Homework 3

Jonathan Heathcote

Due in Class on Tuesday February 28th

In class we outlined two versions of the stochastic growth model: a planner's problem, and an Arrow-Debreu competitive equilibrium. We were working towards showing that allocations in the two setups would be identical.

1. (a) Complete the proof that the sets of equations that characterize (i) the solution to the planner's problem, and (ii) the competitive equilibrium are identical, and thus that one can solve for equilibrium allocations by solving the planner's problem.

   (b) Now consider the following twist on the economy we described in class. Income (from both labor and capital) is taxed at rate τ_t, where $0 < \tau_t < 1$. There is no allowance for depreciation: thus the typical consumer's budget constraint (in the sequence of markets formulation, without state-contingent claims) is

   $$c_t + k_{t+1} = (1 - \tau_t)(r_t k_t + w_t n_t) + (1 - \delta)k_t$$

   Revenues are used for non-valued government purchases G. Consider (i) a planner's problem in which the planner has to set aside a constant amount G of output each period for government purchases, and (ii) a competitive equilibrium in which the tax rate τ_t is such that at each date equilibrium revenue is equal to the same amount G.

      i. Describe the planner's problem and the competitive equilibrium, and the two sets of equations characterizing (i) the planner's solution and (ii) the equilibrium.

      ii. In general, are allocations the same in each case?

      iii. Now suppose the utility function takes the form

      $$u(c, n) = \ln(c) + v(1 - n)$$

      where $v(.)$ is strictly increasing and strictly concave.
      Are allocations the same in the competitive equilibrium and the planner's problem in this case? If so, why? If not, what additional policy instruments would the government need in the decentralized economy to achieve the allocation that solves the planner's problem?

(c) Consider the competitive equilibrium described in part (*b*). Suppose the period utility function for the representative consumer is

$$u(c, l) = \ln(c) + \psi \ln(1 - n)$$

Suppose output, produced by a representative firm, is given by

$$\begin{aligned} y_t &= z_t F(k_t, n_t) \\ F(k, n) &= k^{\theta} n^{1-\theta} \end{aligned}$$

Suppose individuals discount at rate β, and capital depreciates at rate δ. Consider the non-stochastic steady state for this economy. Assume that in the non-stochastic steady state $z = 1$. Suppose we want to calibrate the economy to replicate the following facts:

i. Two-thirds of income goes to labor:

$$\frac{wn}{wn + rk} = \frac{2}{3}$$

ii. People work one-third of the time endowment

$$n = \frac{1}{3}$$

iii. Government spending is 20% of output

$$\frac{G}{Y} = 0.2$$

iv. Investment is 15% of output

$$\frac{x}{Y} = 0.15$$

v. The annual after-tax return to capital, net of depreciation, is 4%

$$(1 - \tau)r - \delta = 0.04$$

What values for β, ψ, θ, δ and τ does this calibration imply?

(d) Define a recursive competitive equilibrium for the economy with taxation. Is the state vector the same as the one for the non-distorted economy?