

Homework 7

JONATHAN HEATHCOTE

due in class on Tuesday April 25th

1. MODEL

Consider the simple RBC model that we discussed in class. A representative agent / social planner solves the following problem:

$$\max_{\{c_t\}\{k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

subject to

$$\begin{aligned} c_t + k_{t+1} &= e^{z_t} k_t^\theta + (1-\delta)k_t \\ z_{t+1} &= \rho z_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2) \end{aligned}$$

where $\{c_t, z_t, k_t\}$ are time t values of consumption, the technology shock and capital respectively.

2. EXERCISE

1. Write down the set of Euler equations and other equations that characterize equilibrium.
2. Compute the steady state of this economy and report it. Use the following parameter values: $\beta = 0.99, \delta = 0.02, \theta = 0.3, \gamma = 2, \rho = 0.95$.
3. Linearize the system of equations by taking first order Taylor series approximations to them around the steady state.
4. Solve the system of equations by following the steps outlined in class. Report:
 - (i) Eigenvectors and eigenvalues for your linearized system
 - (ii) A decision rule for the choice variable (c) as a function of the state variables (k and z)
 - (ii) Laws of motion for the state variables
5. Starting with an initial capital stock equal to 10 percent of the steady state capital stock (computed in step 2) use the law of motion computed in step 4 to plot the evolution of capital and consumption through time (setting $z_t = 0$ for all t) until the economy converges to steady state.

6. Imagine you are interested in knowing how well the model conforms with data.

Let $\sigma_\varepsilon^2 = 0.0001$.

- (a) Simulate the economy for 200 periods and plot the simulated paths for output, consumption and investment
- (b) Hodrick-Prescott filter these series (using a smoothing parameter of 1600) and average over 100 simulations to compute statistics to compare to those on page 30 of the Cooley book, i.e.

percentage std. dev. of output = 1.72%, % std dev consumption = 1.27%,
% std dev investment = 8.24%, $Corr(y_t, c_t) = 0.83$ and $Corr(y_t, i_t) = 0.91$.
How does this model economy compare with the U.S. economy?