in r. From 3.8 it is clear that there are two channels through which a change in r can affect equilibrium steady state Y. It can do so both directly and indirectly by inducing individuals to choose a different value for γ. Differentiating with respect to r (holding γ constant) shows that Y is strictly decreasing in r for $\gamma < A$. Similarly Y is strictly increasing in γ and from 3.2 it is clear that γ is decreasing in r. Thus an increase in r reduces Y whether or not it also decreases γ.

For any given value for π there is a range of values for r consistent with a particular unique γ^*. Denote this range by the open interval $R_{\gamma^*}(\pi) = (r_{\gamma^*}(\pi), r_{\gamma^*-1}(\pi))$. For $r = r_{\gamma^*}(\pi)$ there are two values for γ that give a newborn household the same lifetime utility, γ^* and $\gamma^* + 1$. For a particular choice for π, say $\pi = \widehat{\pi}$, it is clear from 3.8 that for $r \in R_{\gamma^*}(\widehat{\pi})$, the function $T(r; \widehat{\pi})$ is continuous and decreasing. Thus for all values for transfers in the set $T\left(R_{\gamma^*}\left(\widehat{\pi}\right); \widehat{\pi}\right)$ there exists an equilibrium real interest rate $r\left(T, \widehat{\pi}\right)$. Holding constant r and π, a one period increase in γ implies a discrete jump in aggregate transfers. Thus the infimum of $T\left(R_{\gamma^*+1}\left(\widehat{\pi}\right)\right)$ is strictly greater than the supremum of $T\left(R_{\gamma^*}\left(\widehat{\pi}\right)\right)$. However, an equilibrium also exists at $\widehat{\pi}$ for every intermediate value for T with a real interest rate given by $r_{\gamma^*}\left(\widehat{\pi}\right)$. Each of these equilibria corresponds to a unique division of the population between γ^* and $\gamma^* + 1$, the two alternative optimal values for γ at $\left(r_{\gamma^*}\left(\widehat{\pi}\right), \widehat{\pi}\right)$.

The facts that in steady state for any feasible combination of choices for T and π there is a unique equilibrium r and a unique fraction of the population playing each optimal strategy means that for every combination of T and π there are also unique equilibrium values for B and M.

3.3. An example

Variables of interest are graphed in Figure 1 for an economy with the following choices for parameters and policy variables.

Parameters	A	60
	d	0.08
	E	10
Policy variables	T	0
	π	$[0.005, 0.105]$

Note that bond holdings and money balances are aggregates. The average time between bank trips across the population (the average value for γ) is denoted μ_γ.

First I discuss the relationships between variables for ranges of values for π, the inflation rate, such that all households choose the same γ, and therefore μ_γ is constant. I then try to give some intuition for these relationsips for ranges of values for π such that any given household is indifferent at birth between two consecutive integer choices for γ.

For fixed γ and μ_γ, aggregate real money balances do not vary much with π and seignorage $S = \pi M/(1+\pi)$ therefore rises roughly one-for-one with π. For transfers to be held constant rB must be higher in steady states with higher inflation rates so that $T = S - rB$ remains unchanged. Holding γ constant, aggregate bond holdings B depend on π only via the latter's equilibrium relationship with r. Moreover, B is increasing in r, since increasing r increases the bond holdings of older generations. This means that an increase in rB necessitates an increase in r. Thus r is positively related to π over ranges in which γ is constant, and B is also increasing over those intervals.

The average time between trips to the bank is inversely related to the inflation rate. There are ranges for π over which the equilibrium is in mixed strategies in the sense that for some fraction of households the interval between bank trips is γ while for the remainder it is $\gamma + 1$. In Figure 1 an example of this is seen by observing that around $\pi = 0.083$, μ_γ varies smoothly between 1 and 2. Over these ranges for π there is an inverse relationship between equilibrium r and π. This is because for individuals to be indifferent between two different quantities of trips to the bank the nominal interest rate must be roughly constant (see 3.2).

Aggregate money holdings fall over values for π in which the equilibrium is in mixed strategies, since increasing π reduces μ_γ and means that people hold more of their wealth in the form of bonds and less in the form of cash. In terms of the government budget this reduction in the amount of money subject to the inflation tax offsets the effects of a higher inflation tax rate combined with a lower cost of borrowing. When households visit the bank every period, no money is carried from one period into the next and $M = 0$.

Over ranges for π in which the equilibrium is in mixed strategies, the effect on aggregate debt of increasing π depends on two factors. Firstly, increaing π reduces μ_γ allowing households to substitute debt for cash. Secondly, increasing π reduces r over these intervals, reducing the rate at which bond holdings accumulate through a household's lifetime. Whether bond holdings rise or fall with π depends on which effect dominates.

When π exceeds a certain level, the equilibrium r is zero and the average time between bank trips is one period, i.e., individuals go to the bank each period. Aggregate money balances are zero and aggregate bond holdings are independent of π. From 3.12 it is clear that if $\gamma = 0$, then $r = 0$ is the unique real interest rate that ensures $Y = E \Leftrightarrow T = 0$. From 3.11, if $r = 0$ then $B = E(A-1)/2$.

4. Out of Steady State

4.1. The experiment

I consider the effects of a shock to inflation, holding transfers constant. Specifically I assume that the government specifies a path for the price level such that at some date there is a permanent unanticipated change in the rate of inflation. At all dates the real value of transfers to the newborn, T, is constrained to be equal to a fixed constant. Thus the net profits or losses from the governments money and bond profit operations are not permitted to vary. The government implements this policy (assuming it is feasible) by issuing appropriate quantities of money and bonds at each date. I now describe the precise timing of the shock in some detail.

The economy begins in a steady state corresponding to a particular choice for T and π_o, the initial inflation rate. At each date up to and including period t households observe their real money balances and real bond holdings, (both measured in units of the period consumption good) and their life expectancy. They then decide whether or not to visit the asset market, what to trade in the goods market, and how much unsold debt and unspent cash to carry forward. When they make these decisions households assume that the inflation rate and the equilibrium real interest rate will remain constant into the infinite future at the initial steady state levels, π_o and r_o. Between one period and the next the money households carry forward loses value at the rate $1/(1+\pi_o)$ while bonds accumulate interest at the rate $(1+r_o)$.

At the very start of $t+1$ the government announces a new target for the change in the price level between the end of trading in $t+1$ and the start of trading in $t+2$. There is no change in the price level between the time of the announcement and the end of trading in $t+1$. The percentage change in the price level between $t+n$ and $t+n+1$ is announced to be the same as betwen $t+1$ and $t+2$ for all $n \geq 1$. After the announcement at the start of $t+1$ agents re-optimize and choose new time paths for money and bond holdings, given perfect foresight regarding all future prices (including the real interest rate). Since agents are able to return to the asset market before the new inflation rate takes effect, there is no unanticipated inflation in this particular sense. Following trade in the asset and goods markets in $t+1$, prices rise according to the new announced inflation rate π_n between $t+1$ and $t+2$.

4.2. Characterizing optimal post-shock behavior

I describe the algorithm I use to solve for the equilibrium path for this economy following a shock to the path for the price level in the appendix. The method involves supposing that after a certain amount of time the economy converges to the steady state corresponding to the post-shock inflation rate, and solving for a path for the real interest rate such that markets clear along the transition. For the parameter values described below, one feature of the equilibrium is that following the shock different households within the same generation make different decisions. This is consistent with utility maximization because the equilibrium path for the real interest rate is such that households within the same generation (and which are therefore identical at birth) are indifferent between two or more different timing patterns for their visits to the asset market.

Here I describe how to solve the problem of an individual who was born prior to the shock in $t+1$, taking as given a particular timing sequence for bank trips. The problem of an individual born in $t+1$ or later is very similar.

It is helpful to note that provided the return to bonds always strictly dominates the return to holding currency then, in general, individuals will never return to the bank holding a strictly positive amount of real balances. In other words, given that they have perfect foresight, they tailor their withdrawals to exactly match their optimal consumption profiles. The one possible exception is for the first bank trip following the shock. Because individuals re-optimize following the shock they may return to the bank holding strictly positive real balances.

Analogously to the steady state case, optimal consumption between bank trips after the first post-shock bank trip is given by

$$c_{p_j+f} = \frac{w_j}{l_j\left(1+\pi_n\right)^f} \tag{4.1}$$

where p_j is the date of the j^{th} bank trip after the shock, l_j is the length of the j^{th} trip, π_n is the new post-shock inflation rate, and w_j is real balances withdrawn at the start of the j^{th} trip. Note that between bank trips, consumption is independent of r. An individual's utility between trips to the bank is given by

$$U_j = l_j \ln\left(\frac{w_j}{l_j}\right) - \frac{\ln\left(1+\pi_n\right) l_j \left(l_j - 1\right)}{2}$$

Let p_1 be the date of an individual's first bank trip after the shock at the end of period t. Taking as given the quantity of post-shock bank trips $\widehat{Q}$, the lengths of these trips $\{l_j\}_{j=1}^{\widehat{Q}}$, and the expression for consumption between bank trips given in 4.1, an individual at date p_1 solves

$$\max_{\{w_j\}} \sum_{j=1}^{\widehat{Q}} U_j - \widehat{Q}d$$

subject to

$$w_1 + \frac{w_2}{\prod\limits_{f=1}^{l_1}\left(1+r_{p_1+f-1}\right)} + \ldots + \frac{w_{Q_j}}{\prod\limits_{f=1}^{\sum_{j=1}^{\widehat{Q}-1} l_j}\left(1+r_{p_1+f-1}\right)} \leq \prod_{f=t}^{p_1-1}\left(1+r_f\right) b_t + \frac{m_{p_1-1}}{\left(1+\pi_{p_1-1}\right)} = \phi \tag{4.2}$$

The right hand side of 4.2 is real wealth at p_1.

The solution to this problem has a simple form

$$w_1 = \frac{l_1\phi}{\sum\limits_j l_j}, \qquad w_j = \prod_{f=1}^{\sum_{i=1}^{j-1} l_i}\left(1+r_{p_1+f-1}\right)\frac{l_j\phi}{\sum\limits_j l_j},\ j \geq 2 \tag{4.3}$$

where ϕ is given in 4.2.

As discussed above, the only complication emerges when considering behavior prior to the first post-shock bank trip when individuals may choose not to exhaust their real balances. The equations above give optimal consumption given real balances held at the date of the first post-shock bank trip, $m_{p_1-1}/\left(1+\pi_{p_1-1}\right)$. If households return to the asset market immediately following the announcement of the policy change then $p_1 = t+1$ and

$$\frac{m_{p_1-1}}{\left(1+\pi_{p_1-1}\right)} = \frac{m_t}{\left(1+\pi_o\right)}$$

The more complicated case is when $p_1 \geq t+2$. Between $t+1$, the period immediately following the shock and p_1, the date of the first bank trip, optimal consumption will fall at the rate of inflation. Note that an optimal consumption pattern cannot be such that $u'\left(c_{p_1-1}\right) < u'\left(c_{p_1}\right)/\left(1+\pi_{p_1-1}\right)$ - this would imply that an individual had an incentive to reduce consumption in p_1-1, carry extra cash forward and use this to increase consumption in p_1. If $u'\left(c_{p_1-1}\right) > u'\left(c_{p_1}\right)/\left(1+\pi_{p_1-1}\right)$ then the cash in advance constraint is binding and an individual will exhaust his cash prior to returning to the asset market. Thus if an individual is to carry strictly positive domestic real balances into the bank at p_1 it must be the case that

$$u'\left(c_{p_1-1}\right) = \frac{u'\left(c_{p_1}\right)}{\left(1+\pi_{p_1-1}\right)} \Leftrightarrow c_{p_1}\left(1+\pi_n\right) = c_{p_1-1} = \frac{c_{t+1}}{\left(1+\pi_n\right)^{\left((p_1-1)-(t+1)\right)}} \tag{4.4}$$

Using 4.1 and 4.3 we arrive at the following expressions for c_{p_1} and c_{t+1}

$$c_{p_1} = \frac{\prod_{f=t}^{p_1-1}\left(1+r_f\right)b_t + \left(m_{p_1-1}/\left(1+\pi_n\right)\right)}{\sum_j l_j}$$

$$c_{t+1} = \frac{\left(m_t/\left(1+\pi_o\right)\right) - m_{p_1-1}\left(1+\pi_n\right)^{\left((p_1-1)-(t+1)\right)}}{\left(p_1-(t+1)\right)}$$

Substituting these into the previous equation we can solve for m_{p_1-1}

$$\frac{\prod_{f=t}^{p_1-1}\left(1+r_f\right)b_t + \left(m_{p_1-1}/\left(1+\pi_n\right)\right)}{\sum_j l_j}\left(1+\pi_n\right) = \frac{\left(m_t/\left(1+\pi_o\right)\right) - m_{p_1-1}\left(1+\pi_n\right)^{(\eta-1)}}{\eta\left(1+\pi_n\right)^{(\eta-1)}}$$

$$m_{p_1-1} = \max\left\{0, \left(\frac{m_t}{\left(1+\pi_o\right)} - \frac{\eta}{\sum_j l_j + \eta}\left(\frac{m_t}{\left(1+\pi_o\right)} + \left(1+\pi_n\right)^{\eta}\prod_{f=t}^{p_1-1}\left(1+r_f\right)b_t\right)\right)/\left(1+\pi_n\right)^{\eta-1}\right\}$$

where $\eta = \left(p_1-(t+1)\right)$ is the number of periods between the announcement of the new policy and the date of the first subsequent bank trip.

It is important to verify that between any two periods the return to holding bonds strictly dominates the return to holding currency. If this were not the case there would exist a motive to hold money as a store of value.

4.3. Policy parameters

In what follows an 'o' subscript denotes a pre-shock steady state value and a 'n' subscript a value for the variable following the shock. I choose the following parameter and policy values for the pre-shock steady state.[2]

Parameters	A	6
	d	0.0733
	E	10
Policy variables	$T_o = T_n$	0
	π_o	$(1+0.04)^{\frac{1}{4}} - 1$
	π_n	$(1+0.06)^{\frac{1}{4}} - 1$

One feature of these choices is that in both the pre-shock steady state and the steady state corresponding to π_n, the optimal time between bank trips is three. Thus in both steady states households make one return trip to the asset market, and this trip occurs in the middle of their lifetimes.

4.4. Results

I find that the shock to inflation described above implies a large immediate increase in the real interest rate (see Figure 2). The initial large rise is followed by a large fall one period later. After this date the real interest rate oscillates in a narrow range around its value in the steady state associated with the post shock inflation rate, π_n. As time passes, all other real variables converge to their values in this steady state.

The initial movements in the real interest rate may be explained as follows. Aggregate real consumption each period is equal to the constant aggregate endowment. For aggregate nominal spending to sustain the new steeper path for the price level between $t+1$ and $t+2$, aggregate nominal spending at $t+2$ must be higher than it would have been in the absence of the shock. However, the nominal spending of households who do not visit the asset market in $t+2$ is predetermined and independent of the price level in $t+2$. If prices are higher, the real consumption of these households in $t+2$ must be lower than that of households of the same age in the initial steady state. Thus, for the goods market to clear, both the nominal and the real spending of those households *in* the asset market at $t+2$ must increase relative to the initial steady state. To induce households in the market to undertake this increase in spending, the government's monetary policy

[2] In the computational algorithm I set $\beta = 200,000$ (see 6.1).

must drive up the real interest rate in $t+1$, the period of the shock, to thereby increase the real value of the bond holdings (and therefore the consumption) of households returning to the asset market in $t+2$. Because the households that return to the bank in $t+2$ maintain above average consumption until their next visit to the bank, a low value for r_{t+2} is required to ensure that the cumulative interest earnings of households returning to the bank in $t+3$ are not too high.

For a constant real interest rate, the steady state analysis indicated that household utility is maximized by evenly spaced bank trips. This is not necessarily the case following a shock to inflation because movements in the path for the real interest rate introduce an additional consideration into households' portfolio decisions. In particular, given a constant inflation rate, the opportunity cost of holding money is increasing in the period real interest rate. In this example, the equilibrium path for the economy turns out to be in mixed strategies; the path for the real interest rate is such that at birth identical households are indifferent between different timing sequences for their trips to the asset market. The table below describes the percentage of each living generation that visits the asset market at each date. At date $t+1$ the only households in the asset market are the newborn (of age 0), and the households born three periods ago (of age 3) who were planning to return in $t+1$ prior to the announcement of the new inflation rate. Without the shock, all of the households born in $t+1$ would return to the asset market just once in $t+4$. However, because the increase in inflation is associated with a rise in the real interest rate, this generation has an incentive to sell a smaller fraction of its bonds (and thereby receive high interest payments on a larger fraction of its wealth), and to return sooner to the bank. This helps to explain why 16% of the households born in $t+1$ choose to visit the asset market in both $t+2$ and $t+5$, while another 1% return in $t+3$ and $t+5$. Similarly, because r_{t+7} and r_{t+8} are relatively low, a higher than average fraction of the population chooses to visit the bank in $t+7$.

If timing were exogenous, r_{t+1} would have to rise sufficiently for households who are scheduled to return to the bank in $t+2$ to withdraw enough real balances for aggregate nominal spending to increase in line with the new inflation rate. When timing is endogenous, the number of households in the market in $t+2$ is above average, and thus the per-capita real bond holdings and consumption of these households do not have to be as large. This reasoning suggests that endogenizing the timing of asset market visits implies a smaller initial increase in the real interest rate than would be seen if timing were exogenous.

Not surprisingly there is a high correlation between the fraction of the population in the asset market in a given period and aggregate money balances carried out of the period (individuals only return to the asset market to replenish their cash balances). The path for aggregate bond holdings is almost a mirror image of that for real balances. The path for seignorage looks quite similar to that for real balances; seignorage is above average when real balances have increased from the previous period.

% of generation of each age in asset market at each date							% of total population in asset market
	age						
date	5	4	3	2	1	0	
t	0	0	100	0	0	100	33.3
$t+1$	0	0	100	0	0	100	33.3
$t+2$	0	0	100	0	16	100	36.0
$t+3$	0	0	100	1	0	100	33.5
$t+4$	0	0	83	0	0	100	30.5
$t+5$	0	17	100	1	0	100	36.3
$t+6$	0	0	99	0	0	100	33.2
$t+7$	0	1	100	13	0	100	35.7
$t+8$	0	0	87	6	0	100	32.2
$t+9$	0	13	94	0	0	100	34.5
$t+10$	0	6	100	0	0	100	34.3
$t+11$	0	0	100	4	0	100	34.0

5. Conclusion

In the economy described in this paper, over-lapping generations of households choose when to trade assets subject to paying a fixed cost of entering the market. An increase in the rate of inflation that is constrained to be neutral in its effects on the government budget position is found to substantially increase the real interest rate in the period it is announced. However, after two periods, the real interest rate is always close to its value in the steady state corresponding to the new inflation rate.

When households are free to decide when to visit the bank following a shock to inflation, they have an incentive to choose the timing and size of their bond sales such that their money balances are relatively small when the real interest rate is relatively high. Whether a household's potential gain from re-optimizing its portfolio immediately after an inflation shock outweighs the fixed cost of returning to the asset market depends on the household's life expectancy, its wealth, and the fraction of this wealth that is in the form of debt. For example, households which have relatively low total wealth or which have recently visited the asset market will not immediately visit the asset market unless expected price movements are very large. On the other hand, for the newborn generation and for households who had planned to be in the asset market prior to the announcement of the policy change, relatively small expected future interest rate movements can induce immediate changes in the planned pattern of withdrawals. In the example in the paper, a rise in the real interest rate in the period of the shock followed by an expected fall in the next period induces some of the households born at the time of the shock to plan to exhaust their cash balances in the period of the shock, and to return to the asset market in the next period. Older households do not revise the previously planned timing pattern for their asset market trips.

This discussion helps to explain why large deviations from the real interest rate corresponding to the eventual steady state do not persist. As time passes, the fraction of the population born after the shock increases. At birth, these households choose time profiles for money and bond holdings given perfect foresight of future prices. Since newborn households' planned timing decisions are more sensitive to the expected path for the real interest rate than are those of older households, the fraction of households in the asset market at a given date becomes increasingly sensitive to the path of the interest rate around that date. Nominal spending is increasing in the fraction of the population in the asset market since, in Baumol-Tobin fashion, household consumption falls during the period between bank trips. Consequently, as time passes, aggregate nominal spending becomes increasingly sensitive to real interest rate movements. Thus smaller movements in interest rates are required to meet a target inflation rate.

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6. Appendix - The Solution Algorithm

The way I solve for an equilibrium following the shock is as follows.

Step 1

Compute the unique equilibrium real interest rate corresponding to the initial steady state inflation rate and choice for transfers.

Step 2

Make an initial guess at the future path of the real interest rate over the next $A \times g$ periods where $g \geq 1$.

Step 3

Given this guess and the path for inflation solve for the optimal behavior of the representative agent of each generation alive at the time of the shock as follows:

- Pick one generation
- Compute his vector of asset holdings at the start of period $t + 1$.
- Pick a particular timing sequence for bank trips and solve for optimal withdrawals at each trip and optimal consumption between trips (how to do this is discussed in the main body of the paper).
- Compare this level of utility with that of (ideally) all other possible timing sequences.
- Assign a probability to an individual choosing each sequence that is increasing in the level of utility associated with it. Specifically I assume that the probability that an individual chooses a sub-optimal policy, α_s, is given by

$$\alpha_s = \max \left\{ 0.5 - \beta \left(\frac{U_o - U_s}{U_o} \right), 0 \right\} \tag{6.1}$$

  where U_o is lifetime utility from the optimal strategy and U_s is lifetime utility from the sub-optimal strategy. Note that if β is large, $\alpha_s > 0 \Rightarrow U_o \simeq U_s$. Thus as $\beta \to \infty$, this behavior converges to a utility maximizing strategy.

- Repeat for every generation alive in period t and for the representative individual born in $t + 1$ assuming that his real wealth at birth is $E + T$, where T is the policy choice for transfers to the newborn.

Step 4

Solve for aggregate variables in period $t + 2$.

Step 5

Iterate for $A \times z$ periods.

Step 6

Check whether the goods market clears in every period. Aggregate demand for real balances and for real bond holdings are derived by aggregating across optimal individual choices. Assuming that the government conducts its money market operations to satisfy these demands, the markets for money and bonds will clear. Since individuals' budget constraints are always satisfied, by Walras' Law we know that, if the sequence for the real interest rate is such that the goods market clears in each period, then the government's budget constraint must also be satisfied at the target level for transfers (this is a useful error-finding check).

Step 7

If markets clear in each period and the real interest rate appears to be converging to the value corresponding to the steady state for the post-shock parameter values conjecture that the real interest rate sequence constitutes an equilibrium. Otherwise guess a new interest rate sequence and repeat.

Figure 1: Steady state with T=0

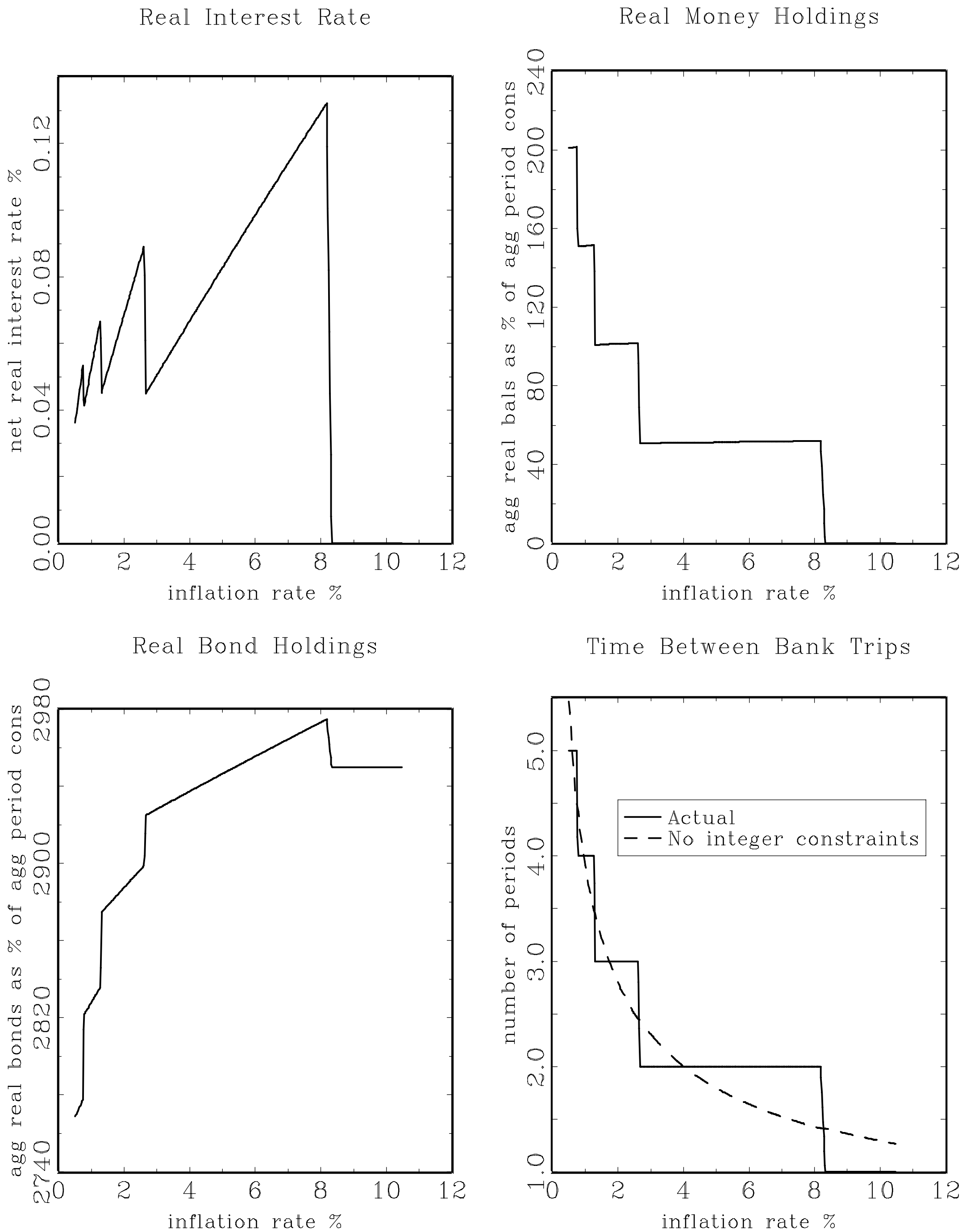

Figure 3: Endogenous timing impulse responses