

*Optimal Progressivity
with Age-Dependent Taxation*

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How progressive should labor income taxation be?

- Arguments **in favor** of progressivity:
 - ▶ Redistribution with respect to unequal initial conditions
 - ▶ Redistribution over life cycle when credit constraints are tight
 - ▶ Public insurance when markets are missing

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 - ▶ Human capital investment distortion
- **Q: Life-cycle** → optimal progressivity should vary with age?
- We take a **Ramsey-approach** to this question

HSV tax-transfer system

$$T(y) = y - \lambda y^{1-\tau}$$

- $\tau > 0 \Rightarrow T'(y) > \frac{T(y)}{y}$ (progressive system)

HSV tax-transfer system

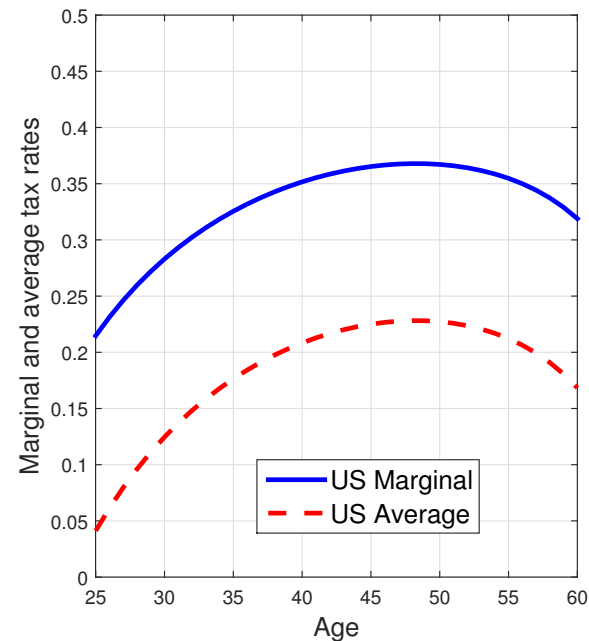
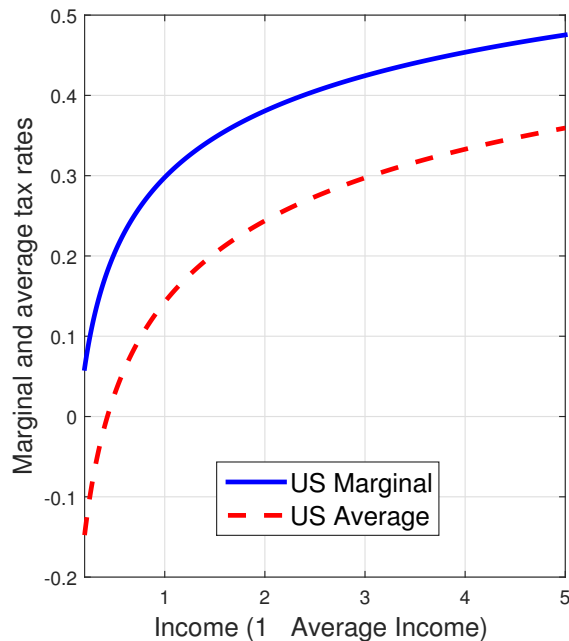
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- It preserves **analytical tractability**
- It **closely approximates** actual US system ($\tau^{US} = 0.181$)



This Paper

- Generalize HSV tax/transfer system to allow **age variation**:

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- **OLG equilibrium model** with:

1. differential disutility of work & learning ability [ex-ante heter.]
2. partial insurance against earnings risk [ex-post uncertainty]
3. age profile for productivity and disutility of work [life cycle]
4. flexible labor supply [static choice]
5. skill investment [dynamic choice]

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- **Extension**: numerically solved model with **saving and borrowing**

ENVIRONMENT

Preferences

- **Preferences** over consumption (c), hours (h), publicly-provided goods (G), and skill-investment (s) effort:

$$U_i = -v_i(s_i) + \mathbb{E}_0 \sum_{a=0}^A \beta^a u_i(c_{ia}, h_{ia}, G)$$

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$$v_i(s_i) = \frac{1}{(\kappa_i)^{1/\psi}} \cdot \frac{s_i^{1+1/\psi}}{1+1/\psi}$$

$$\kappa_i \sim \text{Exp}(1)$$

$$u_i(c_{ia}, h_{ia}, G) = \log c_{ia} - \frac{\exp[(1+\sigma)(\varphi_i + \bar{\varphi}_a)]}{1+\sigma} (h_{ia})^{1+\sigma} + \chi \log G$$

$$\varphi_i \sim \mathcal{N}\left(\frac{v_\varphi}{2}, v_\varphi\right)$$

Individual Wages and Earnings

- **Wages:**

$$\log z_{ia} = x_a + \alpha_{ia} + \varepsilon_{ia}$$

- ▶ x_a deterministic age-productivity profile

- ▶ $\alpha_{ia} = \alpha_{i,a-1} + \omega_{ia}, \quad \omega_{ia} \sim \mathcal{N}\left(-\frac{v_\omega}{2}, v_\omega\right)$ [perm. uninsurable]

- ▶ $\varepsilon_{ia} = \varepsilon_{i,a-1} + \eta_{ia}, \quad \eta_{ia} \sim \mathcal{N}\left(-\frac{v_\eta}{2}, v_\eta\right)$ [private insurance]

- **Pre-tax earnings:**

$$y_{ia} = \underbrace{p(s_i)}_{\text{skill price}} \times \underbrace{\exp(x_a)}_{\text{deterministic age-profile}} \times \underbrace{\exp(\alpha_{ia} + \varepsilon_{ia})}_{\text{shocks}} \times \underbrace{h_{ia}}_{\text{hours}}$$

- **Asset markets:**

- ▶ Within-period insurance against ε , but no inter-temporal trade

Technology

- **Output** a CES aggregator over continuum of skill types s :

$$Y = \left[\int_0^{\infty} N(s)^{\frac{\theta-1}{\theta}} ds \right]^{\frac{\theta}{\theta-1}}, \quad \theta \in [1, \infty)$$

- **Skill price:** $p(s) =$ marginal product of $N(s)$

$$\log p(s) = \frac{1}{\theta} \log Y - \frac{1}{\theta} \log N(s)$$

- Aggregate **resource constraint:**

$$Y = \sum_{a=0}^A \int_{i=0}^1 c_{i,a} di + G$$

Government

- Government budget constraint (no government debt):

$$G = \sum_{a=0}^A \int_0^1 [y_i - \lambda_a y_i^{1-\tau_a}] di$$

- Planner chooses policy **once and for all** s.t. balanced budget

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- Government chooses G , or equivalently $g \equiv \frac{G}{Y}$
 - ▶ Optimal public good provision: $g^* = \frac{\chi}{1+\chi}$
 - ▶ **Samuelson condition**: $MRS_{C,G} = MRT_{C,G} = 1$

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- Government **chooses vector** $\{\lambda_a, \tau_a\}_{a=0}^A$

Skill Prices and Skill Investment

- Skill price has the **Mincerian form**:

$$\log p(s) = \pi_0(\bar{\tau}) + \pi_1(\bar{\tau})s(\kappa; \bar{\tau})$$

- Closed form expressions for $\pi_0(\bar{\tau})$ and $\pi_1(\bar{\tau})$
- Optimal **skill investment linear in κ** :

$$s(\kappa; \bar{\tau}) = [(1 - \bar{\tau}) \pi_1(\bar{\tau})]^\psi \cdot \kappa$$

where: $\bar{\tau} = \frac{1-\beta}{1-\beta^{A+1}} \sum_{a=0}^A \beta^a \tau_a$

- Distribution of $p(s)$ is **Pareto with parameter θ**

Equilibrium c, h Allocations

$$\log c_a = \log \lambda_a + (1 - \tau_a) \left[\frac{\log(1 - \tau_a)}{1 + \sigma} - (\varphi + \bar{\varphi}_a) + \log p(s) + x_a + \alpha \right] + \mathcal{C}_a$$

- Unaffected by individual insurable shocks ε
- \mathcal{C}_a is increasing in $v_{\varepsilon a} \Rightarrow$ higher productive efficiency

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$$\log h_a = \frac{\log(1 - \tau_a)}{1 + \sigma} - (\varphi + \bar{\varphi}_a) + \left(\frac{1 - \tau_a}{\sigma + \tau_a} \right) \varepsilon - \frac{1}{\sigma + \tau_a} \mathcal{C}_a$$

- log-utility \rightarrow hours unaffected by $\{\lambda_a, p(s), x_a, \alpha\}$
- $\frac{1 - \tau_a}{\sigma + \tau_a}$ is the **tax-modified Frisch elasticity**

SOCIAL WELFARE

Social Welfare Function

- Planner puts equal weight on all currently alive agents, discounts utility of future cohorts at rate $\gamma = \beta$
- Start with policy that maximizes steady state welfare
- Then consider policy that maximizes welfare including transition
 - ▶ Transition driven by irreversible skill choice of existing cohorts
- Easy to optimize over large vector of policy choices because social welfare has a closed-form

QUALITATIVE ANALYSIS

Optimal Policy: Conditions for Age Invariance of τ_a

1. Optimal $\{\tau_a^*, \lambda_a^*\}$ are age-invariant iff:
 - (a) $\theta = \infty$ or $\beta = 1$: no skill investment or no discounting
 - (b) $v_\eta = 0$: flat profile of insurable productivity dispersion
 - (c) $v_\omega = 0$: flat profile of uninsurable productivity dispersion
 - (d) $\{x_a, \bar{\varphi}_a\}$ constant: flat profile of average efficiency / disutility

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3. Given any profile for $\{\tau_a\}$, the optimal profile for $\{\lambda_a^*\}$ equates average consumption by age

Optimal Age-Varying Progressivity

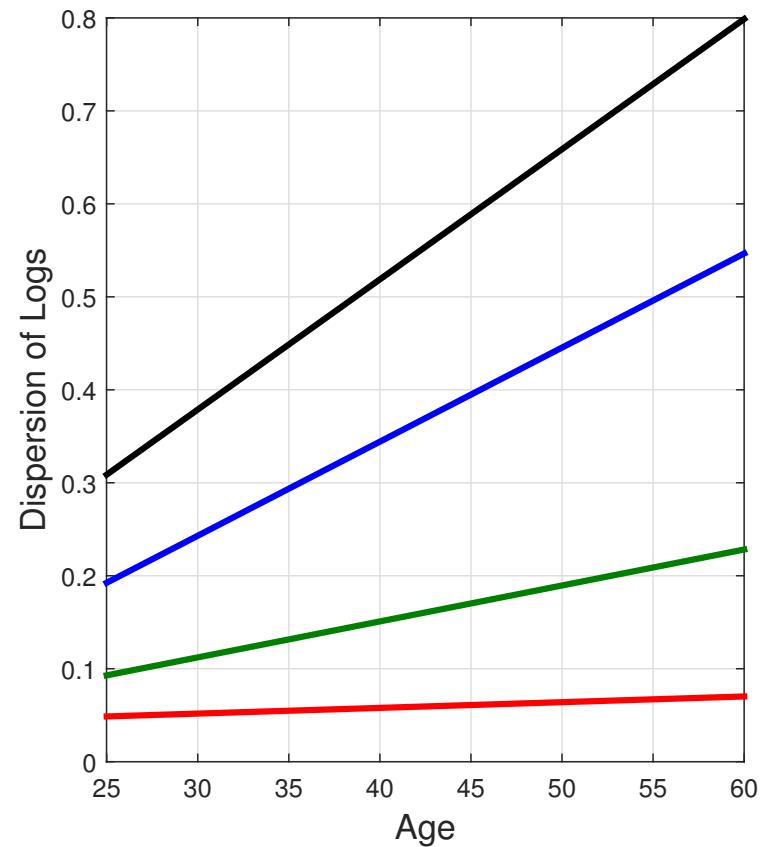
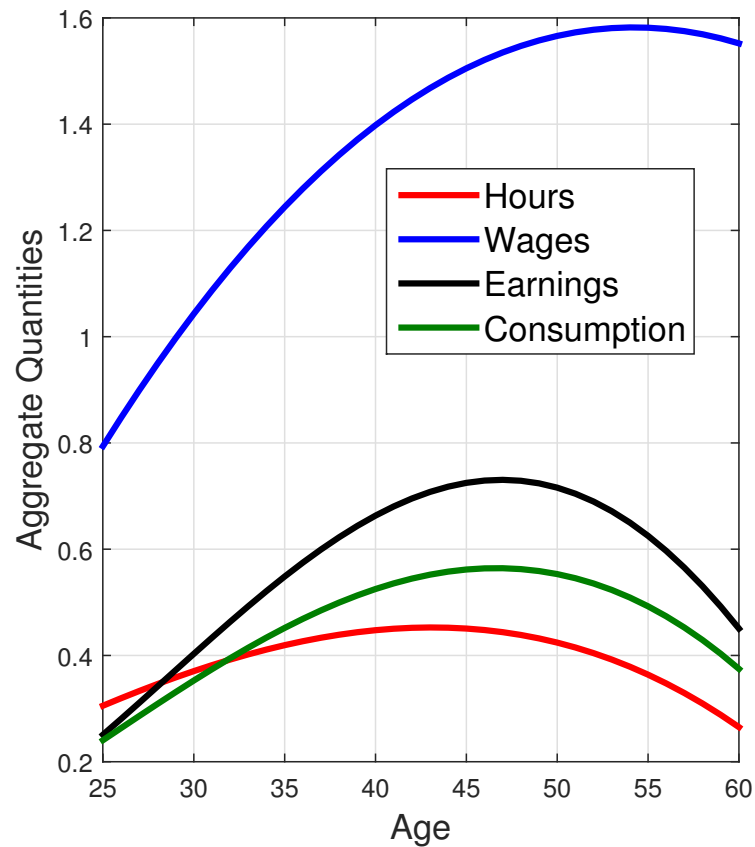
- Four separate channels that shape age profile of progressivity:
 - Individual Discounting Channel**
Lower β implies steeper optimal profile $\{\tau_a^\}$*
 - Insurable Risk Channel**
Permanent insurable risk ($v_\eta > 0$) tilts optimal $\{\tau_a^\}$ towards zero more at old ages than at young ages*
 - Uninsurable Risk Channel**
Permanent uninsurable risk ($v_\omega > 0$) implies optimal profile $\{\tau_a^\}$ increasing in age*
 - Life-Cycle Channel**
Upward-sloping age profile of efficiency net of disutility of work $\{x_a - \bar{\varphi}_a\}$ implies decreasing optimal profile $\{\tau_a^\}$*

CALIBRATION

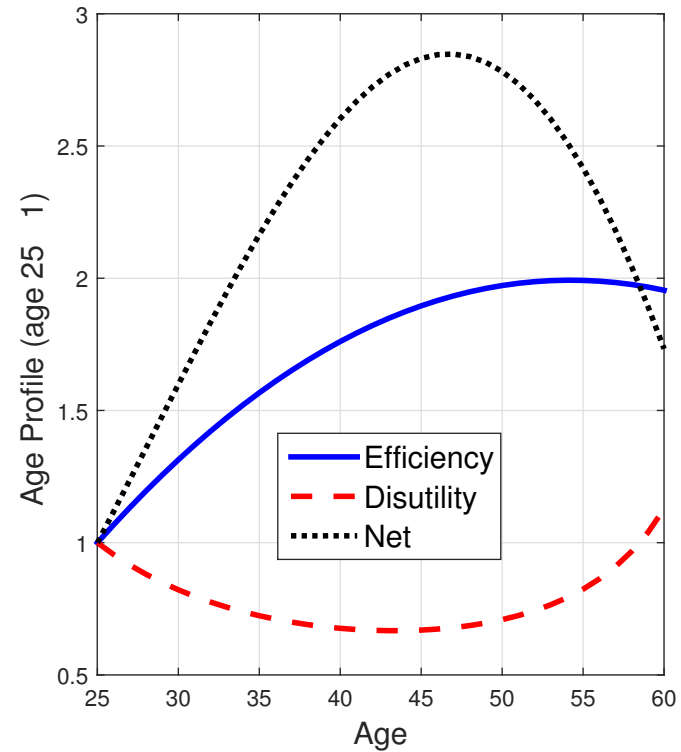
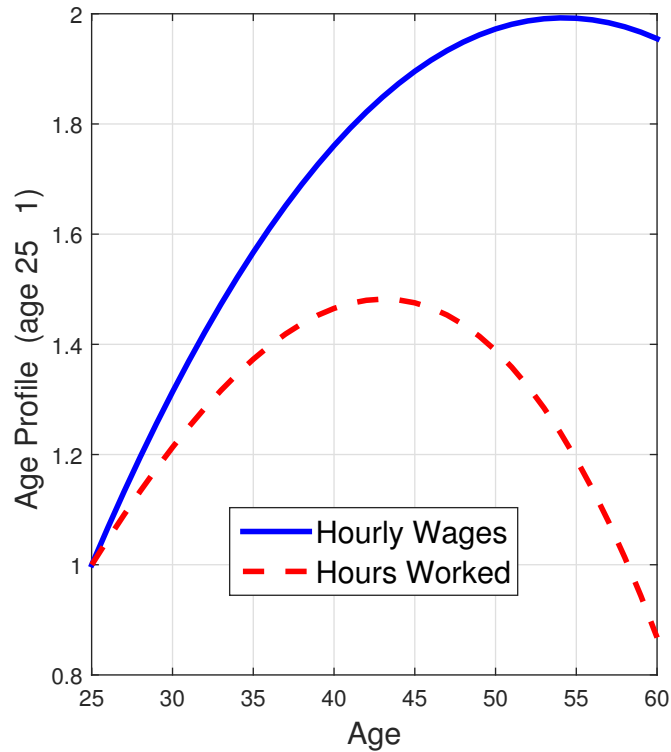
Parameterization

- Parameter vector $\{\chi, \tau^{US}, \sigma, \psi, \theta, v_\varphi, v_\omega, v_{\varepsilon 0}, v_\eta\}$ and $\{x_a, \bar{\varphi}_a\}$
- Assume observed $G/Y = 0.19 = g^*$ $\rightarrow \chi = 0.233$
- US progressivity estimated on micro data $\rightarrow \tau^{US} = 0.181$
- Frisch elasticity (micro-evidence ~ 0.5) $\rightarrow \sigma = 2$
- Price-elasticity of skill investment $\rightarrow \psi = 0.65$
- $var_0(\log c) \rightarrow \theta = 3.12$
- $var(\log h) \rightarrow v_\varphi = 0.035$
- $cov(\log w, \log c) \rightarrow v_\omega = 0.0058$
- $cov(\log w, \log h) \rightarrow v_{\varepsilon,0} = 0.09, v_\eta = 0.044$
- $\{x_a, \bar{\varphi}_a\}$ estimated to match age profiles wages of and hours

Life-cycle Means and Variances



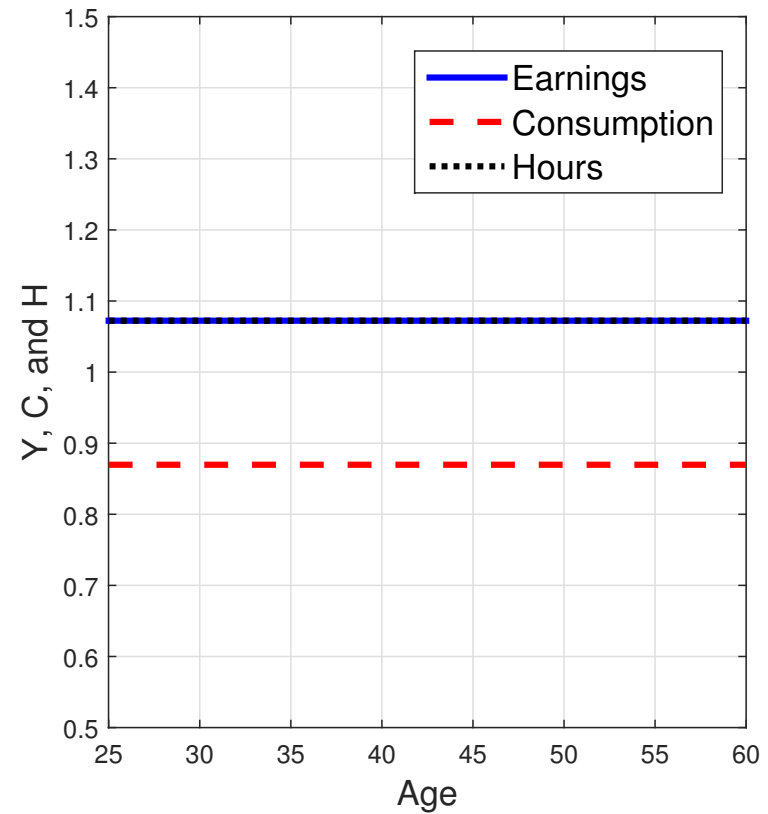
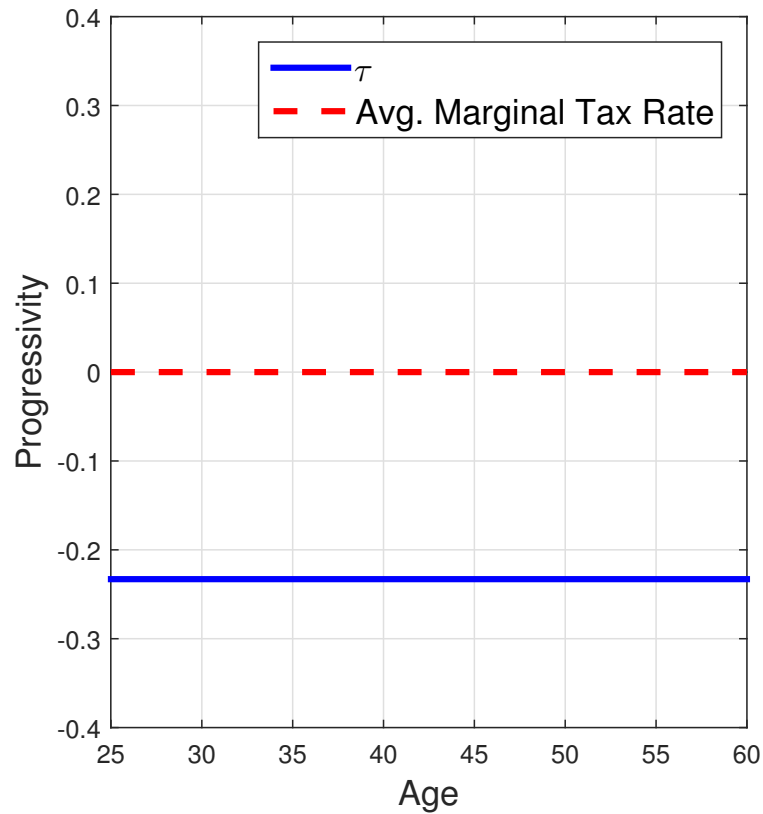
Age Profile for Efficiency and Disutility of Work



- Note: Net effect strongly hump-shaped

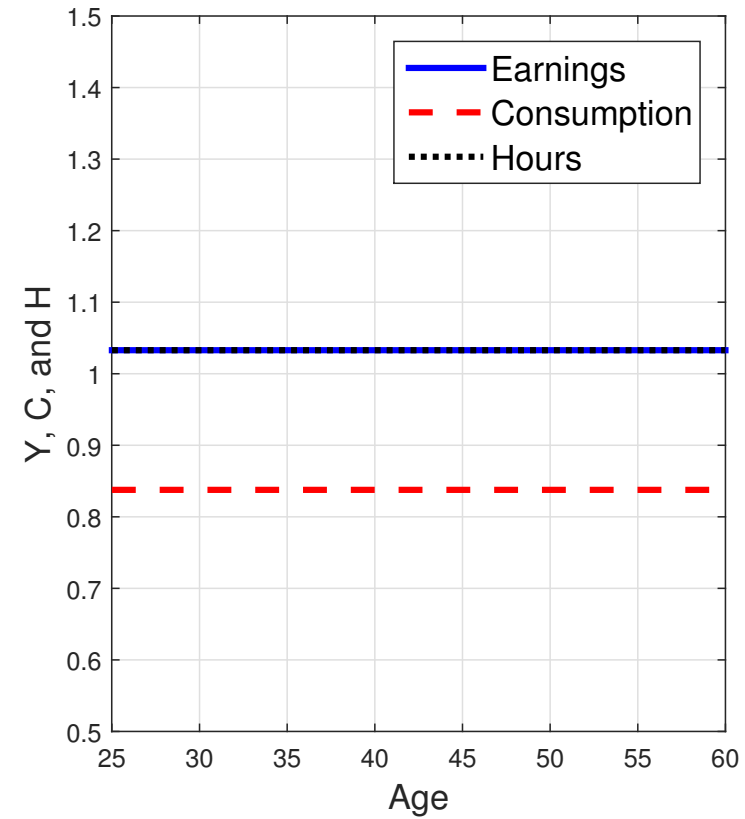
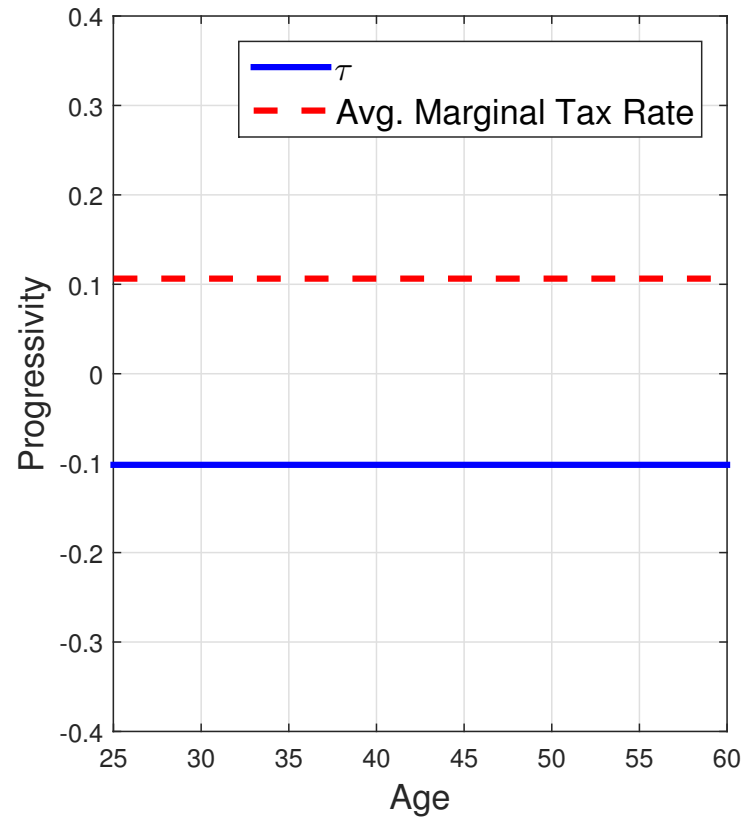
QUANTITATIVE RESULTS

Representative Agent



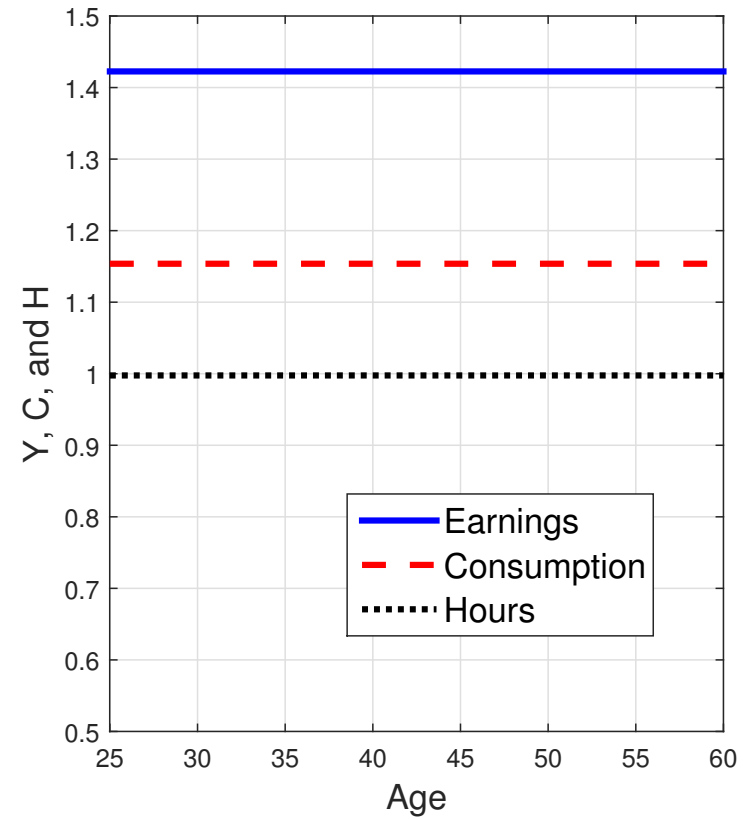
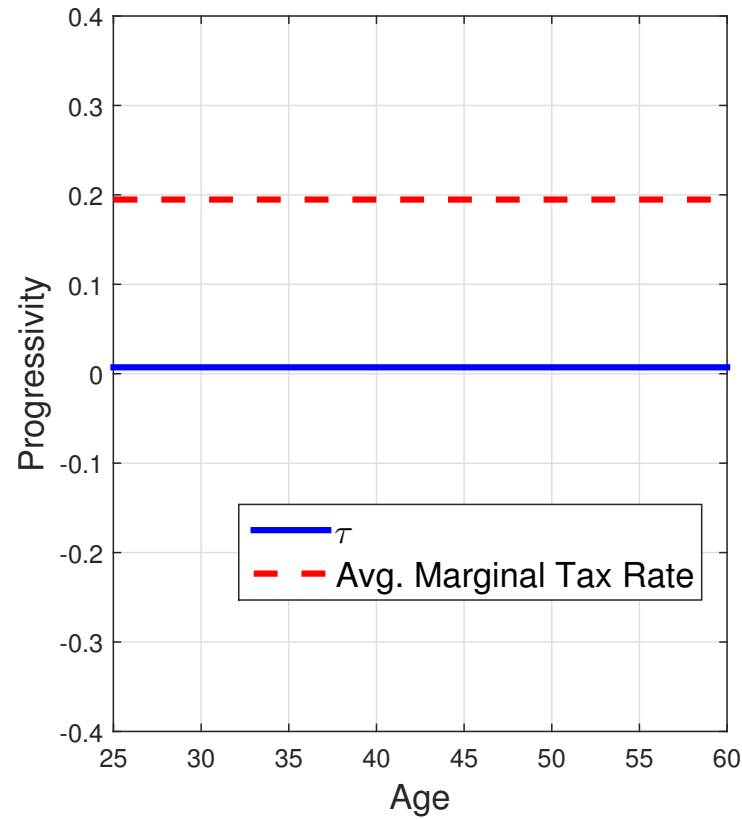
- Optimality: $\tau_a^* = -\chi$

Add Heterogeneity in Disutility of Work (φ)



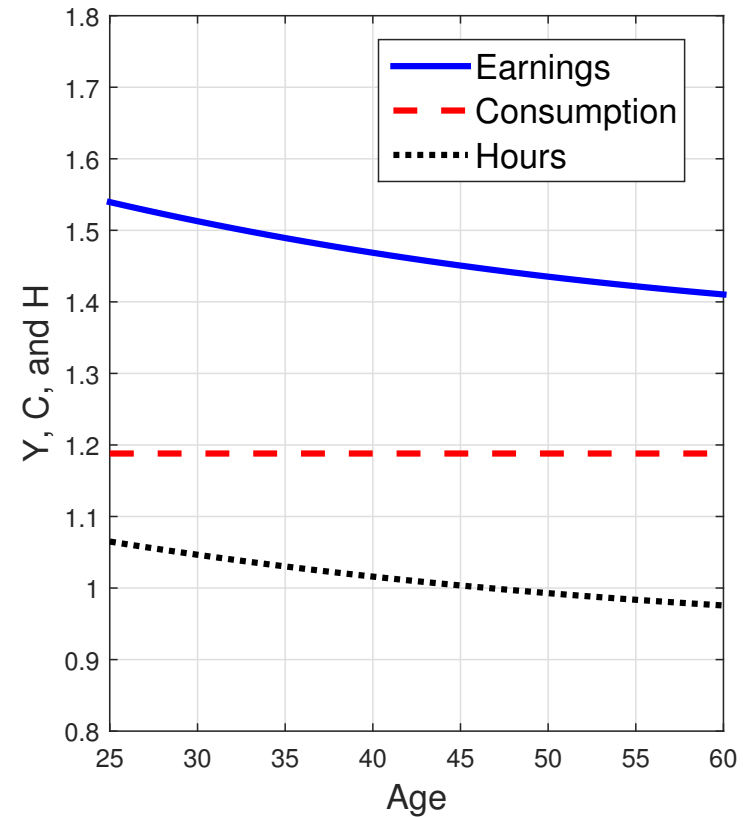
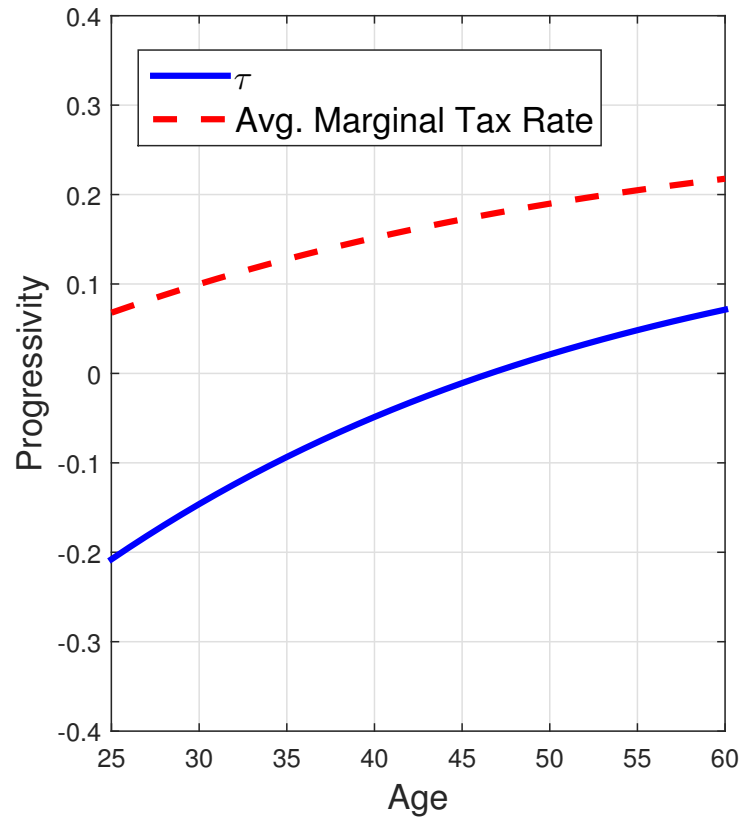
- τ_a^* still flat but shifted up (redistribution) \Rightarrow lower labor supply

Add Heterogeneity in Ability (θ finite)



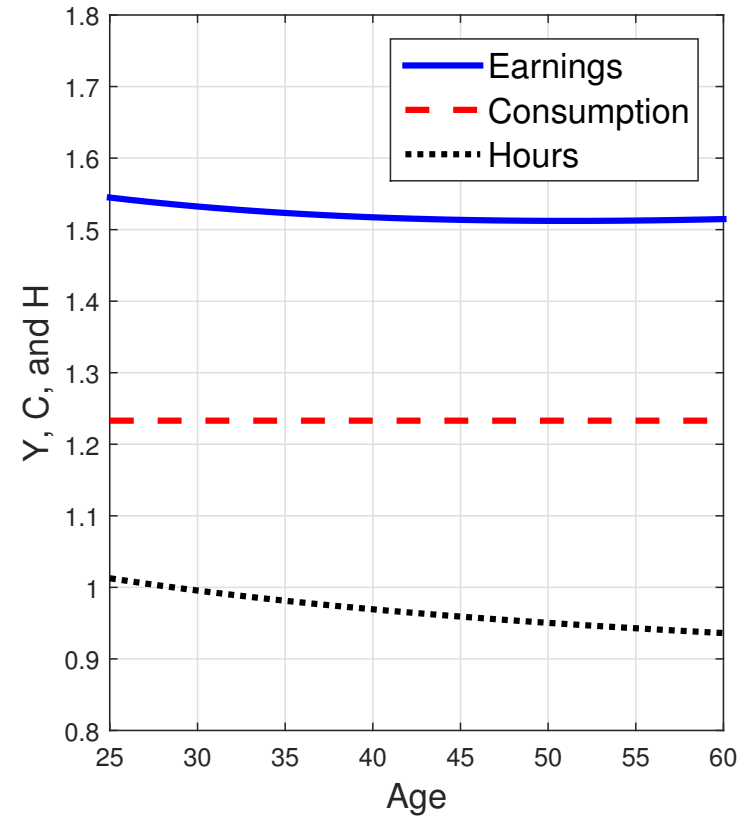
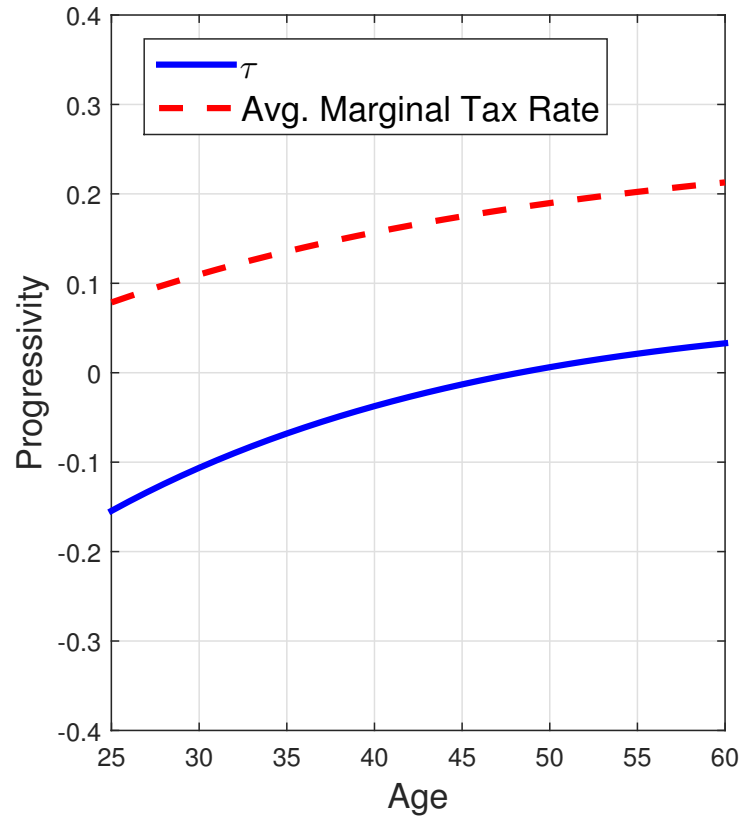
- τ_a^* still flat but shifted further up (redistribution > distortion)

Add Discounting ($\beta < 1$)



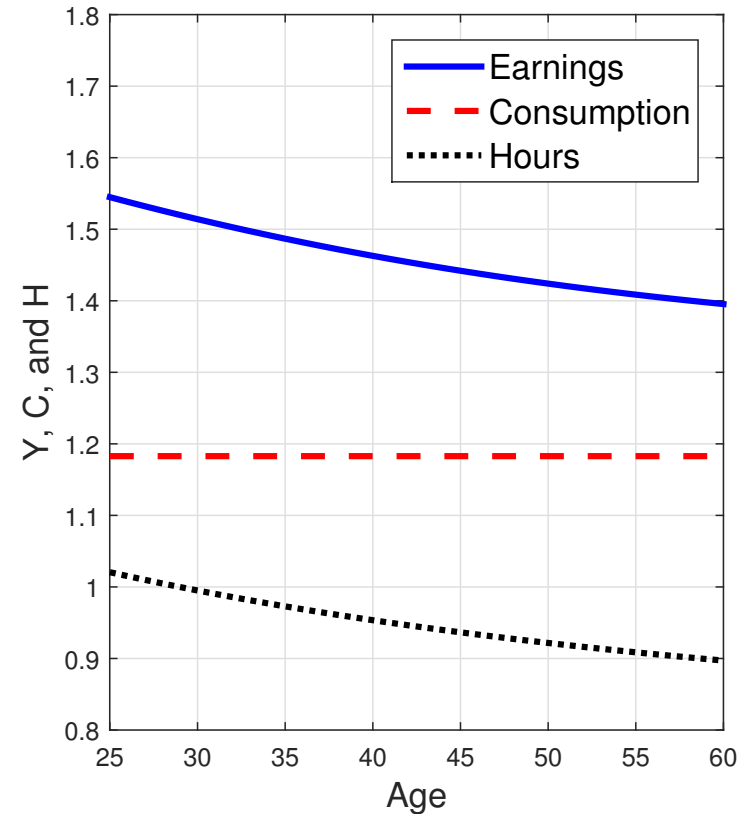
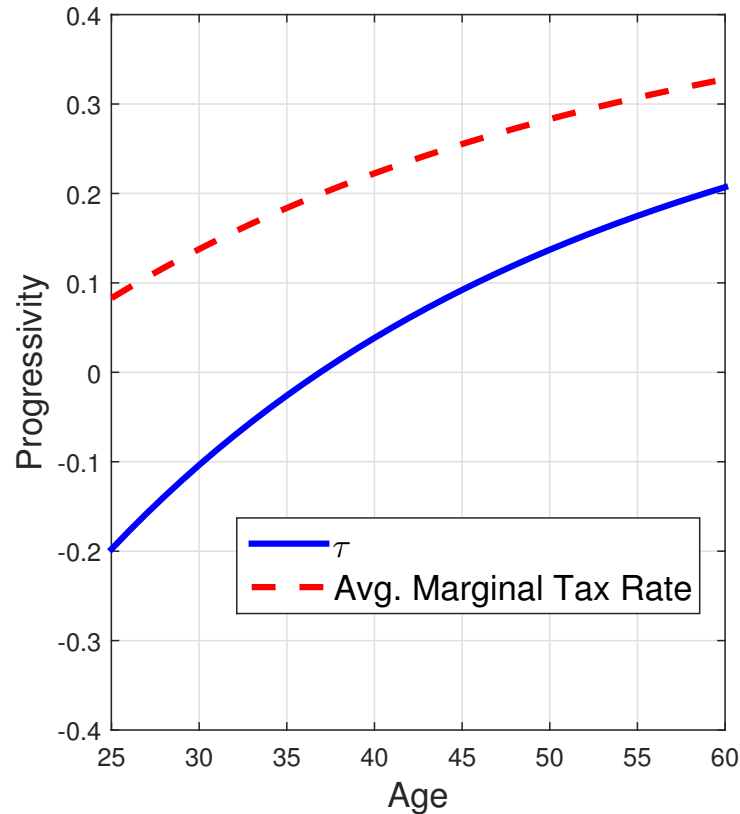
- Skill choice depends on $\bar{\tau} = \frac{1-\beta}{1-\beta^{A+1}} \sum_{a=0}^A \beta^a \tau_a$

Add Insurable Risk ($v_\varepsilon > 0$)



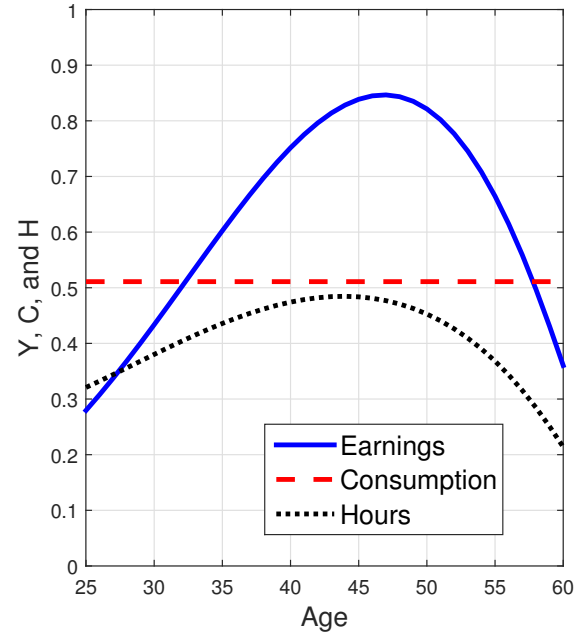
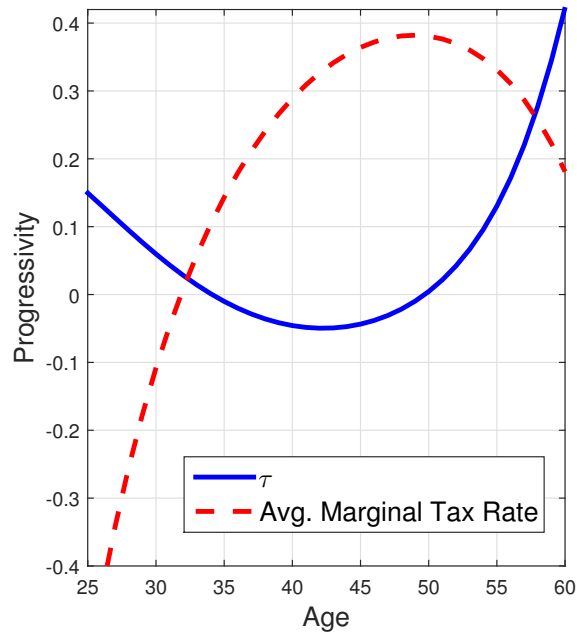
- Profile for τ_a^* is flattened towards zero

Add Uninsurable Risk ($v_\omega > 0$)



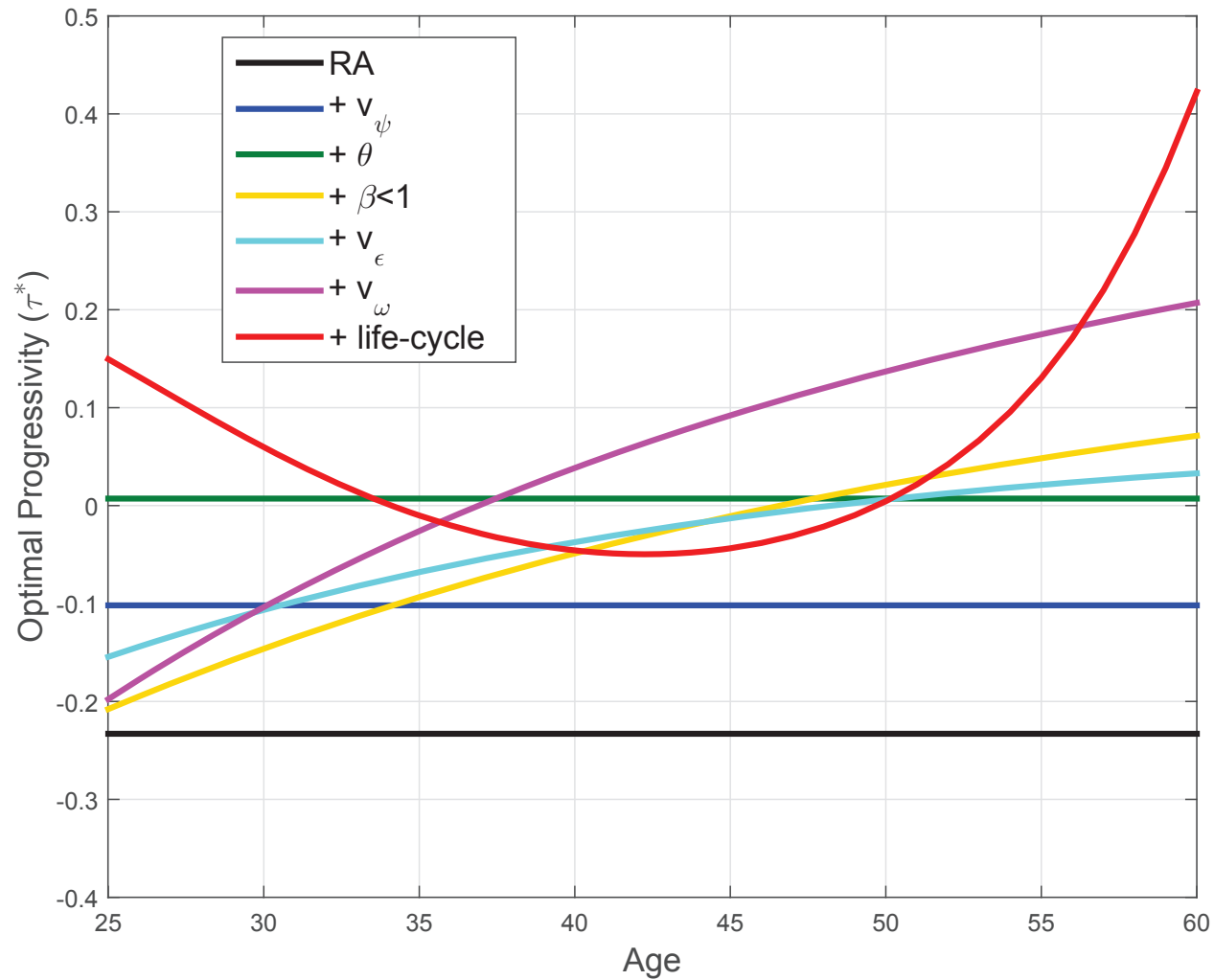
- Profile for τ_a^* steeper: more redistribution needed later in life since uninsurable risk cumulates

Add Life Cycle $\{x_a, \bar{\varphi}_a\}$

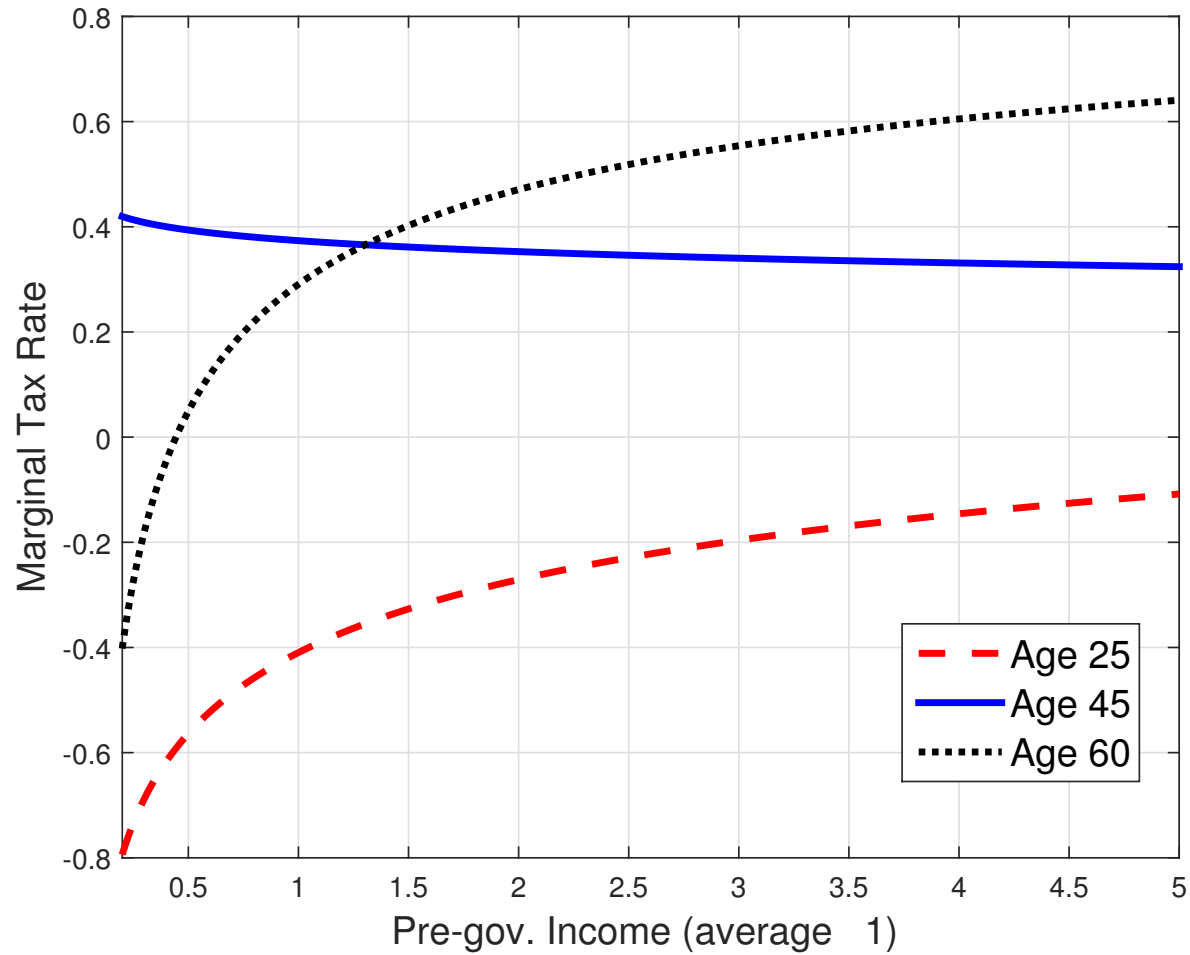


- $x_a - \bar{\varphi}_a$ hump-shaped + τ_a distorts labor supply $\Rightarrow \tau_a$ U-shaped
- If $\{x_a, \bar{\varphi}_a\}$ is the only source of heterogeneity, then optimal $\{\tau_a^*\}$ equates labor wedge by age (tax smoothing)
- Recall $\{\lambda_a^*\}$ equates average C by age (resource wedge)

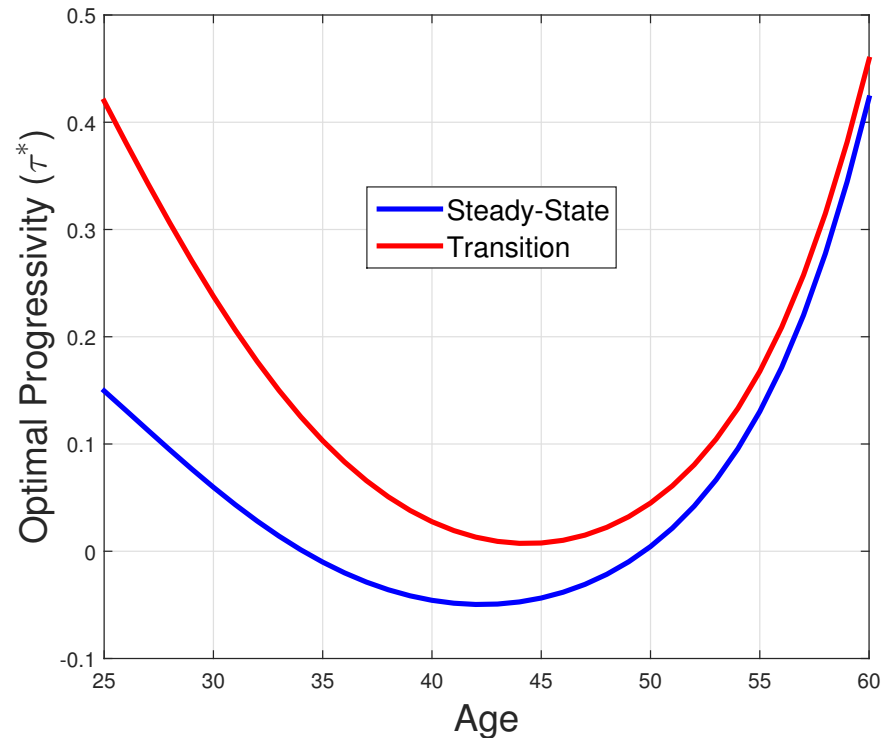
All Channels: Summary



All Channels: Marginal Tax Rates by Age



All Channels: SS vs Transition



- Sunk skill investment channel: $\Rightarrow \tau_a^*$ higher at all ages
- Planner discounting channel $\Rightarrow \tau_a^*$ increases more for young

Welfare Gains

- Equivalent variation in lifetime consumption

INTERTEMPORAL TRADE

Introducing Borrowing and Lending

- Two modifications to baseline model:
 1. **Non-contingent bonds in zero net supply** s.t. credit limit
 2. No insurable productivity shocks ($v_{\varepsilon,0} = v_{\eta} = 0$)

Introducing Borrowing and Lending

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 1. **Non-contingent bonds in zero net supply** s.t. credit limit
 2. No insurable productivity shocks ($v_{\varepsilon,0} = v_{\eta} = 0$)
- **Numerical solution:**
 1. Skill investment decision rules unchanged
 2. Solve numerically for hours worked, savings, **interest rate**
 3. Search for optimal $\{\tau_a\}$ as 5th order polynomial of age

Value of the Credit Limit

- We use **SCF 2013** data to compute it for US households 25-60
 - (a) Limit on each credit card
 - (b) Limit on HELOC
 - (c) Installment loans for vehicle and durables (not education)
 - (d) Other debt
- Credit limit: $(a) + (b) + 2 \times (c) + 2 \times (d)$
- The 90th percentile (conditional on > 0) is $1.7 \times$ annual Y
- We set it to $2 \times Y$

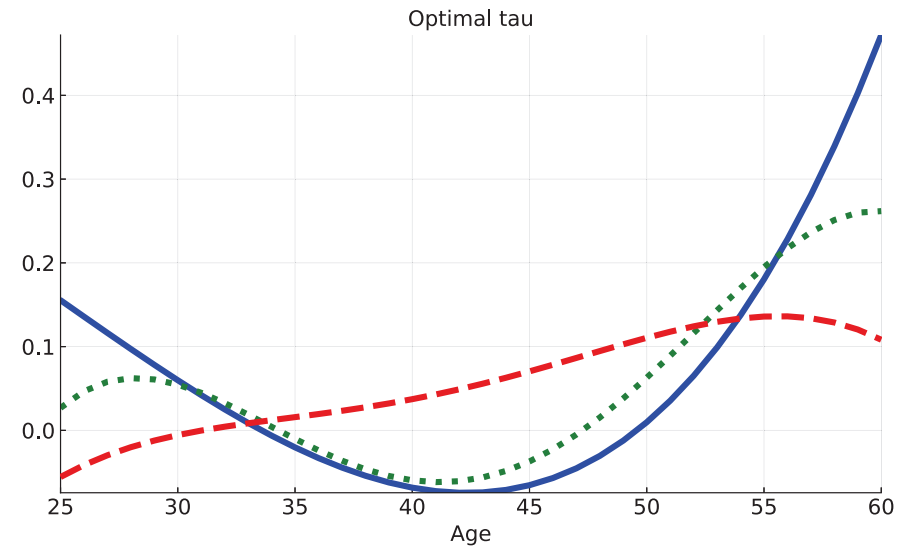
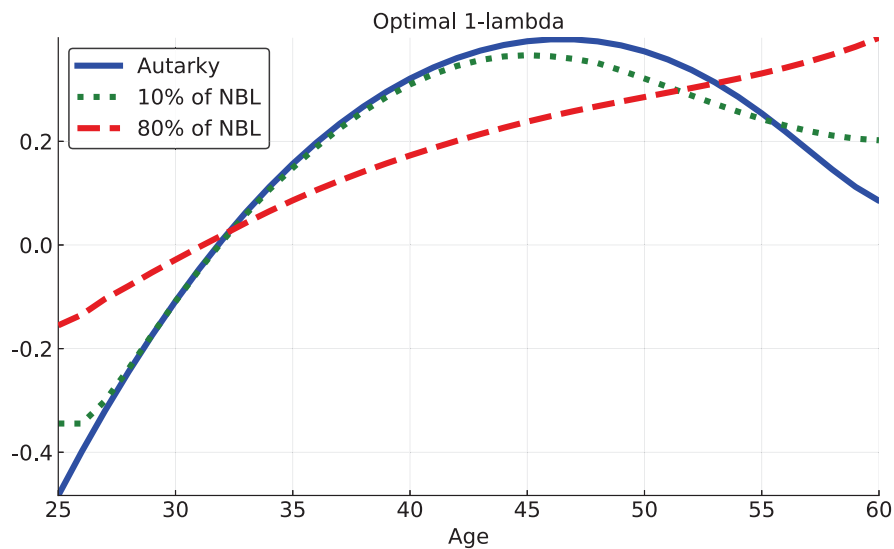
Findings from Numerical Experiment

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- With **flat** $\{x_a, \bar{\varphi}_a\}$, negligible differences with / without wealth
 - ▶ Ex-ante heter. and permanent shocks \Rightarrow bond of little use
- With **empirical profile of** $\{x_a, \bar{\varphi}_a\}$, young households borrow to smooth consumption over the life-cycle
 - ▶ $\{\lambda_a^*\}$ flatter: less need for intergenerational redistribution
 - ▶ $\{\tau_a^*\}$: less U-shaped as a consequence
 - ▶ How much? It depends on credit limit

Optimal Progressivity with Borrowing/Saving



- Shape of τ_a^* profile is closer to the case w/o life-cycle, but flatter

THANKS!

HSV tax-transfer system

- It preserves **tractability** \Rightarrow forces at work are transparent
- But is this specification **too restrictive?**
- **Static setting**: optimal policy in this comes class close to Mirrlees
 - ▶ Heathcote and Tsujiyama, 2018
- **Dynamic setting**: possible welfare gains if **taxes age-varying**
 - ▶ Weinzierl 2009, Farhi & Werning 2013, Golosov, Troshkin & Tsyvinski, 2016, Karabarbounis, 2016, Ndiaye 2017
- Age is observable!