Notes on "Should Robots by Taxed?" (not much here except the algebra behind some of the expressions in the paper)

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1 Basic Idea

Model with two types of workers, routine and non-routine

Production requires routine tasks and non-routine tasks

Routine tasks can be done by routine workers, or by robots

Diamond-Mirrlees result: Don't tax intermediate inputs

Imagine a factory producing something

You can tax the output, but don't mess with the choices inside the factory gates – the factory will figure out the most efficient production method, and taxes on inputs will only distort these choices

But Diamond Mirrlees result relies on being able to tax different goods at different linear rates

Two features of GRT make taxing robots potentially desirable:

1. Cannot tax routine and non-routine workers at different rates since type is private information – necessary to break Diamond Mirrlees

2. Routine and non-routine workers are imperfect substitutes in production

The logic for taxing robots is as follows:

1. Planner wants to compress income between routine and non-routine workers

2. But income taxes are distortionary, as usual

3. If the planner taxes robots, it will increase demand for routine workers to do routine tasks, which will narrow the wage and income gap between the two types of workers => less need to redistribute through taxation (argument relies on routine and non-routine workers being imperfect substitutes)

2 Static Model

π_n and π_r non-routine and routine workers

Utility is

$$U_j = u(c_j, l_j) + v(G)$$

Budget constraint

$$c_j \leq w_j l_j - T(w_j l_j)$$

Costs ϕ units of output to produce a robot

Robot producers solve

$$\max_X \{p_x X - \phi X\}$$

which implies $p_x = \phi$

So robot prices are essentially exogenous – worker wages will be (a bit) more endogenous

Production function

$$Y = A \left(\left[\int_0^1 y_i^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} \right)^{(1-\alpha)} N_n^{\alpha}$$

where each i is a different task, and y_i is output of task i. Each task can be done by a routine worker or by robots

$$y_i = \begin{array}{cc} \kappa_i x_i & \text{if } i \text{ automated} \\ \lambda_i n_i & \text{if } i \text{ not automated} \end{array}$$

Assume $\frac{\kappa_i}{\lambda_i}$ weakly decreasing in i. So tasks will be automated up to some threshold m and then done by routine workers after that

(Actually will assume $\kappa_i = \lambda_i$)

$$Y = A \left(\left[\int_0^m (\kappa_i x_i)^{\frac{\rho-1}{\rho}} di + \int_m^1 (\lambda_i n_i)^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} \right)^{(1-\alpha)} N_n^{\alpha}$$

Firm maximizes profits given by

$$Y - w_n N_n - w_r \int_m^1 n_i di - (1 + \tau^x)\phi \int_m^1 x_i di$$

FOCs

$$\begin{aligned} w_n &= \frac{\alpha Y}{N_n} \\ w_r &= ... \\ (1 + \tau^x)\phi &= ... \end{aligned}$$

$\kappa_i = \lambda_i = 1$

This immediately implies $x_i = x$ and $n_i = n$ for all i with

$$\begin{aligned} mx &= X \\ (1-m)n &= N_r \end{aligned}$$

Optimal automation choices

$$m = \begin{array}{ccc} 0 & if & w_r < (1+\tau^x)\phi \\ [0,1] & if & w_r = (1+\tau^x)\phi \\ 1 & if & w^r > (1+\tau^x)\phi \end{array}$$

Note that the middle case is not really knife edge, because w_r is endogenous – more automation will tend to depress w_r.

Focus on the middle case (call this an equilibrium with automation)

Then $y_i = y$ for all i, with

$$1y = x = n = X + N_r$$

and

$$m = \frac{X}{X + N_r}$$

Production function simplifies to

$$Y = A(X + N_r)^{1-\alpha} N_n^{\alpha}$$

Government budget constraint

$$G \leq \pi_r T(w_r l_r) + \pi_n T(w_n l_n) + \tau^x p_x X$$

Equilibrium is $\{c_j, l_j, G, N_j, X, x, n, m\}$, prices $\{w_j, p_x\}$ and a tax system $T(\cdot), \tau^x$ s.t.

1. Workers solve their problems given prices and taxes
2. Firms solve their problems
3. The GBC is satisfied
4. Markets clear

The market clearing conditions for labor, robots and output are

$$\begin{aligned} N_n &= \pi_n l_n \\ N_r &= \pi_r l_r \end{aligned}$$

$$\begin{aligned} mx &= X \\ (1-m)n &= N_r \end{aligned}$$

$$\pi_r c_r + \pi_n c_n + G \leq Y - \phi X$$

With automation, we have

$$w_r = (1+\tau^x)\phi$$

so routine wages are pinned down by technology and the robot tax. Routine wages increase directly with the robot tax

So the FOC for employing robots is

$$(1+\tau^x)\phi = (1-\alpha)A\left(X + \pi_r l_r\right)^{-\alpha} N_n^{\alpha}$$

which we can write in terms of m using $X = mx = m(X + \pi_r l_r)$ which implies

$$X = \frac{m\pi_r l_r}{1-m}$$

$$\begin{aligned}
(1+\tau^x)\phi &= (1-\alpha)A\left(\frac{m\pi_r l_r}{1-m} + \pi_r l_r\right)^{-\alpha} N_n^{\alpha} \\
m &= 1 - \left[\frac{(1+\tau^x)\phi}{(1-\alpha)A}\right]^{\frac{1}{\alpha}} \frac{\pi_r l_r}{\pi_n l_n}
\end{aligned}$$

The FOC for non-routine workers can also be solved for. Note that

$$\begin{aligned}
w_n N_n &= \alpha Y \\
w_r N_r &= (1-\alpha)(1-m)Y
\end{aligned}$$

so

$$\begin{aligned}
\frac{w_n}{w_r} &= \frac{\alpha}{1-\alpha}\frac{1}{1-m}\frac{N_r}{N_n} \\
\frac{w_n}{w_r} &= \frac{\alpha}{1-\alpha}\frac{1}{\left[\frac{(1+\tau^x)\phi}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\frac{\pi_r l_r}{\pi_n l_n}}\frac{N_r}{N_n} \\
w_n &= \frac{\alpha}{1-\alpha}\frac{1}{\left[\frac{(1+\tau^x)\phi}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}}(1+\tau^x)\phi \\
&= \alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left[(1+\tau^x)\phi\right]^{\frac{\alpha-1}{\alpha}}
\end{aligned}$$

so non-routine worker's wages are also pinned down by technology and taxes. Why is that?

Note also that a tax on robots reduces w_n/w_r.

Output available for private and public consumption is

$$\begin{aligned}
Y-\phi X &= \pi_n w_n l_n+\pi_r w_r l_r+\phi(1+\tau^x)X-\phi X \\
&= \pi_n w_n l_n+\pi_r w_r l_r+\tau^x\phi\left(\frac{m\pi_r l_r}{1-m}\right) \\
&= \pi_n w_n l_n+\pi_r w_r l_r+\tau^x\phi\pi_r l_r\left(\left[\frac{(1+\tau^x)\phi}{(1-\alpha)A}\right]^{-\frac{1}{\alpha}}\left(\frac{\pi_r l_r}{\pi_n l_n}\right)^{-1}-1\right) \\
&= \pi_n w_n l_n+\pi_r w_r l_r+\tau^x\frac{\phi\pi_n}{\alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left[(1+\tau^x)\phi\right]^{\frac{\alpha-1}{\alpha}}}w_n l_n\left[\frac{(1+\tau^x)\phi}{(1-\alpha)A}\right]^{-\frac{1}{\alpha}}-\tau^x\frac{\phi\pi_r}{(1+\tau^x)\phi}w_r l_r \\
&= \pi_n w_n l_n+\pi_r w_r l_r+\tau^x\frac{(1-\alpha)\pi_n}{\alpha(1+\tau^x)}w_n l_n-\tau^x\frac{\pi_r}{(1+\tau^x)}w_r l_r \\
&= \frac{(\alpha+\tau^x)\pi_n w_n l_n}{\alpha(1+\tau^x)}+\frac{\pi_r w_r l_r}{(1+\tau^x)}
\end{aligned}$$

2.1 HSV Taxation

$$T(wl)=wl-\lambda(wl)^{1-\gamma}$$

$$\begin{aligned}
c &= wl-T(wl) \\
&= \lambda(wl)^{1-\gamma}
\end{aligned}$$

$$u(c,l)+v(G)=\log(c)-\zeta\frac{l^{1+\upsilon}}{1+\upsilon}+\chi\log(G)$$

Optimality for hours gives

$$l=\left(\frac{1-\gamma}{\zeta}\right)^{\frac{1}{1+\upsilon}}$$

so hours are independent of wages (balanced growth property)

Thus

$$c_j=\lambda(w_j l)^{1-\gamma}$$

which implies (assuming $\tau^x=0$)

$$\frac{c_r}{c_n}=\left(\frac{w_r}{w_n}\right)^{1-\gamma}=\frac{\phi^{\frac{1-\gamma}{\alpha}}}{\left[\alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]^{1-\gamma}}$$

The equilibrium level of automation is

$$1-\left[\frac{\phi}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\frac{\pi_r}{\pi_n}$$

Suppose ϕ goes down over time, so robots are becoming cheaper to produce

$$\begin{aligned} w_r &= \phi \\ \ln(w_r) &= \ln\phi = \ln\left(\frac{1}{\phi^{-1}}\right) \end{aligned}$$

$$\frac{d\log(w_r l)}{d\log(\phi^{-1})} = -1$$

Similarly

$$\frac{d\log(w_n l)}{d\log(\phi^{-1})} = \frac{1-\alpha}{\alpha}$$

And

$$\frac{d\log\left(c_n/c_r\right)}{d\log\left(\phi^{-1}\right)} = \frac{1-\gamma}{\alpha}$$

Within the HSV class of taxation, one could compute the optimal value for γ, and explore how it varies with ϕ.

They focus instead of fully optimal non-linear taxation, a la Mirrlees

Optimal Taxation

Assume planner seeks to maximize

$$\pi_r\omega_r\left[u(c_r,l_r)+v(G)\right]+\pi_n\omega_n\left[u(c_n,l_n)+v(G)\right]$$

If the planner's only constraint is the resource constraint, it will not want to tax robots

use type-specific lump-sum transfers

2.2 Mirrlees problem

Focus on case when automation interior, and where, with $\tau^x = 0$, non-routine workers earn higher wage than routine workers

Natural Mirrlees setup would not have any taxes appearing in the problem formulation

Here they assume the planner must choose a linear robot tax τ^x

Given that, firms' problems and all the equilibrium relations derived previously – up to the determination of l_r and l_n – will be preserved

So we can write the resource constraint as

$$\pi_r c_r + \pi_n c_n \leq \frac{(\alpha+\tau^x)\pi_n w_n l_n}{\alpha(1+\tau^x)} + \frac{\pi_r w_r l_r}{(1+\tau^x)}$$

There are also incentive constraints

$$u(c_n,l_n) \geq u(c_r,\frac{w_r}{w_n}l_r)$$

In these constraints, wages are given by

$$w_r = (1+\tau^x)\phi$$

$$w_n = \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \left[(1+\tau^x)\phi\right]^{\frac{\alpha-1}{\alpha}}$$

So the planner understands that by changing τ^x it will change wages and change output.

Substituting the expressions for wages into the RHS of the resource constraint gives

$$\begin{aligned}
& \frac{(\alpha+\tau^x)\pi_n w_n l_n}{\alpha(1+\tau^x)} + \frac{\pi_r w_r l_r}{(1+\tau^x)} \\
= & \frac{(\alpha+\tau^x)\pi_n \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \left[(1+\tau^x)\phi\right]^{\frac{\alpha-1}{\alpha}} l_n}{\alpha(1+\tau^x)} + \frac{\pi_r (1+\tau^x)\phi l_r}{(1+\tau^x)} \\
= & \frac{(\alpha+\tau^x)}{\alpha(1+\tau^x)^{\frac{1}{\alpha}}} \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \phi^{\frac{\alpha-1}{\alpha}} \pi_n l_n + \phi \pi_r l_r
\end{aligned}$$

The first term is where the tax on robots distorts output.

Note that

$$\frac{\partial}{\partial \tau^x} \left(\frac{(\alpha+\tau^x)}{\alpha(1+\tau^x)^{\frac{1}{\alpha}}} \right) = -\frac{1}{\alpha^2} \tau^x \frac{1-\alpha}{(\tau^x+1)^{\frac{1}{\alpha}(\alpha+1)}}$$

so a higher robot tax reduces productivity. Note also that at $\tau^x = 0$, the marginal impact on output from raising the tax is zero. So on the margin a tax on robots is not distortionary.

But we know that at $\tau^x = 0$, raising τ^x has a first order effect on the wage differential raising $\frac{w_r}{w_n}$. This means that if the non-routine worker was previously tempted to pick the allocation intended for the routine worker, he is no longer tempted to do so, because he would now need to work more hours to deliver income $w_r l_r$ (hours would be $\frac{w_r}{w_n} l_r$). Because the IC constraint is relaxed, the planner can achieve a more desirable allocation (e.g. more redistribution).