Inter-generational Redistribution in the Great Recession

Andrew Glover Jonanthan Heathcote Dirk Krueger
Jose-Victor Rios-Rull

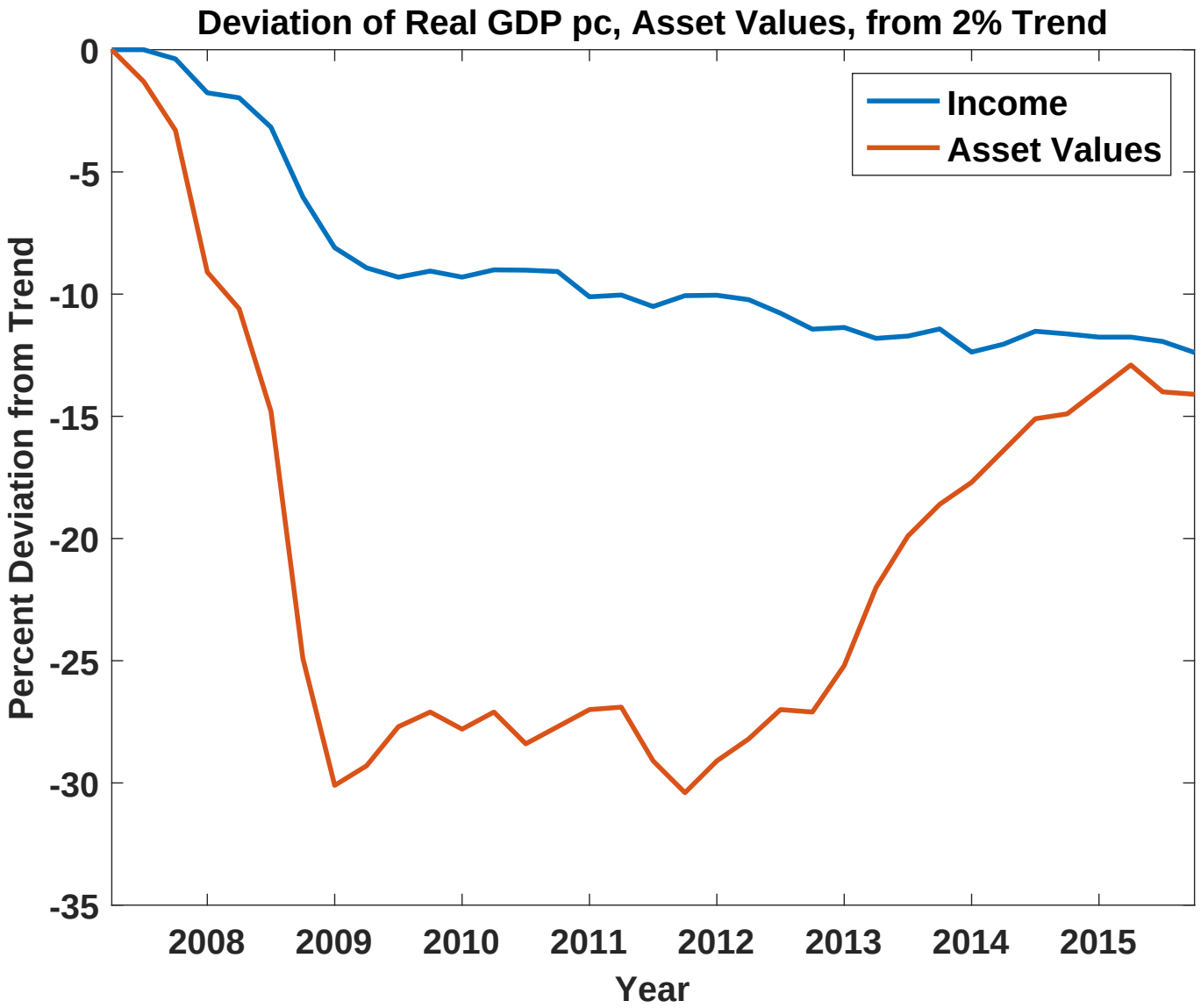
University of Texas
Minneapolis FED and CEPR
University of Pennsylvania, CEPR, CFS, NBER and Netspar
University of Pennsylvania, CEPR and NBER

Seminar at the University of Bonn
June 2017

Introduction

- Salient features of the great recession:
  - Large fall in output and labor incomes.
  - Larger fall in asset prices (stocks, houses).
- Research Question: What are the distributional consequences for households at different stages of the life cycle?

Motivating Facts: Aggregate Data

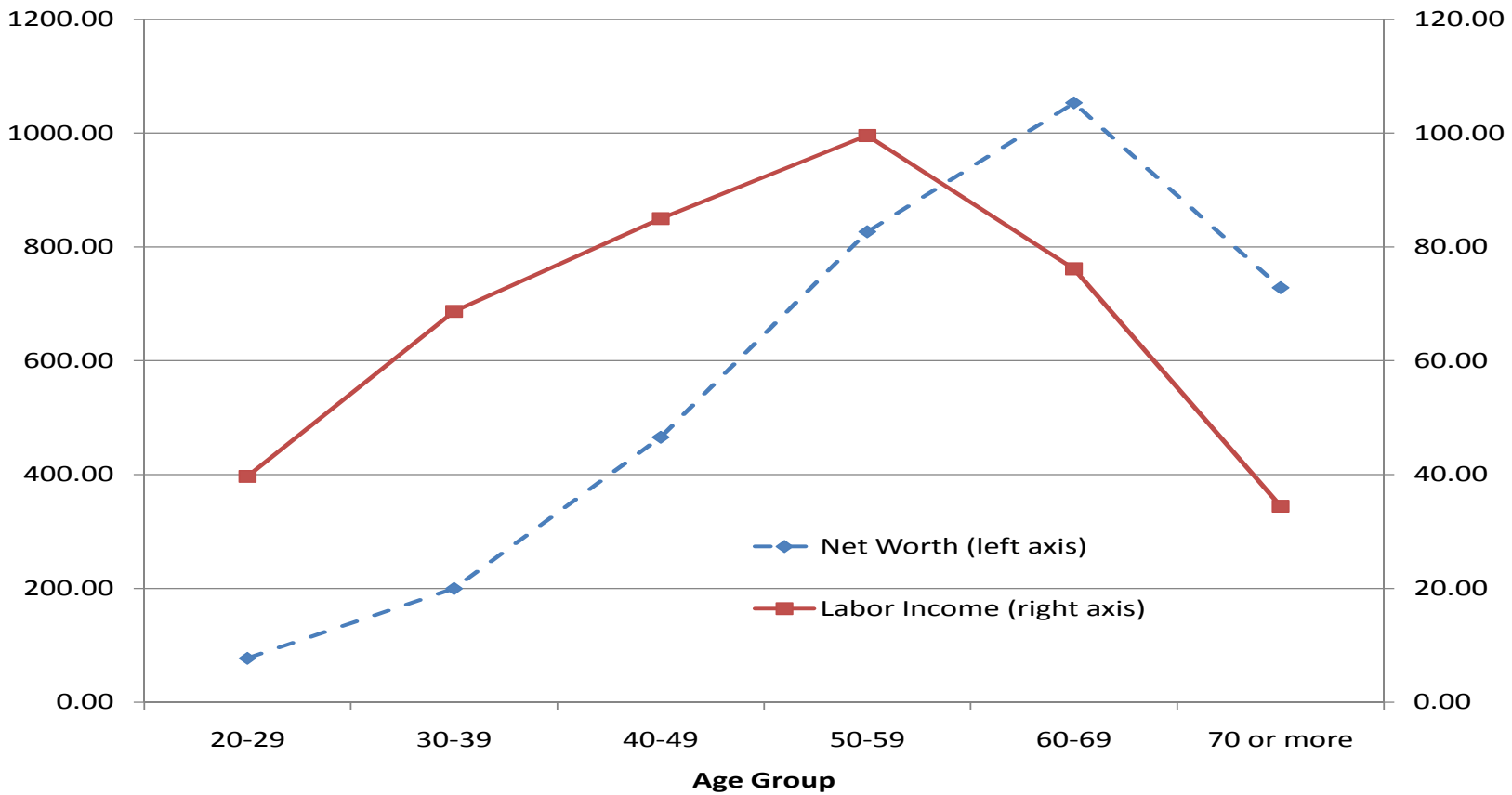


Motivating Facts

- Why focus on age dimension?
  - Labor income and wealth vary substantially by age.
  - Portfolio composition (risky versus riskless assets) varies substantially by age.
  - Labor income losses in great recession vary substantially by age.
- (1) - (3) $\Longrightarrow$ Wealth and welfare losses vary substantially by age.

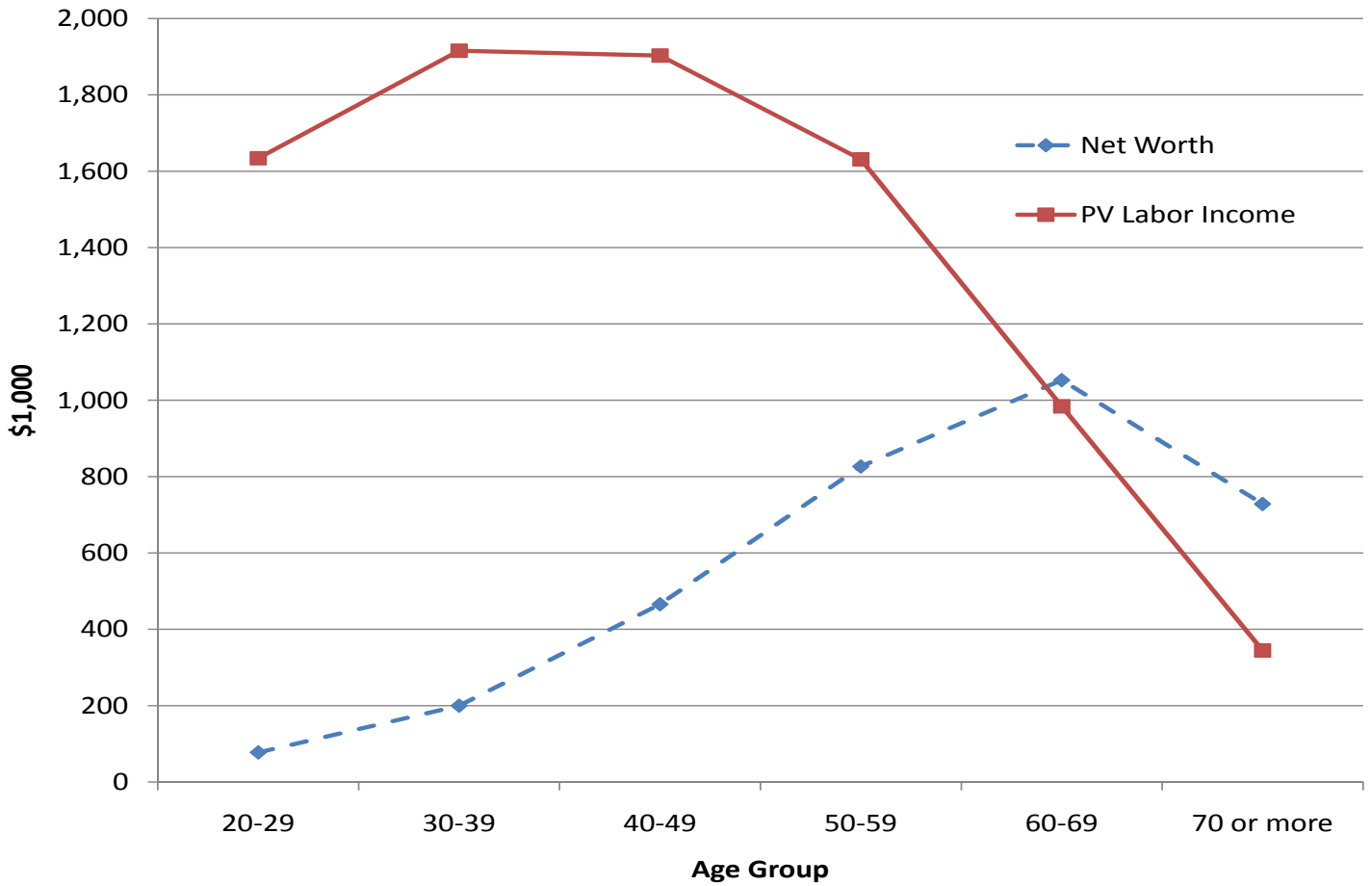
Motivating Facts: Income and Wealth Over Life Cycle

Figure: Labor Income and Net Worth by Age, SCF 2007 ($1,000)



Motivating Facts: Income and Wealth Over Life Cycle

Figure: Present Value Labor Income and Net Worth by Age



Motivating Facts: Income and Wealth Over the Life Cycle (2007 SCF, $1,000)

Age	Total Income	Labor Income	Asset Income	Assets	Debts	Net Worth
All	83.43	70.07	13.36	659.00	103.34	555.66
20-29	38.83	39.68	-0.85	130.66	53.30	77.36
30-39	69.83	68.68	1.15	335.87	136.12	199.75
40-49	93.40	84.97	8.43	598.21	132.62	465.59
50-59	117.97	99.56	18.41	959.77	133.24	826.53
60-69	109.06	76.15	32.90	1156.96	104.10	1052.86
70+	57.56	34.46	23.11	756.76	28.48	728.28

Motivating Facts: Portfolio Shares by Age from 2007 SCF (in %)

Age Head	(1) Stk	(2) Res. RE	(3) Non bus.	(4) Non RE	(5) Risky NW	(6) Bond +CD	(7) Car	(8) Oth.	(9) Debt	(10) Safe NW
All	30.3	47.0	12.9	3.8	94.0	17.0	3.5	4.2	-18.6	6.0
20-29	13.2	77.7	43.3	1.3	135.5	13.7	15.3	4.5	-68.9	-35.5
30-39	26.3	96.5	12.7	5.0	140.4	13.8	9.7	4.2	-68.2	-40.4
40-49	30.4	57.6	12.6	3.8	104.4	15.2	4.4	4.5	-28.5	-4.4
50-59	32.7	42.4	13.5	3.7	92.4	17.0	2.8	4.0	-16.1	7.7
60-69	32.2	35.6	13.4	4.1	85.3	17.5	2.4	4.7	-9.9	14.7
70+	27.1	39.8	9.0	3.3	79.2	19.3	1.8	3.7	-3.9	20.8

Risky Net Worth (5) is equal to sum of columns (1)+(2)+(3)+(4). Safe Net Worth (10) is sum of columns (6)+(7)+(8)+(9). Total Net Worth is sum of (5)+(10)

Motivating Facts: Capital Losses by Age Group

Infl. adj. capital losses from 2007:2 to 2009:1-2013:4 ($1,000, 2007)								
Age of Head	Stocks	Res. RA	Nonc. bus.	Nonres. prop.	Total	(%)net worth	(%) inc.	Total/ 2009Q1
All	30.6	64.4	15.1	6.5	116.5	21.0	139.6	154.5
20-29	1.9	14.8	7.1	0.3	24.0	31.1	61.9	24.5
30-39	9.5	47.5	5.4	3.0	65.4	32.8	93.7	73.0
40-49	25.7	66.1	12.3	5.4	109.6	23.5	117.3	139.8
50-59	49.1	86.4	23.6	9.4	168.5	20.4	142.8	232.3
60-69	61.5	92.4	29.8	13.3	197.0	18.7	180.6	278.9
70+	35.9	71.4	13.8	7.4	128.5	17.6	223.2	173.9

- Capital losses concentrated among older households

Motivating Facts: Change in Labor Income 2007 to 2010, Relative to Trend, CPS

	(%)
pc earnings	-9.8
20-29	-14.3
30-39	-12.6
40-49	-10.3
50-59	-11.1
60-69	-6.0
70+	-1.4

- Current earnings losses concentrated among younger households

Motivating Facts

- Why focus on age dimension?
  - Labor income and wealth vary substantially by age.
  - Portfolio composition (risky versus riskless assets) varies substantially by age.
  - Labor income losses in great recession vary substantially by age.
- (1) - (3) $\Longrightarrow$ Wealth and welfare losses vary substantially by age.

The Plan for Remainder of Talk

- The Approach
  - Construct and compute a quantitative OLG model with aggregate risk.
  - Calibrate it to life cycle facts from 2007 SCF.
  - Engineer a great recession.
- Questions:
  - Can model generate magnitude of asset price declines as observed in the data?
  - Can the model generate realistic age profile of asset portfolios?
  - How are wealth and welfare losses from great recession distributed across different age cohorts?

Key Channel

- Young cohorts have lots of future labor income, few financial assets.
- Hurt by lower current wages, might benefit from lower asset prices.
- Welfare consequences of downturn depend on:
  - Size of labor income asset price decline
  - Its persistence
  - Behavioral response of households (consumption-savings and portfolio allocation choices).
- Therefore want labor income, asset prices and household choices to be jointly and *endogenously* determined in a quantitative life cycle model.

Related Literature

- OLG economies with aggregate risk:
  - Asset pricing: Labadie (1986), Huffman (1987), Constantinides, Donaldson and Mehra (2002), Storesletten, Telmer and Yaron (2007), Kubler and Schmedders (2015)
  - Allocations: a) Business cycles: Rios-Rull (1994, 1996), Gomes, Michaelides and Polkovnichenko (2010), b) Intergenerational risk sharing: Bohn (1998), Shiller (1999), Demange (2002), Smetters (2006), Krueger and Kubler (2006), Ball and Mankiw (2007), Miyazaki, Sato and Yamada (2009), Olovsson (2010).
- Redistributional consequences by age of other aggregate shocks:
  - Inflation: Doepke and Schneider (2006a,b), Meh, Rios-Rull and Terajima (2010).
  - Demographics: Demange and Laroque (1999), Rios-Rull (2001), Abel (2003), Attanasio, Kitao and Violante (2007), Krueger and Ludwig (2007).
- Consumption disasters: Barro (2006, 2009), Nakamura, Steinsson, Barro and Ursua (2013), Gourio (2010).

An OLG Model with Aggregate Risk

- Labor income and asset prices driven by aggregate shock $z \in Z = \{z_n, z_r, z_d\}$.
- z follows Markov process with transition matrix $\Gamma_{z,z'}$.
- Households live for I periods. Supply one unit of time, relative labor efficiency (income) $\{\varepsilon_i(z)\}_{i=1}^I$. Normalize $\sum_i \varepsilon_i(z) = L = 1$.
- Time discount factors $\{\beta_i\}_{i=1}^I$ vary with age. Utility function $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. Wealth distribution $A = \{A_i\}_{i=1}^I$. No bequests.
- Technology

$$Y(z) = zK^{\theta}L^{1-\theta} = z$$

- Supply of fixed factor (land, capital) normalized to $K = 1$. Labor income (wages) equals $w(z) = (1-\theta)z$. Capital income equals θz.
- Market Structure: Ownership shares of K traded at price $p(z, A)$. Exogenous net supply B of corporate bonds, price $q(z, A)$.

▸ Details of the Model

Calibration Strategy

- Model period 10 years. Agents enter at age 20, live for 6 periods.
- Aggregate endowment process $z \in Z = \{z_n, z_r, z_d\}, \Gamma_{z,z'}$ derived directly from aggregate time series data. In Great Recession (z_r) output falls 9.84%.
- Life cycle profiles $\{\beta_i, \varepsilon_i(z)\}$ chosen so that model with $z = z_n$ matches life cycle earnings and net worth profiles from 2007 SCF.
- Choose $(\theta = 30\%, B = 0.07)$ s.t. model matches 2007 SCF aggregate wealth to earnings ratio (7.88), share of risky assets (91.8%).
- Choose $\sigma = 4.24$ s.t. model ξ lines up with Great Recession $\xi = 26.8\%/9.84\% = 2.7$.

▸ Details of the Calibration

The Model: Overview and Stochastic Structure

- OLG model with aggregate risk.
- Labor income and asset prices driven by same aggregate shock.
- Aggregate technology shock $z \in Z = \{z_n, z_r, z_d\}$
- z follows Markov process with transition matrix $\Gamma_{z,z'}$.

The Model: Households

- Households live for I periods.
- Households endowed with 1 unit of time, supplied to the market inelastically.
- Labor efficiency profile $\{\varepsilon_i(z)\}_{i=1}^{I}$ varies with age and state of the cycle. Normalize so that $\sum_i \varepsilon_i(z) = 1$ for all $z \in Z$.
- No initial wealth, no bequests.
- Time discount factors $\{\beta_i\}_{i=1}^{I}$ vary with age. Period utility function is CRRA $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$

The Model: Technology

- Production
$$Y(z) = zK^{\theta}L^{1-\theta}$$

- Total supply of labor normalized to $L = 1$.

- Supply of fixed factor (land, capital) normalized to $K = 1$. Shares of ownership of fixed factor traded. Total number of shares equal to 1.

- Thus output equals $Y(z) = z$. Labor income (wages) equals $w(z) = (1 - \theta)z$. Capital income equals θz.

Discussion of the Assumptions I: Housing

- Can re-interpret the model as explicit model of housing. Assume:
  - Fixed supply 1 of perfectly divisible houses. Competitive rental markets.
  - Cobb Douglas utility over non-durables, housing services $\frac{\left(c^{\nu} s^{1-\nu}\right)^{1-\sigma}}{1-\sigma}$
  - Households can freely invest in three assets: bonds, stocks, houses.
- Results: rents are proportional to dividends, housing prices proportional to stock prices.
- Thus model with housing has exactly the same asset pricing and welfare implications as our model without explicit housing.

Discussion of the Assumptions II: Unemployment

- In recession labor incomes fall because real wages $w(z) = (1-\theta)z$ fall, whereas hours worked $L = 1$ remain constant.

- Could equivalently assume that labor income in recession falls due to reduction in hours worked $L(z)$:

$$Y(z) = L(z)^{1-\theta}$$

- As long as $L(z_r)/L(z_n) = (z_r/z_n)^{\frac{1}{1-\theta}}$ model with TFP shocks z and model with aggregate shocks to hours worked $L(z)$ (or aggregate shocks to unemployment) are isomorphic.

The Model: Market Structure

- Exogenous net supply B of corporate bonds. Unit supply of shares.
- Aggregate state of the economy (z, A), where $A = (A_1, \ldots, A_I)$ denotes the beginning of period wealth distribution across age cohorts.
- Stock price $p(z, A)$, bond price $q(z, A)$.
- Stocks pay dividends $d(z, A) = \theta z - [1 - q(z, A)]\, B$
- Aggregate (start of period) wealth:
  $W(z, A) = p(z, A) + d(z, A) + B$

Recursive Problem of the Household

- State space (i, a, z, A), where a is the individual share of total wealth held by the household.

$$\begin{aligned} v_i(a,z,A) &= \max_{c\geq 0, y, \lambda, a'} \left\{ u(c) + \beta_{i+1} \sum_{z' \in Z} \Gamma_{z,z'} v_{i+1}(a', z', A') \right\} \\ c + y &= \varepsilon_i(z) w(z) + W(z,A) a \\ a' W(z', A') &= \left(\lambda \frac{p(z', A') + d(z', A')}{p(z, A)} + (1-\lambda) \frac{1}{q(z, A)} \right) y \\ A' &= G(z, A, z') \end{aligned}$$

- Policy functions $c_i(a,z,A), y_i(a,z,A), \lambda_i(a,z,A)$ and $a'_i(a,z,A,z')$.

Equilibrium: Markets, Prices and Aggregation

- Labor market: wages $w(z) = (1-\theta)z$ and $\sum_{i=1}^{I} \varepsilon_i(z) = L = 1$.
- Financial Markets: Share prices $p(z, S)$ and bond prices $q(z, A)$

$$\begin{aligned} \sum_{i=1}^{I} y_i(A_i, z, A)\lambda_i(A_i, z, A) &= p(z, A) \\ \sum_{i=1}^{I} y_i(A_i, z, A)\left[1 - \lambda_i(A_i, z, A)\right] &= q(z, A)B \end{aligned}$$

- Law of Motion: $A_1' = 0$ and $A_{i+1}' = G_{i+1}(z, A, z') = a_i'(A_i, z, A, z')$.

Developing Intuition: A Three Period Model

- Key assumptions:
  - Households only productive when young: $\varepsilon_1 = 1, \varepsilon_2 = \varepsilon_3 = 0$.
  - Households derive no utility from consumption when young. By construction young save everything.
  - Only stocks are traded: $B = 0$.
  - Aggregate shock can only take two values: $Z = \{z_r, z_n\}$.
- State (z, A) where $A = A_3$ is share of assets held by old. Share of wealth held by middle-aged is $1 - A$.
- Only middle-aged make meaningful decision: how many of their shares to sell.
- Note: wealth distribution irrelevant in Rep. Agent model or 2 period OLG model.

Developing Intuition: A Three Period Model

- Measure of asset price collapse:

$$\xi(A) = \frac{\log(p(z_r, A)/p(z_n, A))}{\log(z_r/z_n)}$$

  Note: in RA economy with $CRRA = \sigma$, iid z shocks: $\xi^{RA} = \sigma$.
- Choice of middle-aged: purchase shares $A' = G(z, A)$, at $p(z, A)$
- Consumption when middle aged and old:

$$\begin{aligned} c_m(z, A) &= (1 - A)\,(p(z, A) + \theta z) - G(z, A)p(z, A) \\ c_o(z, A; z', A') &= G(z, A)p(z', A') \end{aligned}$$

- Euler equation

$$u'\left[(1 - A)\,(p(z, A) + \theta z) - G(z, A)p(z, A)\right]$$
$$= \beta \sum_{z'} \Gamma_{z,z'} \frac{[p(z', A') + \theta z']}{p(z, A)} u'\left[G(z, A)p(z', A')\right]$$

- Second equation: young's labor income equals their share purchase

$$[1 - G(z, A)]p(z, A) = (1 - \theta)z$$

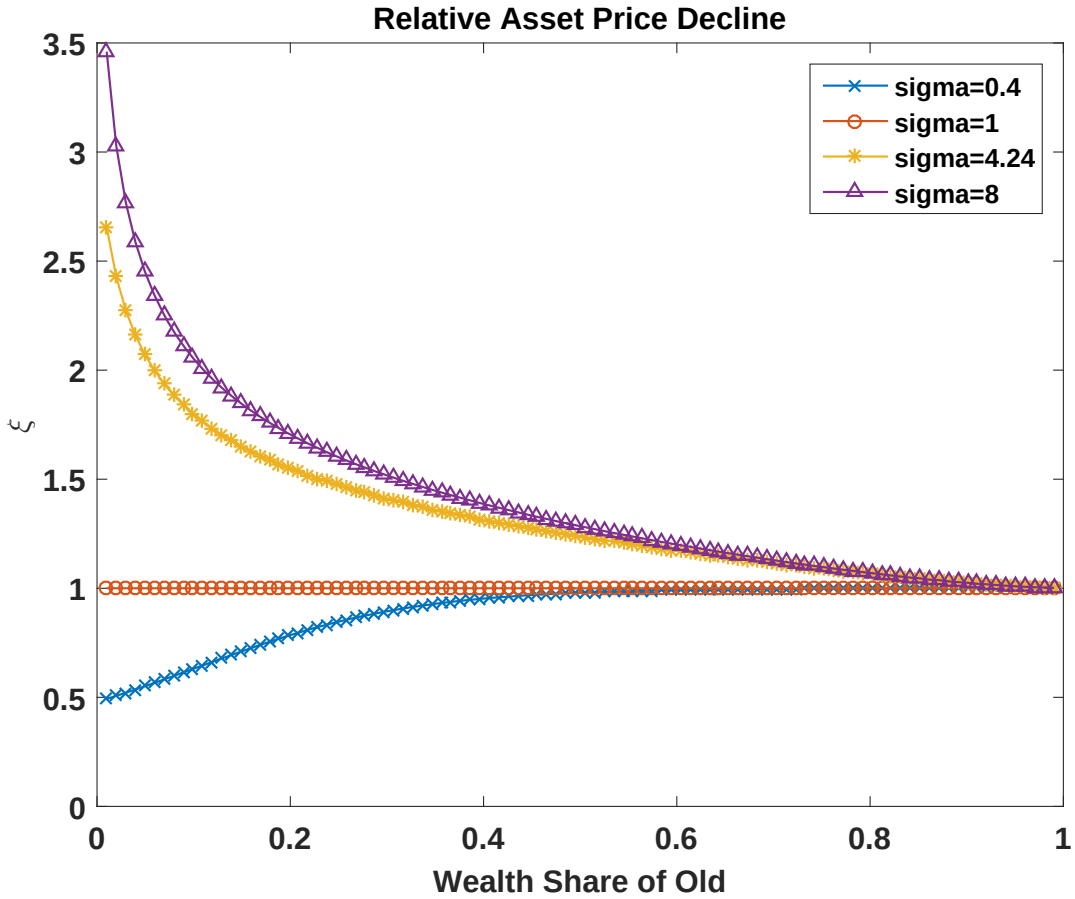
Developing Intuition: A Three Period Model

- Solution is pair of functional equations in the unknown functions $p(z, A), G(z, A)$.
- Consumption, welfare can be calculated from $p(z, A), G(z, A)$.
- Note: for log-utility complete analytical characterization of RCE:
  - Asset prices are proportional to output z, that is $\xi = 1$.
  - Wealth distribution $(1 - A, A)$ does not respond to shock z.
  - Consumption of all generations move one for one with z.
  - If z is iid, then young are exactly indifferent between being born into a Great Recession and being born into normal times.

▸ More on the Log-Case

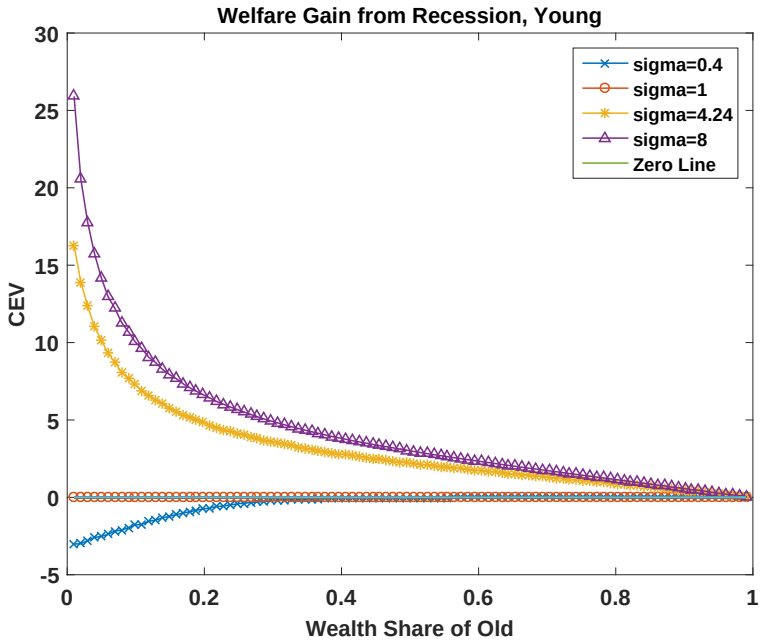
- Now: display (numerical) solution for $\sigma \neq 1$. Other parameters consistent with calibration of full model (e.g. income falls 9.84%)

Asset Price Decline Relative to Output



- The more households dislike consumption fluctuations (the higher σ) the larger is the fall in p relative to z in the recession.
- When $IES = 1/\sigma < 1$ a larger wealth share of the middle-aged (smaller A) translates into greater asset price collapse $\xi(A)$.

Welfare Consequences of Recessions for the Young



- Welfare measured as % consumption equivalent variation (positive numbers indicate welfare gains from recession).
- Welfare consequences mirror the elasticity of asset prices to output. Young can easily win from Great Recession. But in the simple model:
  - Young do not value consumption in Great Recession.
  - Young not disproportionally affected by labor income declines.
  - Middle-aged (and old) only have access to risky assets.

Calibration Overview

- Model period 10 years. Agents enter at age 20, live for 6 periods.
- Aggregate endowment process $z \in Z = \{z_n, z_r, z_d\}, \Gamma_{z,z'}$ derived directly from aggregate time series data. In Great Recession (z_r) output falls 9.84%.
- Life cycle profiles $\{\beta_i, \varepsilon_i(z)\}$ chosen so that model with $z = z_n$ matches life cycle earnings and net worth profiles from 2007 SCF.
- Choose $(\theta = 30\%, B = 0.07)$ s.t. model matches 2007 SCF aggregate wealth to earnings ratio (7.88), share of risky assets (91.8%).
- Choose $\sigma = 4.24$ s.t. model ξ lines up with Great Recession $\xi = 26.8\%/9.84\% = 2.7$.

Calibration: Endowment Process

- States $z \in Z = \{z_n, z_r, z_d\}$. Normal times $z_n = 1$, Great Recession $z_r < 1$, Great Depression $z_d < z_r$.
  - Set z_r s.t. transition from z_n to z_r involves output decline of 9.84% (average 2009-2013 deviation from 2% growth trend).
  - Set z_d s. t. output in z_d is 28.9% below z_n, (average 1932-1936 deviation from trend).
- Transition matrix Γ
  - Impose (perhaps arbitrary) restrictions $\Gamma_{n,d} = \Gamma_{r,r} = \Gamma_{d,r} = 0$. Note: makes markets sequentially complete with two assets.
  - Choose $\Gamma_{n,r}, \Gamma_{r,d}$ such that unconditional probability of Great Recession is 13.7% and Great Depression is 2.84% (as estimated from Maddison data, 1800-2010.)

$$z = \begin{pmatrix} 1.0000 \\ 0.9016 \\ 0.7109 \end{pmatrix}, \quad \Gamma_{z,z'} = \begin{array}{c} \\ z \\ \\ \end{array} \begin{pmatrix} 0.835 & 0.165 & 0.000 \\ 0.793 & 0.000 & 0.207 \\ 1.000 & 0.000 & 0.000 \end{pmatrix} \begin{array}{c} \\ \\ \\ z' \end{array}$$

Calibration: Earnings Losses in Great Recession

- Estimate age-specific earnings declines (relative to aggregate trend) from 2007 to 2010 using CPS data to obtain $\{\varepsilon_i(z_r)\}_{i=1}^{I}$.

Age	(%)
20-29	-14.3
30-39	-12.6
40-49	-10.3
50-59	-11.1
60-69	-6.0
70+	-1.4
pc earnings	-9.8

Calibration: Model with Exogenous Portfolios

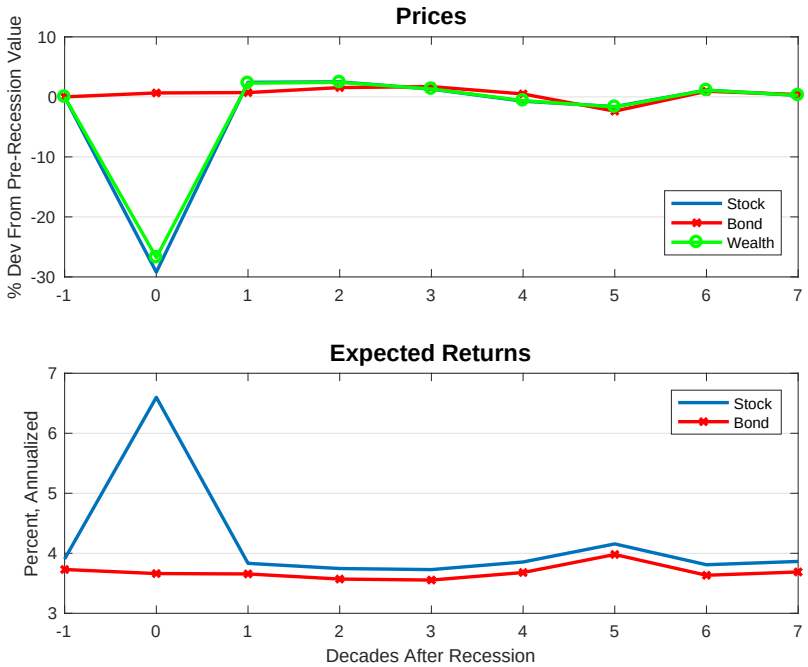
- Alternative version of the model in which savings is a choice, but in which the portfolio shares are exogenous.
- New parameters: age-varying portfolio shares $\{\lambda_i(z)\}_{i=1}^{I}$.
- Set equal to age-specific shares of risky assets from SCF:

Age	$\lambda_i(\%)$
20-29	135
30-39	140
40-49	104
50-59	92
60-69	85
70+	79
Aggr.	94

Results

- Asset Prices in a Great Recession
- Portfolio Choices
- Welfare Results
- Quantifying the Asset Price Channel
- Results with Exogenous Portfolios
- Importance of Asymmetric Earnings Losses
- Accounting for Intracohort Heterogeneity

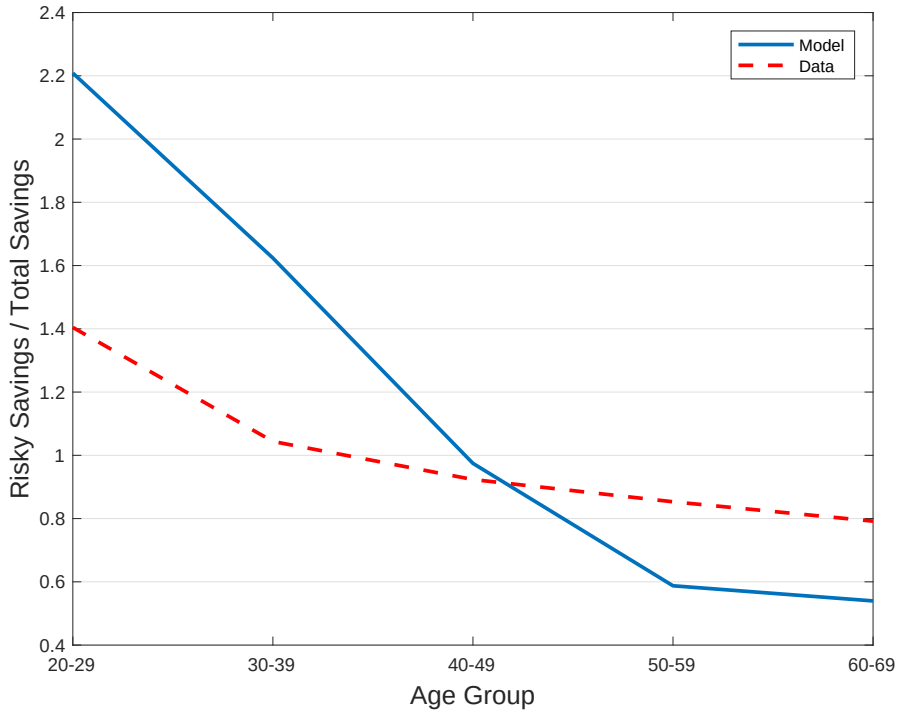
Asset Price Declines



- Thought experiment: Following long period of normal times, Great Recession for 10 years with $\Delta z = 9.8\%$, then recovery.
- p falls by 29.2% ($\sigma > 1$ is key), price of bonds q barely moves.
  - Positive expected consumption growth (q should fall)
  - But: Increase in income *risk* $\Longrightarrow$ precautionary savings up. Keeps q from falling, risk free rate from rising (as in actual Great Recession).

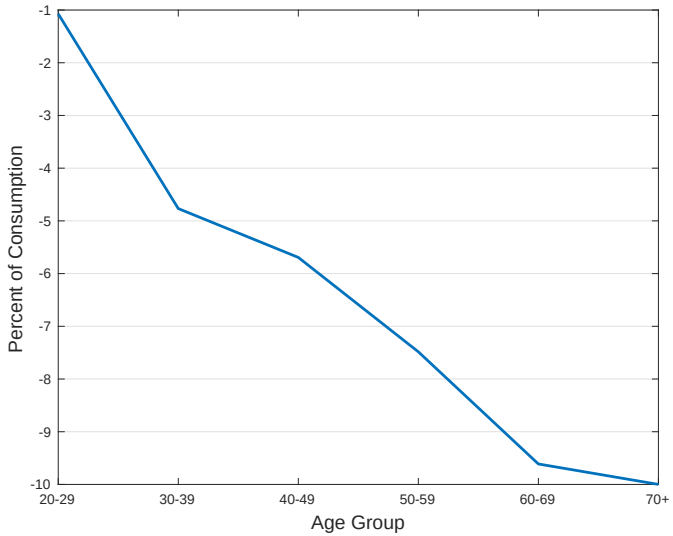
▸ Standard Asset Pricing Statistics

Portfolio Shares in Risky Assets: Models and Data



- Share of risky assets in portfolio declines strongly with age. Why?
  - Markets sequentially complete $\Longrightarrow$ All households *born prior to recession* share recession consumption risk perfectly.
  - For same risk exposure, young require more leveraged portfolios.
- Endogenous portfolio shares depend too strongly on age. Will also consider model with exogenous (and factual) portfolios.

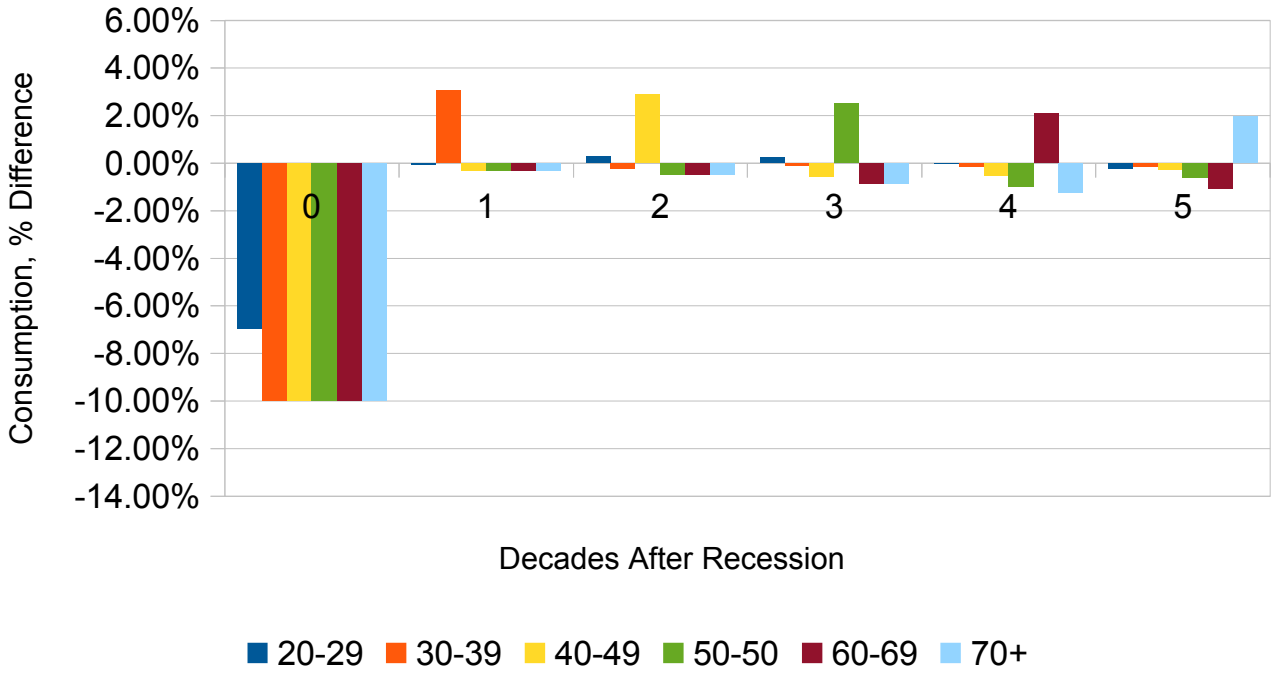
Welfare Losses from the Great Recession



Age	Δ Welf.
20-29	**-1.07%**
30-39	-4.78%
40-49	-5.69%
50-59	-7.48%
60-69	**-9.61%**
70+	**-10.00%**

▸ Wealth-Based Welfare Measure

Exploring the Welfare Losses: Consumption



- Immediate age-specific consumption response to recession symmetric (-10%) across generations alive prior to recession.
- Newborns see smaller consumption drop (relative to no recession (-7.0%) percent. Permanent consumption advantage.

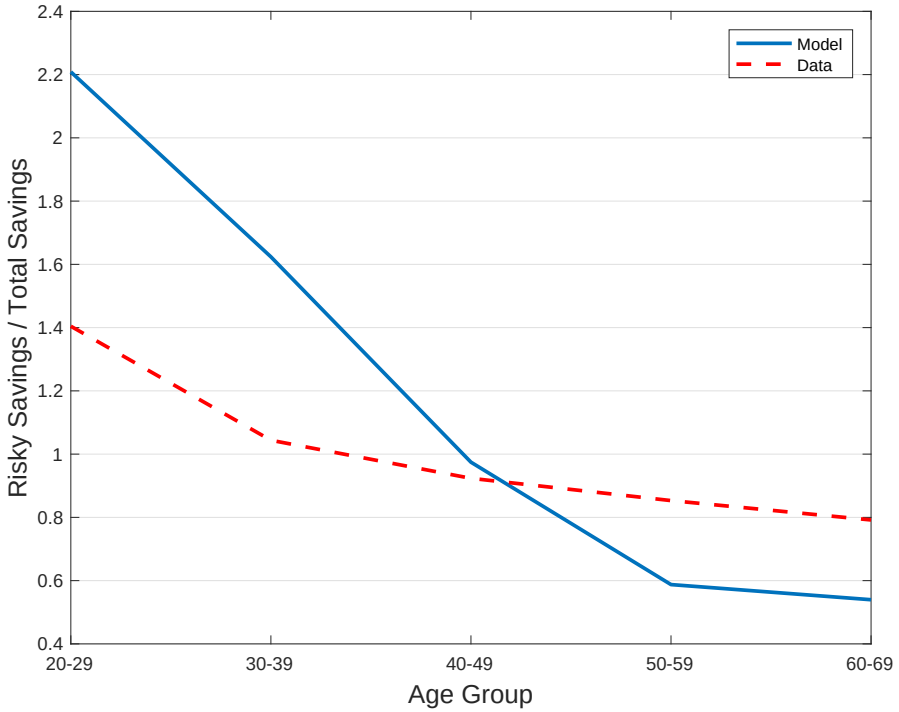
Importance of Asset Pricing Channel?

- Partial equilibrium scenarios with constant return to saving, equal to the R of aggregate market portfolio in pre-recession period.

  1. Hold wealth distrib. constant at start of recession [No Wealth Δ].
  2. Fall in age-specific wealth implied by asset price fall [Wealth Δ].

	Asset Pricing Model		
Age Group	Baseline General Eq.	No Wealth Δ Partial Eq.	Wealth Δ Partial Eq.
20-29	-1.07	-6.53	-6.53
30-39	-4.78	-7.19	-14.03
40-49	-5.69	-6.90	-17.40
50-59	-7.48	-6.55	-16.33
60-69	-9.61	-3.38	-11.27
70+	-10.00	-0.58	-10.00

Exogenous Portfolios



- Now households are forced to hold empirical portfolios (from 2007 SCF). Still make consumption-savings decisions.
- Key plus: more realistic capital losses in Great Recession
- Key minus: Asset price movements do not reflect time-varying appetite for taking on aggregate risk.

Exogenous Portfolios

- Elasticity of Asset Prices to Output. Key: bond prices fall a lot too (big increase in risk-free rate in recession).

Asset	Endog.	Exog.
Wealth	2.72	2.02
Stock	2.97	2.08
Bond	-0.07	1.31

- Welfare losses across age. Key: more significant welfare losses of very young, very old.

Age	Endog.	Exog.
20-29	-1.07%	-2.39%
30-39	-4.78%	-2.91%
40-49	-5.69%	-2.54%
50-59	-7.48%	-7.30%
60-69	-9.61%	-13.73%
70+	-10.00%	-11.37%

Welfare Losses from Recession by Age: Symmetric Earnings Losses

- Given asset pricing channel, why do the young actually lose?
- Answer: because they are especially hard-hit by the Great Recession in the labor market.

Age	Bench.	Sym. Δ Earn.
20-29	-1.07%	0.32%
30-39	-4.78%	-5.04%
40-49	-5.69%	-5.90%
50-59	-7.48%	-7.64%
60-69	-9.61%	-9.74%
70+	-10.00%	-10.09%

Incorporating (Limited) Intra-Cohort Heterogeneity

- Assume the wealthy are passive investors.
- Calibrate model to bottom 90% earnings, wealth life cycle profile.
- Requires (on average) less patient individuals.
- Overall: asset price mechanism less relevant to bottom 90%.

	Economy	
Age Group	Baseline	Low Wealth
20-29	-1.07%	-5.12%
30-39	-4.78%	-6.76%
40-49	-5.69%	-7.23%
50-59	-7.48%	-8.20%
60-69	-9.61%	-9.57%
70+	-10.00%	-9.88%

Conclusion

- We have explored the implications for asset prices of large recessions. Can rationalize large price drops if IES $1/\sigma << 1$
- We have explored the portfolio implications of the model. It can account for (too much of the) relatively risky portfolios of young and relatively safe portfolios of the old in the data.
- We have explored the redistributive implications of such recessions. Old lose a lot, young little. Might have gained if it wasn't for the dismal labor market.
- Heterogeneity within young generation?
    - Winners likely not the ones that do not participate in the asset market ....

Conclusion

- We have explored the implications for asset prices of large recessions. Can rationalize large price drops if IES $1/\sigma << 1$
- We have explored the portfolio implications of the model. It can account for (too much of the) relatively risky portfolios of young and relatively safe portfolios of the old in the data.
- We have explored the redistributive implications of such recessions. Old lose a lot, young little. Might have gained if it wasn't for the dismal labor market.
- Heterogeneity within young generation?
    - Winners likely not the ones that do not participate in the asset market ....
    - ... bud rather those who plan to have large wealth-to-income ratio in their 50's.

What Is This Useful For?

- Policy implications?
  - By construction nothing can be done about the recession itself.
  - But: government can of course affect distribution of welfare losses or gains.
  - E.g. by purchasing assets at distressed prices (TARP?) government may have mitigated welfare losses of elderly at expense of welfare gains of young.
  - Same might be true for expansion of outstanding government debt.

THANK YOU FOR COMING AND LISTENING

Logarithmic Utility ($\sigma = 1$)

Proposition

Let $\sigma = 1$ and $\varepsilon_i(z) = \varepsilon_i \ \forall z$. Then there exists a recursive competitive equilibrium such that

- *The distribution of wealth $A = \bar{A} = (\bar{A}_1, \ldots, \bar{A}_I)$ is constant over time: $\forall z, z', i = 1, \ldots, I-1$*

$$\begin{aligned} G_{i+1}(z, \bar{A}, z') &= a_i'(z, \bar{A}, z', \bar{A}_i) = \bar{A}_{i+1} \\ G_1(z, \bar{A}, z') &= 0 \qquad \forall z, z' \end{aligned}$$

- *Aggregate wealth is proportional to the aggregate shock: $\forall z$*

$$p(z, \bar{A}) + q(z, \bar{A})B = z\Psi$$

- *Asset Portfolios are identical across age groups:*

$$\lambda_i(z, \bar{A}, \bar{A}_i) = \lambda(z) = \frac{p(z)}{z\Psi} \qquad \forall z, \ \forall i = 1, \ldots, I-1.$$

- *Consumption and savings at each age are given by:*

$$\begin{aligned} c_i(z, \bar{A}, \bar{A}_i) &= z\left[(1-\theta)\varepsilon_i + \theta\bar{A}_i + \left(\bar{A}_i - \bar{A}_{i+1}\right)\Psi\right], \\ y_i(z, \bar{A}, \bar{A}_i) &= z\bar{A}_{i+1}\Psi \qquad \forall z, \ \forall i = 1, \ldots, I-1. \end{aligned}$$

back

Logarithmic Utility ($\sigma = 1$)

Proposition

Let $\sigma = 1$ and $\varepsilon_i(z) = \varepsilon_i \ \forall z$. Then there exists a recursive competitive equilibrium with the following properties:

- *Stock and bond prices are given by*

$$\begin{aligned} p(z, \bar{A}) &= p(z) = z\Psi - B\frac{z}{R}\sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \\ q(z, \bar{A}) &= q(z) = \frac{z}{R}\sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'} \qquad \forall z. \end{aligned}$$

  where $R = (\Psi + \theta)/\Psi$.

- *The equity premium is given by*

$$R\sum_z \frac{\Pi_z}{z}\left\{\frac{\sum_{z' \in Z} \Gamma_{z,z'} z' - \left(\sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'}\right)^{-1}}{1 - \frac{B}{R\Psi}\sum_{z' \in Z} \Gamma_{z,z'} \frac{1}{z'}}\right\}$$

back

Logarithmic Utility ($\sigma = 1$)

Proposition

If z is iid then for all $z \in Z$

$$\begin{aligned} p(z) &= z\left(\Psi - \frac{B}{R}\sum_{z' \in Z} \Pi_{z'} \frac{1}{z'}\right) \\ q(z) &= z\left(\frac{1}{R}\sum_{z' \in Z} \Pi_{z'} \frac{1}{z'}\right) \end{aligned}$$

and the average equity premium is given by

$$R\frac{\left(\sum_z \frac{\Pi_z}{z}\sum_z \Pi_z z - 1\right)}{\left(1 - \frac{B}{R\Psi}\sum_z \frac{\Pi_z}{z}\right)}$$

Proposition

In the limit as $\Gamma_{z,z} \to 1\ \forall z$ (perfectly persistent shocks), $q(z) \to R^{-1}$ and $p(z) \to z\Psi - BR^{-1}$.

back

Wealth-Based Welfare Measures

- Wealth-based welfare measure invariant to remaining lifetime horizon.
- How much must *wealth* be reduced in the no-recession state for households to be indifferent between life with or without the recession in the current period?
- Normalize wealth measure by pc consumption in normal times.

back

Age	Bench.	Sym. Δ Earn.	Exog.
20-29	-1.98%	0.60%	-3.90%
30-39	-11.20%	-11.87%	-6.30%
40-49	-15.79%	-16.38%	-6.83%
50-59	-22.83%	-23.31%	-20.39%
60-69	-25.90%	-26.24%	-35.77%
70+	-14.95%	-15.08%	-19.11%

Standard Asset Pricing Statistics

	Return Stats: Benchmark Model		
Asset	Average	Std. Dev.	Corr. w/ Stock
Stock	4.50%	31.2%	1.00
Bond	4.09%	25.3%	0.79
	Return Stats: Model w/o Great Depr.		
Asset	Average	Std. Dev.	Corr. w/ Stock
Stock	4.41%	16.6%	1.00
Bond	3.68%	1.2%	-0.07
	Return Stats: Data		
Asset	Average	Std. Dev.	Corr. w/ Stock
Stock	6.62%	36.4%	1.00
Bond	2.29%	30.4%	0.01

back

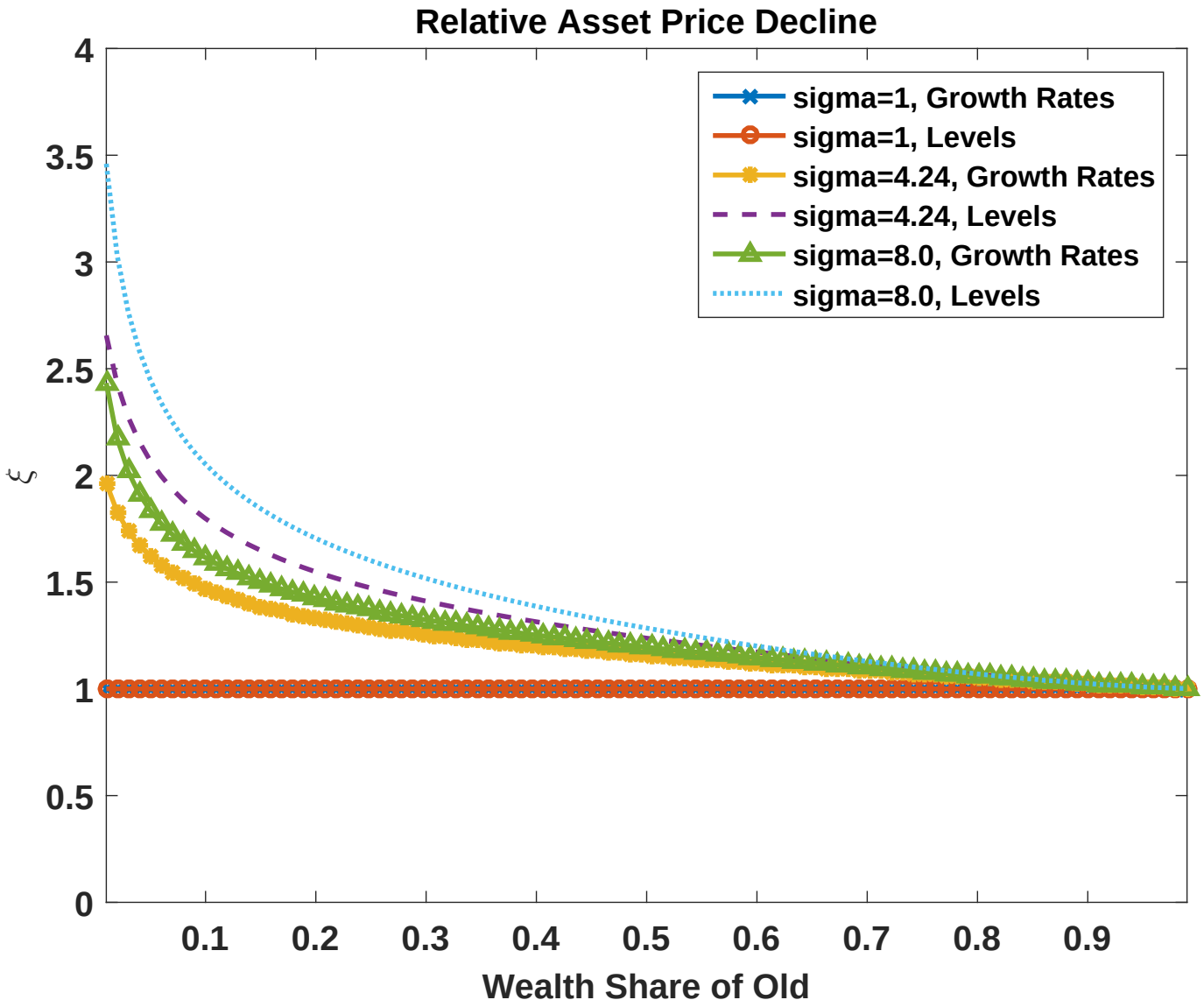
Implications for the Dynamics of the Wealth Distribution: Model vs. Data

	Model End. Portf.			Model Exog. Portf.			Data		
Age	PreR	Rec.	Reco	PreR	Rec.	Reco	2007	2010	2013
20-29	0.0	0.0	0.0	0.0	0.0	0.0	2.3	1.3	1.5
30-39	2.6	-1.4	6.0	4.9	3.9	4.5	6.0	4.2	6.1
40-49	9.9	4.6	12.0	13.6	13.0	12.5	13.9	14.0	14.3
50-59	24.9	24.1	23.4	25.2	25.2	24.8	24.7	24.5	22.9
60-69	36.9	42.6	32.8	33.0	33.7	32.7	31.5	32.7	30.5
70+	25.6	30.1	25.8	23.3	24.2	25.6	21.7	23.4	24.7

Level- or Growth Rate Shocks?

- So far aggregate output z mean reverting, thus in a great recession output and asset prices are expected to recover.
- Robustness to permanent shocks to z? Consider 3-period model but assume that $g' = z'/z$ follows Markov process with $\Gamma_{g,g'}$.
- Calibrate s.t. output falls 9.83% in recession.
- Three basic results
  - For given risk aversion, ξ comparable to model with trend-stationary output if (*and only if*) output. growth over ten or twenty years *negatively* correlated, as in U.S. data (corr ≈ -0.55).
  - Absolute welfare losses from the great recession significantly larger in the stochastic growth economy (for all but oldest generation).
  - *Relative welfare losses* by age are comparable in both economies.

Asset Prices: Two Economies



Relative Welfare Losses by Age: Two Economies

	Economy	
Age Group	Shocks to z	Shocks to z'/z
Old (absolute)	-12.3%	-11.4%
Middle (absolute)	-3.7%	-6.0%
Young (absolute)	2.9%	-5.0%
Middle rel.to Old	8.6%	5.4%
Young rel. to Old	15.2%	6.4%

Incorporating (Limited) Intra-Cohort Heterogeneity

- Are welfare losses of "average household" within an age group representative? Now consider limited intra-cohort heterogeneity.
- Two types of households: a wealthy type and a low-wealth type.
- Assume that wealthy type accounts for a fixed fraction κ_y of aggregate labor earnings, passively holds a fixed fraction κ_a of aggregate debt, equity.
- Thus the wealthy consume a fixed fraction $(1-\theta)\kappa_y + \kappa_a\theta$ of aggregate output at each date.
- Assets are priced by the low-wealth type, and prices fluctuate such that this type always demands $(1-\kappa_a)$ shares and $\kappa_a B$ bonds.
- In essence: recalibration of a model with lower income- and wealth households. Key difference: wealth-to-income ratio is lower among asset pricers now.
- Results fairly unchanged relative to baseline model, but asset price channel somewhat less important.

Incorporating (Limited) Intra-Cohort Heterogeneity

	Economy	
Age Group	Baseline	Low Wealth
20-29	-1.07%	-5.12%
30-39	-4.78%	-6.76%
40-49	-5.69%	-7.23%
50-59	-7.48%	-8.20%
60-69	-9.61%	-9.57%
70+	-10.00%	-9.88%