Partial Answers: Final Practice Questions

May 5, 2006

Question 1

Solve for the price of stocks, p_t

Let's assume that the total income of the young is y_t, so per capita income of the young is $y_t/(1-\delta)$

(we could assume y_t is per capita income for the young - the expressions would end up slightly different, but with exactly the same flavor)

Given the conjecture for individual consumption rules, aggregate consumption is given by

$$\begin{aligned}
C_t &= (1-\delta)\sum_{a=1}^{\infty}\delta^{a-1}c_t^a \\
&= (1-\delta)\left[(1-\beta\delta)\frac{y_t}{1-\delta}+\delta(1-\beta\delta)(s_t^2(p_t+d_t))+\delta^2(1-\beta\delta)(s_t^3(p_t+d_t))+...\right] \\
&= (1-\delta)(1-\beta\delta)\left[\frac{y_t}{1-\delta}+\delta(s_t^2(p_t+d_t))+\delta^2(s_t^3(p_t+d_t))+...\right] \\
&= (1-\beta\delta)y_t+(1-\delta)(1-\beta\delta)(p_t+d_t)\left[\delta(s_t^2)+\delta^2(s_t^3)+...\right] \\
&= (1-\beta\delta)(y_t+p_t+d_t)
\end{aligned}$$

where in the last step we use the fact that the old, in total, hold all the shares, so

$$(1-\delta)\left[\delta(s_t^2)+\delta^2(s_t^3)+...\right]=1$$

From market clearing

$$C_t = y_t + d_t$$

Combining the two expressions for C_t gives the equilibrium price

$$p_t = \frac{\beta\delta(y_t+d_t)}{1-\beta\delta}$$

This expression is quite interesting: if the income of the young (the stock buyers) is larger, so is the equilibrium price. If current dividends - the income of the old stock sellers - is larger, the price is also larger

Verify that given this price, the conjectured decision rule for consumption satisfies the agent's first order condition for stock purchases.

One way to write the budget constraint for an individual of age $a > 1$ is

$$c_t^a + p_t \delta s_{t+1}^{a+1} = s_t^a (p_t + d_t)$$

where w_t^a is the individual's wealth at age a, and s_t^a is the number of shares held at age a (note that the agent only needs to buy δ shares at age a to receive 1 share at age $a+1$, if he survives)

The inter-temporal first order condition for stock purchases is

$$u'(c_t)\delta p_t = \beta\delta E_t \left[u'(c_{t+1})(p_{t+1} + d_{t+1}) \right]$$

Let us check whether this is satisfied for the young:

$$\begin{aligned} c_t^1 &= (1-\beta\delta)\frac{y_t}{1-\delta} \\ s_{t+1}^2 &= \frac{\frac{y_t}{1-\delta}\left(1-(1-\beta\delta)\right)}{p_t\delta} \\ &= \frac{\frac{y_t}{1-\delta}\beta\delta}{\frac{\beta\delta(y_t+d_t)}{1-\beta\delta}\delta} \end{aligned}$$

$$\begin{aligned} c_{t+1}^2 &= (1-\beta\delta)(s_{t+1}^2(p_{t+1}+d_{t+1})) \\ &= (1-\beta\delta)\left(\frac{\frac{y_t}{1-\delta}\beta\delta}{\frac{\beta\delta(y_t+d_t)}{1-\beta\delta}\delta} \left(\frac{\beta\delta(y_{t+1}+d_{t+1})}{1-\beta\delta} + d_{t+1} \right) \right) \end{aligned}$$

It is straightforward to plug these expressions into the inter-temporal FOC, and to verify that is is satisfied

$$\frac{\delta}{(1-\beta\delta)\frac{y_t}{1-\delta}} \frac{\beta\delta(y_t+d_t)}{1-\beta\delta} = \beta\delta E \left[\frac{\left(\frac{\beta\delta(y_{t+1}+d_{t+1})}{1-\beta\delta} + d_{t+1} \right)}{(1-\beta\delta)\left(\frac{\frac{y_t}{1-\delta}\beta\delta}{\frac{\beta\delta(y_t+d_t)}{1-\beta\delta}\delta} \left(\frac{\beta\delta(y_{t+1}+d_{t+1})}{1-\beta\delta} + d_{t+1} \right) \right)} \right]$$

One can do something similar for older agents.